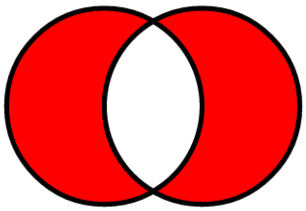
Alin Tomescu, *6.840 Theory of Computation (Fall 2013)*, taught by Prof. Michael Sipser

DFAs and CFGs

NFA/DFAs

Symmetric difference: $A \,\Delta\, B = (A - B) \cup (B - A) = (A \cap \bar{B}) \cup (B \cap \bar{A})$

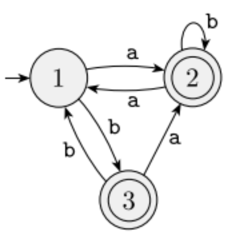

← $A \,\Delta\, B$ is the red part (A is the left circle, B is the right circle)

NFA to DFA conversion can result in exponential state blow up: k NFA states → 2^k DFA states

If a k-state NFA rejects any string, it will have to reject a string of length $\leq 2^k$, because if you convert the NFA to a DFA and take the complement, you get a DFA for the complement of the NFA's language. But this is a 2^k state DFA, which will have to accept a string of length $\leq 2^k$ if it ever accepts something (⇔ if the NFA ever rejects something).

If a **k_1-state NFA and a k_2-state NFA both accept some string**, then the shortest such string has length $\leq m_1 m_2$ (because, "we can always remove a segment of the string where a repeated state occurs in both accepting computations of the two NFAs and the number of pairs of states is $m_1 m_2$", *Prof. Sipser*).

Converting **DFAs to regular expressions** can kind of blow up in size exponentially. See below:



← produces ⇒ $(\mathrm{a}(\mathrm{aa} \cup \mathrm{b})^*\mathrm{ab} \cup \mathrm{b})((\mathrm{ba} \cup \mathrm{a})(\mathrm{aa} \cup \mathrm{b})^*\mathrm{ab} \cup \mathrm{bb})^*((\mathrm{ba} \cup \mathrm{a})(\mathrm{aa} \cup \mathrm{b})^* \cup \varepsilon) \cup \mathrm{a}(\mathrm{aa} \cup \mathrm{b})^*$

CFGs

Closure properties

$$CFG \cap REG \text{ (intersection with regular languages)}$$

because you can build a PDA that keeps track of the DFA for the regular language and also continues to do the initial PDA's work.

$$CFG_1 \cup CFG_2 \text{ (union)}$$

$$CFG^R \text{ (reversal)}$$

$$CFG_1 \cdot CFG_2 \text{ (concatenation)}$$

$$CFG^* \text{ (kleene star)}$$

CFGs are NOT closed under intersection, difference, and complement.

For intersection, consider the following counter-example:

$$A = \{0^m 1^n 2^n \mid n \geq 0\}, B = \{0^m 1^m 2^n\}$$

Then $A \cap B = \{0^n 1^n 2^n \mid n \geq 0\}$, which can be proven to be non-context free using the pumping lemma.

Chomsky normal form

Chomsky normal form grammars only have productions of the form $A \to BC \mid terminal$ and $S \to \varepsilon$. Thus, any string w in the grammar can be derived in at most $2|w| - 1$ steps.