# Bayesian Inference of Temporal Task Specifications from Demonstrations


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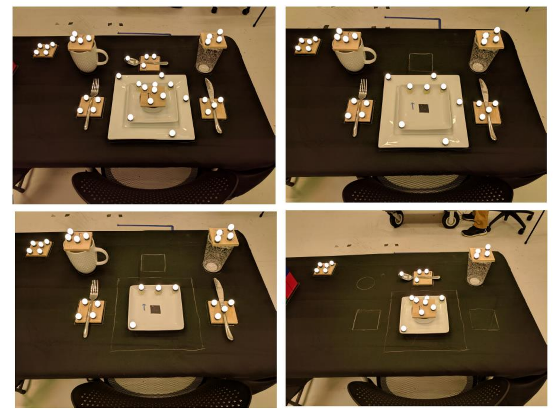
## Motivation

Specifications for setting a dinner table:

- Necessity of object placements
- Correct object positions
- No collisions
- Placement orders



**Not Markovian in object poses**

Expressible in Linear temporal logic (LTL), a flexible specification language used in:

- Synthesis of verifiable controllers[1]
- Reinforcement learning[2]
- Goal description in symbolic planning[3]

**Aim:** Infer task specifications from demonstrations

**Approach**: Bayesian specification inference for task specifications as LTL formula

## Bayesian Formulation

$$P(\varphi|\boldsymbol{D}) = \frac{P(\varphi)P(\boldsymbol{D}|\varphi)}{\sum_{\varphi\in\boldsymbol{\varphi}} P(\varphi)P(\boldsymbol{D}|\varphi)}$$

- $P(\varphi)$ must have positive support over all relevant formulas.
- $P(\boldsymbol{D}|\varphi)$ is the likelihood distribution that honors the size principle:
  - Large likelihood for complex formula.
  - Small likelihood for simple formula
  - Number of conjunctions a measure of formula complexity
- Probabilistic programming languages for sampling based inference[4]

## References


[1] H. Kress-Gazit, G. E. Fainekos and G. J. Pappas, "Temporal-Logic-Based Reactive Mission and Motion Planning," in IEEE Transactions on Robotics, vol. 25, no. 6, Dec. 2009.

[2] X. Li, C. I. Vasile and C. Belta, "Reinforcement learning with temporal logic rewards," *2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Vancouver, BC.

[3] Camacho, A., Triantafillou, E., Muise, C.J., Baier, J.A. and McIlraith, S.A., "Non-Deterministic Planning with Temporally Extended Goals: LTL over Finite and Infinite Traces", AAAI 2017.

[4] Goodman, N.D. and Stuhlmüller, A., 2014. The design and implementation of probabilistic programming languages.

[5]Matthew B. Dwyer, George S. Avrunin, and James C. Corbett. 1999. Patterns in property specifications for finite-state verification. In Proceedings of the 21st international conference on Software engineering (ICSE '99). ACM, New York, NY, USA.


## Acknowledgements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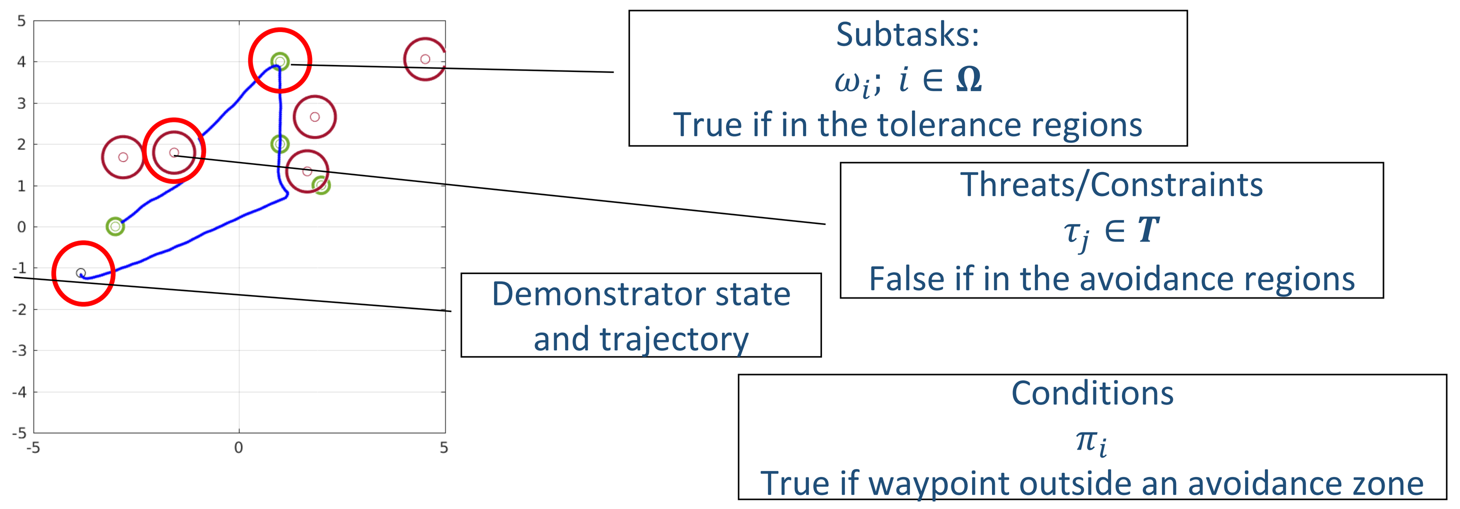
## Formula Template




- Every possible LTL formula forms an intractable hypothesis space
- Complex specifications are constructed as compositions of simpler sub-formulas[5]

**Key Idea:** Define hypothesis space as a conjunction of three templates

**Global satisfaction:**

$$\varphi_{global} = \bigwedge_{\tau\in\boldsymbol{\tau}} \boldsymbol{G}\tau \qquad \boldsymbol{\tau} \subseteq \boldsymbol{T} \qquad P(\varphi_{global}) = P(\boldsymbol{\tau}),\quad Supp(P(\boldsymbol{\tau})) = \wp(\boldsymbol{T})$$

**Eventual Completion:**

$$\varphi_{eventual} = \bigwedge_{i\in\boldsymbol{W_1}} (\pi_i \rightarrow \boldsymbol{F}\omega_i) \qquad \boldsymbol{W_1} \subseteq \boldsymbol{\Omega} \qquad P(\varphi_{eventual}) = P(\boldsymbol{W_1}),\quad Supp(P(\boldsymbol{W_1})) = \wp(\boldsymbol{\Omega})$$

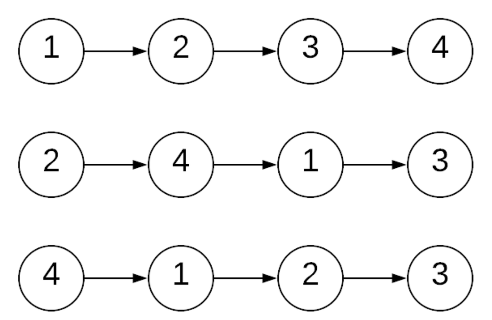
**Ordering:**

$$\varphi_{order} = \bigwedge_{w_1,w_2\in\boldsymbol{W_2}} \pi_i \rightarrow (\neg\omega_{w_2}\boldsymbol{U}\omega_{w_1}) \qquad \boldsymbol{W_2} = \{(w_1,w_2): w_1\in\boldsymbol{\Omega}, w_2\in desc(w_1)\}$$

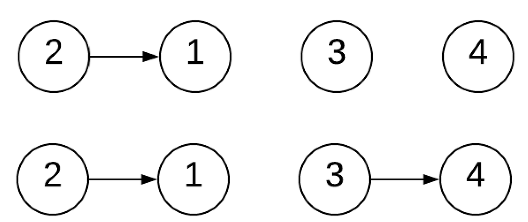
$Supp(P(\varphi_{order})) = supp(P(\boldsymbol{W_2}))$ = All directed acyclic graphs (DAG) over $\boldsymbol{\Omega}$

**We consider three restrictions**

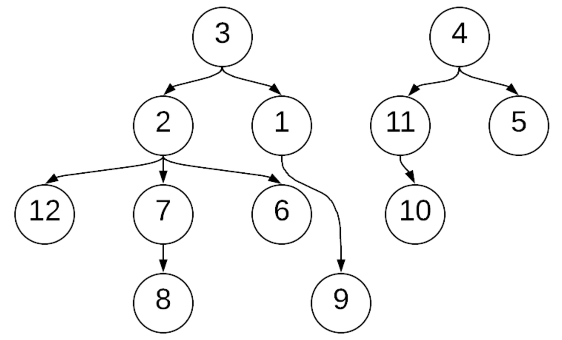
**Linear Chains**



**Set of linear chains**



**Forest of subtasks**



## Priors

$$P(\varphi) = P(\varphi_{global})P(\varphi_{eventual})P(\varphi_{order})$$

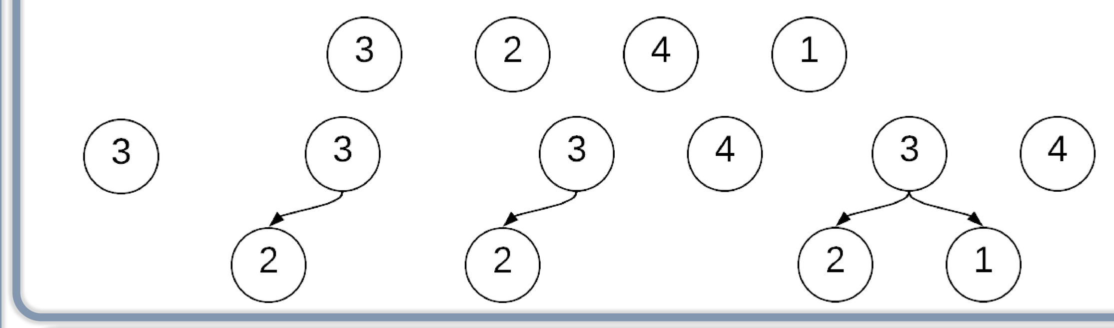
| Prior | $\varphi_{global}$ | $\varphi_{eventually}$ | $\varphi_{order}$ |
|---|---|---|---|
| Prior 1 | SampleSubset($\mathbf{T}$) | SampleSubset($\boldsymbol{\Omega}$) | RandomPermutation($\boldsymbol{\Omega}$) |
| Prior 2 | SampleSubset($\mathbf{T}$) | SampleSubset($\boldsymbol{\Omega}$) | SetOfLinearChains($\boldsymbol{\Omega}$) |
| Prior 3 | SampleSubset($\mathbf{T}$) | SampleSubset($\boldsymbol{\Omega}$) | ForestOfSubtasks($\boldsymbol{\Omega}$) |

### Prior distributions induced by probabilistic program

1. RandomPermuation($\boldsymbol{\Omega}$)
2. For $i \in \boldsymbol{\Omega}$ do:
   I. Insert into forest:
      i. Sample Categorical([Trees], NewTree)
      ii. if NewTree create new tree as sibling
      iii. Else, Insert into tree for descendant forest of selected tree

Categorical probabilities proportional to tree size

Probability of starting a new tree proportional to $N_{new}$



## Likelihood Function

**Complexity-based likelihood function (CB):**

$$\frac{P([\boldsymbol{\alpha}] \vDash \varphi_1|\varphi_1)}{P([\boldsymbol{\alpha}] \vDash \varphi_2|\varphi_2)} = \frac{2^{N_{conj_1}}}{2^{N_{conj_2}}}$$

$$\frac{P([\boldsymbol{\alpha}] \neg\vDash \varphi_1|\varphi_1)}{P([\boldsymbol{\alpha}] \vDash \varphi_1|\varphi_1)} = \frac{\epsilon}{2^{N_{conj_1}}}$$

**Complexity-independent likelihood function (CI):**

$$\frac{P([\boldsymbol{x}] \vDash \varphi|\varphi)}{P([\boldsymbol{x}]\neg\vDash \varphi|\varphi)} = M$$

$M$ is a large number

## Metrics

If ground-truth specification defined by $\boldsymbol{\tau}^*, \boldsymbol{W}_1^*, \boldsymbol{W}_2^*$, compute IOU as follows:

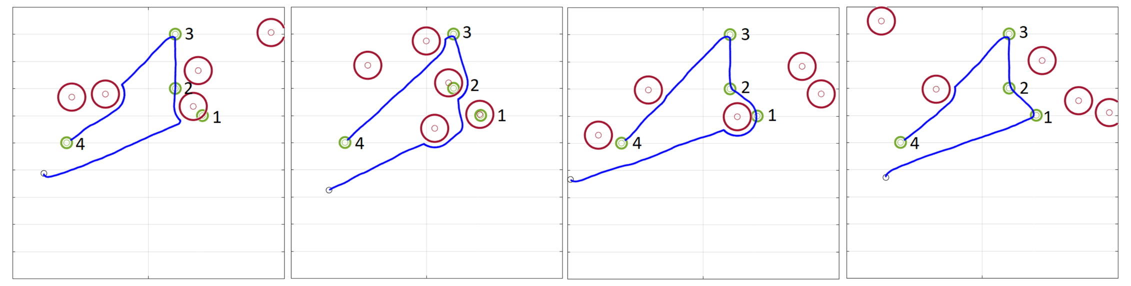
$$L(\varphi) = \frac{|\boldsymbol{\tau} \cap \boldsymbol{\tau}^*| + |\boldsymbol{W_1} \cap \boldsymbol{W}_1^*| + |\boldsymbol{W_2} \cap \boldsymbol{W}_2^*|}{|\boldsymbol{\tau} \cup \boldsymbol{\tau}^*| + |\boldsymbol{W_1} \cup \boldsymbol{W}_1^*| + |\boldsymbol{W_2} \cup \boldsymbol{W}_2^*|}$$

Over the posterior $P(\varphi|\boldsymbol{D})$, compute

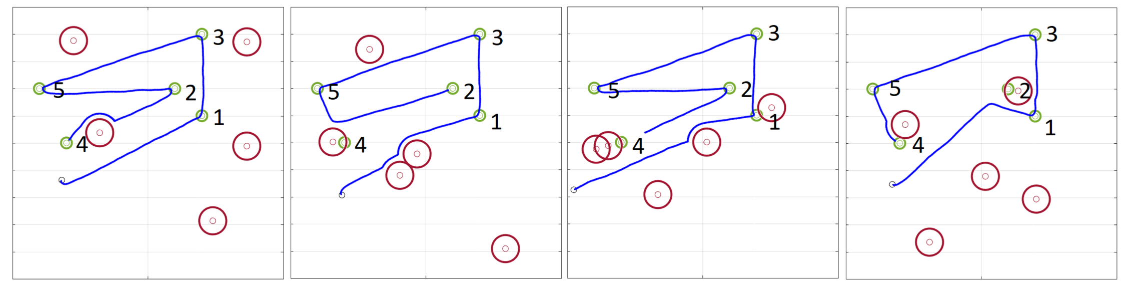
## Results

**Synthetic Domain**

Scenario 1
Visit POI in order
[1,2,3,4,5]

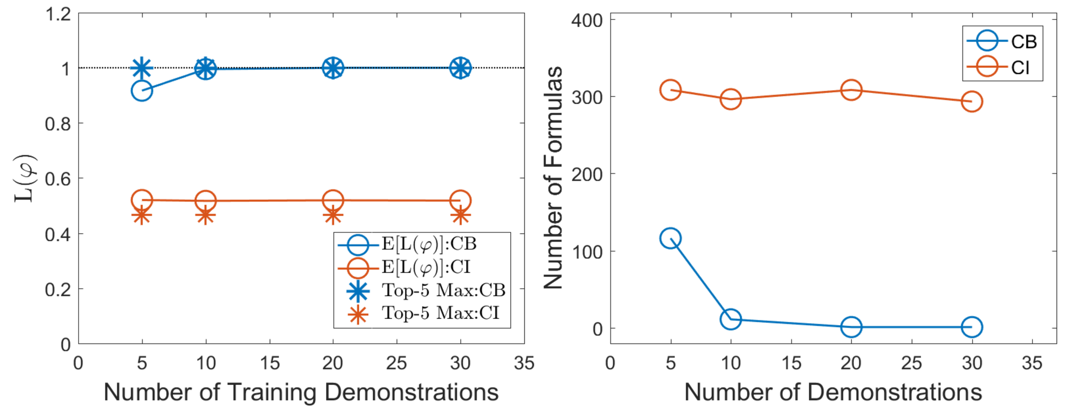


Scenario 2
Visit POI in order
{[1,3,5],[2],[4]}



**CB vs CI: Effect of Size principle**

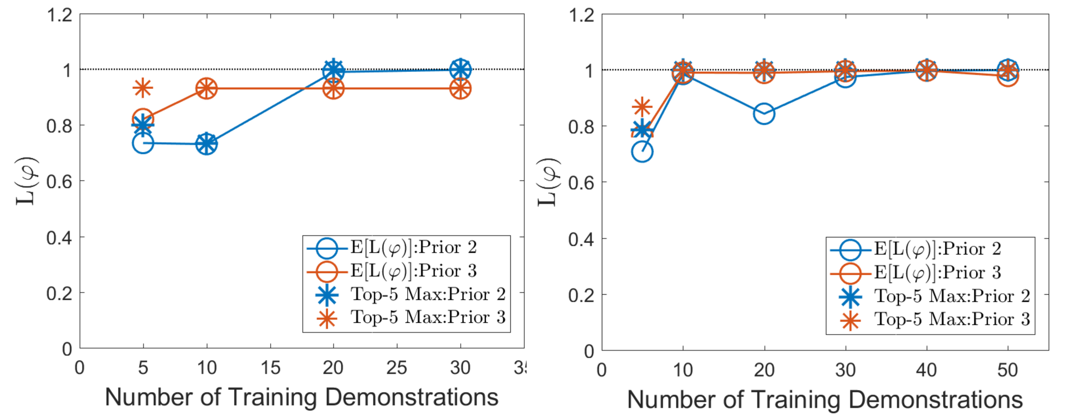
Scenario 1 (Fully ordered) using Prior 1 (Linear Chains)




- CB performs better with more training examples
- CB posterior has greater confidence in predictions

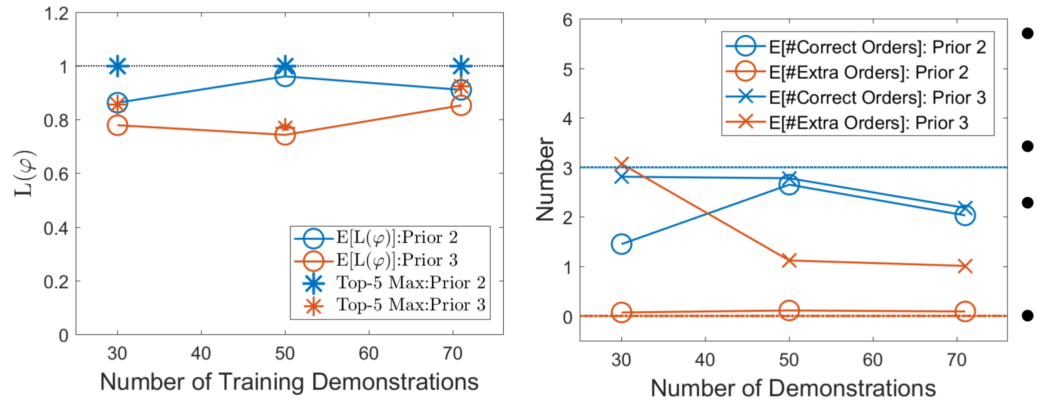
**Efficacy across scenarios**

Scenario 1: Priors 2 and 3 Scenario 2: Priors 2 and 3




- >90% similarity to ground truth with more than 10 demonstrations

**Dinner Table Domain**




- 6 Demonstrators with 4 configurations
- 8 Dinner table objects
- >90% similarity to ground truth
- Prior 3 shows inductive bias towards longer task chains