Sublinear Time Algorithms April 22, 2019

Homework 5

Lecturer: Ronitt Rubinfeld *Due Date: May 6, 2019*

Turn in your solution to each *problem on a separate sheet of paper, with your name on each one.*

1. In the following questions, assume that all input graphs start out with unique IDs.
   (a) Given a graph of max degree at most Δ, show that the edges can be decomposed into at most Δ oriented forests (where each node has outdegree at most 1, the roots have outdegree 0, and edges point along the path to a root). Show that given a node, the edge in oriented forest i and the direction of the edge, can be computed in $O(\Delta)$ sequential time.
   (b) Give a distributed algorithm for 6-coloring trees. Assume that the tree can be viewed as a rooted tree in which children know who their parent is. For full credit, your algorithm should run in $k = O(\log^* n)$ rounds. Note that this gives an LCA for 6-coloring trees which runs in $2^{O(\log^* n)} = O(\log^* n)$ probes. *Hint: Consider algorithms in which a node u looks at its parent v and recolors itself based on the location of the first bit which differs between u and v.*
   (c) Given graph G along with a c-coloring of the nodes (assume you can query the coloring of any node in 1 step). Show how to find an MIS in c distributed rounds.
   (d) Combine the above to give an LCA for 6^{Δ} coloring a degree at most Δ graph G.

2. In class, we gave an LCA for the spanner problem that works for graphs of max degree at most $n^{3/4}$. Show how to construct an LCA for the spanner problem for any graph. For full credit, your runtime should still be $O(n^{3/4})$ per query.

   Hint: (1) Handle the nodes that have degree between $\sqrt{n}$ and $n^{3/4}$ with a different setting of parameters for determining centers. (2) For nodes of degree at least $n^{3/4}$, partition the edges into groups of size $n^{3/4}$, and add a rule 3 edge (u, v) whenever v introduces u to a new cluster within its partition (this will allow more edges in the final graph, but show that it won't destroy the sparsity of the spanner).

3. Say that $f : \{0, 1\}^n \to \{0, ..., n\}$ is *monotone* if for all x, y such that $x_i \le y_i$ for $i = 1, \ldots, n$, then $f(x) \le f(y)$. Show that distinguishing whether f is monotone from the case that f is ϵ-far from monotone (i.e., there is no monotone g such that f and g differ on at most ϵ-fraction of the domain $\{0, 1\}^n$) requires $\Omega(n)$ queries. Hint: reduce from the communication complexity problem of disjointness. Another hint: Let $|x|$ be the number of 1's in x. Let Alice define $p(x)$ to be -1 if the parity of the input bits in her set is 1, and 1 if the parity is 0. Let Bob define $q(x)$ similarly. Let them compute $h(x) = 2|x| + p(x) + q(x)$.