Alin Tomescu, *6.840 Theory of Computation (Fall 2013)*, taught by Prof. Michael Sipser

Reductions

$3SAT \leq_p CLIQUE$

Construction: The three literals in a clause become a group of three nodes in the graph (n literals → n nodes, so repeats will occur). All nodes (literals) will be connected except: literals from the same clause, and complementary literals like x_2 and $\bar{x}_2$. For an expression ϕ with k clauses, a k-clique will correspond to the literals that need to be set to true in the expression.

Why it works: Because you are encoding all the ways you could pick one variable from each clause such that if all selected variables are set to true, then the expression is true.

Example: For $\phi = (x_1 \vee x_1 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_2) \wedge (\bar{x}_1 \vee x_2 \vee x_2)$, we would get:

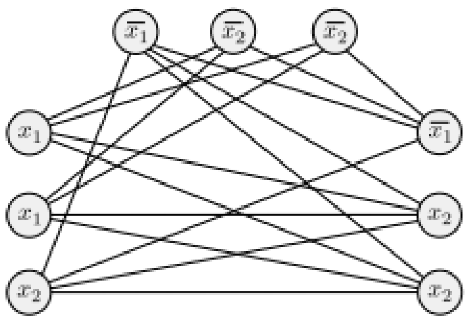
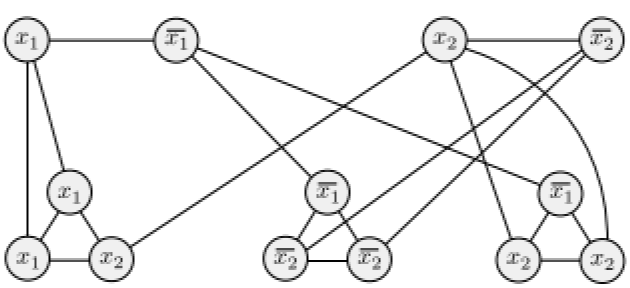


Figure 1: SAT to CLIQUE reduction (from Sipser 3rd. Ed)

$3SAT \leq_p VERTEX\text{-}COVER$

Construction: For each variable (not literal), we create an $x - \bar{x}$ pair in the graph (two nodes connected by an edge). For each clause we create group of 3 nodes connected all connect by 3 edges (so a clique), where the nodes are labeled just like the literals in the clause. We connect the nodes in the clause *gadget* to the nodes with the same labels in the variable *gadget.* For an expression ϕ, a vertex cover will correspond to a satisfying assignment. The VC will consist of nodes from the variable gadgets and of nodes from the clause gadgets. The nodes in the VC from the variable gadgets will give the truth assignment. For example, if x_2 from the variable gadgets, is in the VC then x_2 would be set to $true$ in the SAT expression. The nodes in the VC from the clause gadgets would be set to false. The VC should be of size $2k + v$, where k is the number of clauses in ϕ and v is the number of variables. This is because the VC will have one node from each variable gadget and 2 nodes from each clause gadget.

Example: $\phi = (x_1 \vee x_1 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_2) \wedge (\bar{x}_1 \vee x_2 \vee x_2)$



Other reductions

$SAT \leq_p 3SAT$

$HAMPATH \leq_p 3SAT$

$HAMPATH \leq_p UHAMPATH$

$HAMPATH \ \leq_p LONGESTPATH$

$3SAT \leq_p SUBSET\text{-}SUM$

$TQBF \leq_p GG$