Designing Efficient Map-Reduce Algorithms

Review of Map-Reduce
A Common Mistake
Size/Communication Trade-Off
Specific Tradeoffs

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Review of Map-Reduce

Mappers and Reducers
Key-Value Pairs
Example Application: Join

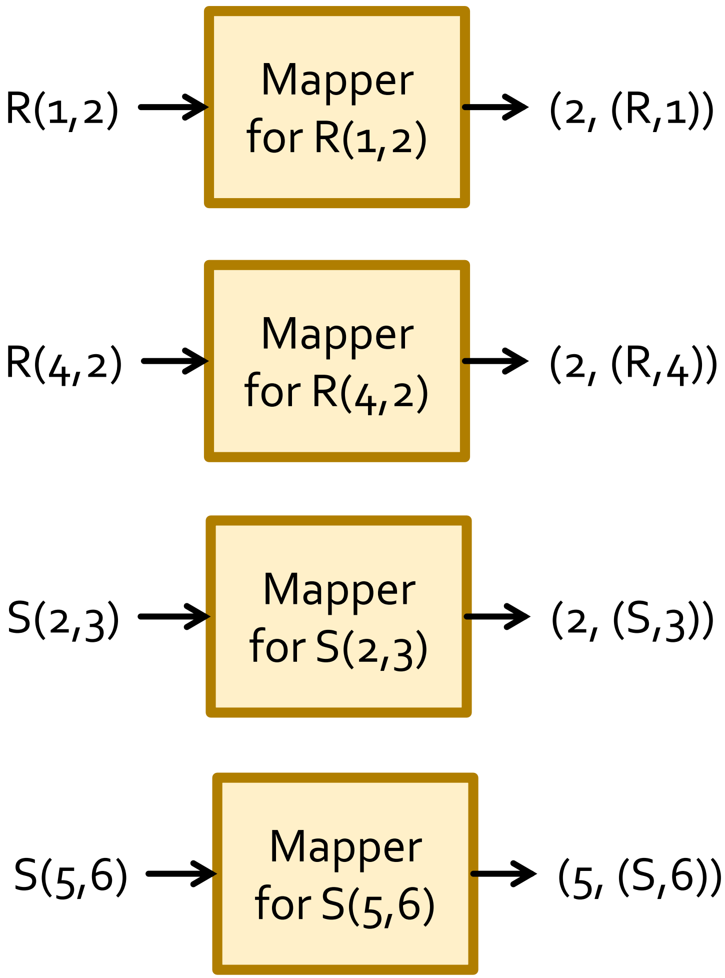
Mappers and Reducers

- Map-Reduce job =
  - Map function (inputs -> key-value pairs) +
    - Keys not unique!
  - Reduce function (key and list of values -> outputs).
- Map and Reduce Tasks apply Map or Reduce function to (typically) many inputs.
  - Unit of parallelism.
- *Mapper* = application of the Map function to a single input.
- *Reducer* = application of the Reduce function to a single key-(list of values) pair.

Example: Natural Join

- Join of R(A,B) with S(B,C) is the set of tuples (a,b,c) such that (a,b) is in R and (b,c) is in S.
- Mappers need to send R(a,b) and S(b,c) to the same reducer, so they can be joined there.
- Mapper output: key = B-value, value = relation and other component (A or C).
  - Example: R(1,2) -> (2, (R,1))
    S(2,3) -> (2, (S,3))

Mapping Tuples



Grouping Phase

- There is a reducer for each key.
- Every key-value pair generated by any mapper is sent to the reducer for its key.

Mapping Tuples

Mapper for R(1,2) — (2, (R,1))

Mapper for R(4,2) — (2, (R,4))

Mapper for S(2,3) — (2, (S,3))

Mapper for S(5,6) — (5, (S,6))

Reducer for B = 2

Reducer for B = 5

Constructing Value-Lists

- The input to each reducer is organized by the system into a pair:
  - The key.
  - The list of values associated with that key.

The Value-List Format

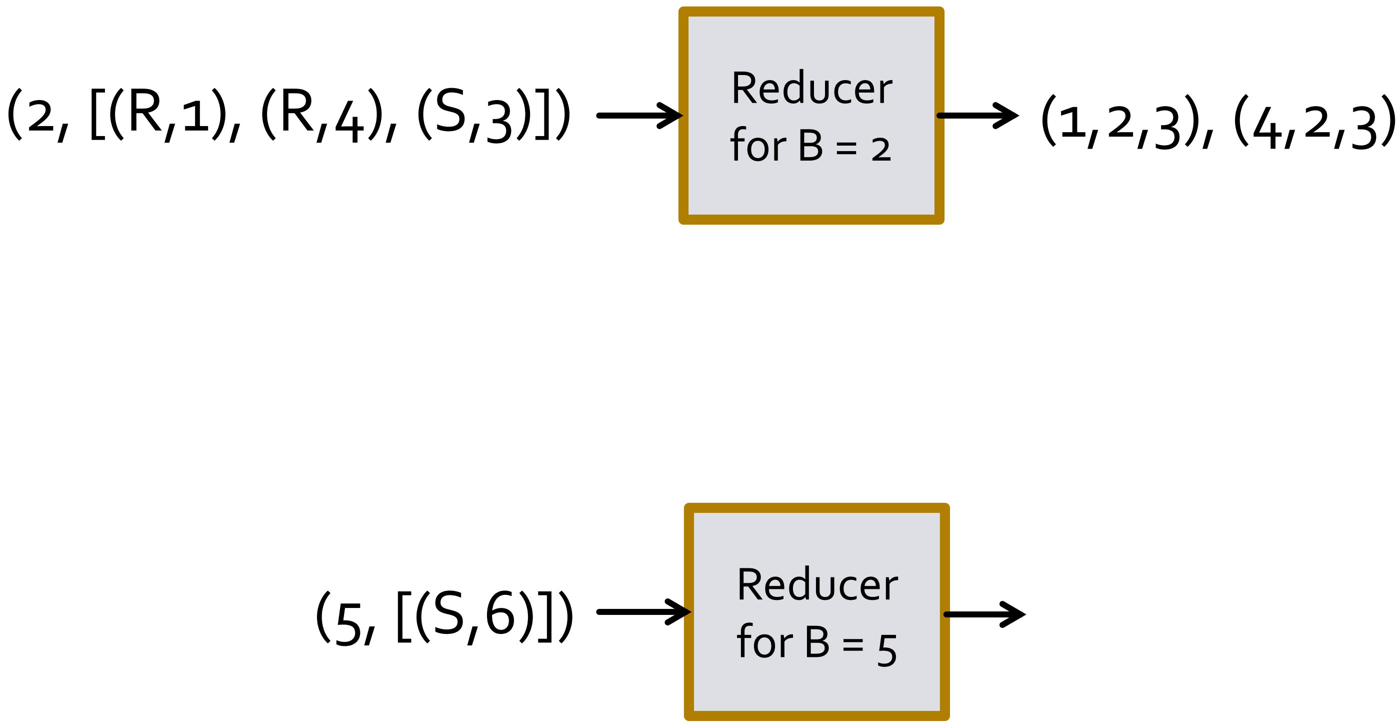
(2, [(R,1), (R,4), (S,3)]) → Reducer for B = 2

(5, [(S,6)]) → Reducer for B = 5

The Reduce Function for Join

- Given key *b* and a list of values that are either (R, a_i) or (S, c_j), output each triple (a_i, b, c_j).
  - Thus, the number of outputs made by a reducer is the product of the number of R's on the list and the number of S's on the list.

Output of the Reducers



Motivating Example

The Drug Interaction Problem
A Failed Attempt
Lowering the Communication

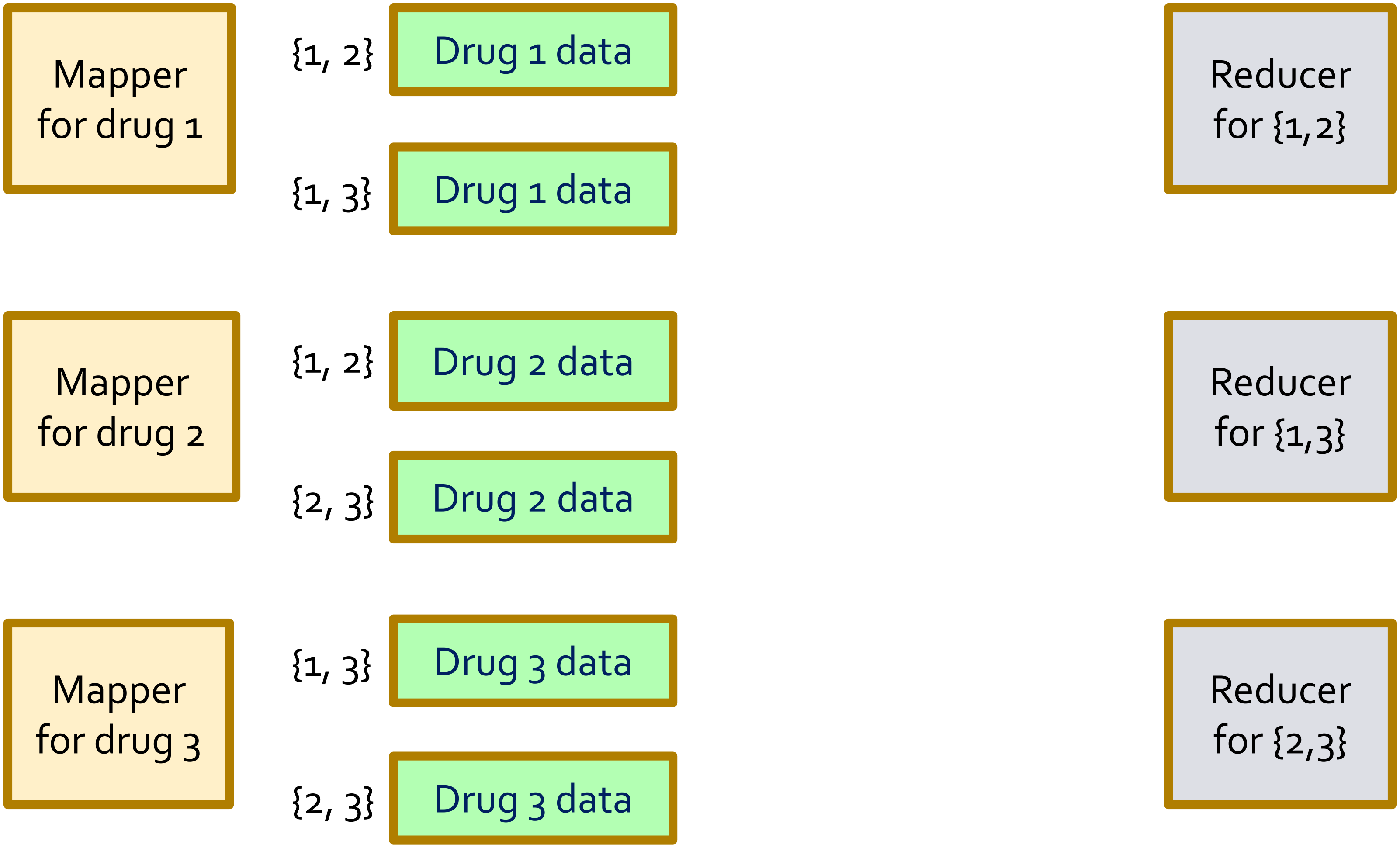
The Drug-Interaction Problem

- Data consists of records for 3000 drugs.
  - List of patients taking, dates, diagnoses.
  - About 1M of data per drug.
- Problem is to find drug interactions.
  - Example: two drugs that when taken together increase the risk of heart attack.
- Must examine each pair of drugs and compare their data.

Initial Map-Reduce Algorithm

- The first attempt used the following plan:
  - Key = set of two drugs $\{i, j\}$.
  - Value = the record for one of these drugs.
- Given drug i and its record R_i, the mapper generates all key-value pairs $(\{i, j\}, R_i)$, where j is any other drug besides i.
- Each reducer receives its key and a list of the two records for that pair: $(\{i, j\}, [R_i, R_j])$.

Example: Three Drugs



Example: Three Drugs

Mapper for drug 1

{1, 2} Drug 1 data

{1, 3} Drug 1 data

Mapper for drug 2

{1, 2} Drug 2 data

{2, 3} Drug 2 data

Mapper for drug 3

{1, 3} Drug 3 data

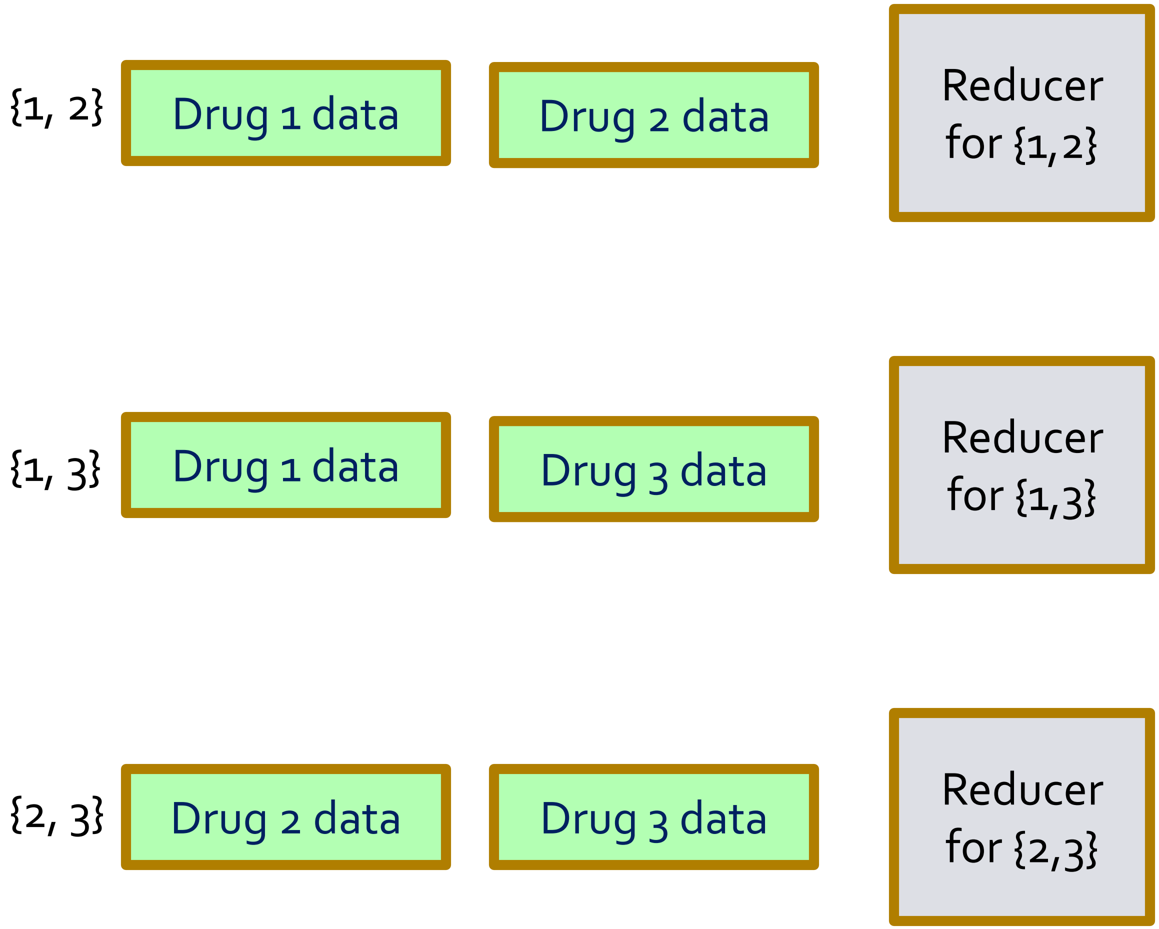
{2, 3} Drug 3 data

Reducer for {1,2}

Reducer for {1,3}

Reducer for {2,3}

Example: Three Drugs



What Went Wrong?

- 3000 drugs
- times 2999 key-value pairs per drug
- times 1,000,000 bytes per key-value pair
- = 9 terabytes communicated over a 1Gb Ethernet
- = 90,000 seconds of network use.

The Improved Algorithm

- They grouped the drugs into 30 groups of 100 drugs each.
  - Say G_1 = drugs 1-100, G_2 = drugs 101-200,..., G_{30} = drugs 2901-3000.
  - Let g(i) = the number of the group into which drug *i* goes.

The Map Function

- A key is a set of two group numbers.
- The mapper for drug *i* produces 29 key-value pairs.
  - Each key is the set containing g(i) and one of the other group numbers.
  - The value is a pair consisting of the drug number i and the megabyte-long record for drug *i*.

The Reduce Function

- The reducer for pair of groups $\{m, n\}$ gets that key and a list of 200 drug records – the drugs belonging to groups m and n.
- Its job is to compare each record from group m with each record from group n.
  - Special case: also compare records in group n with each other, if $m = n+1$ or if $n = 30$ and $m = 1$.
- Notice each pair of records is compared at exactly one reducer, so the total computation is not increased.

The New Communication Cost

- The big difference is in the communication requirement.
- Now, each of 3000 drugs’ 1MB records is replicated 29 times.
  - Communication cost = 87GB, vs. 9TB.

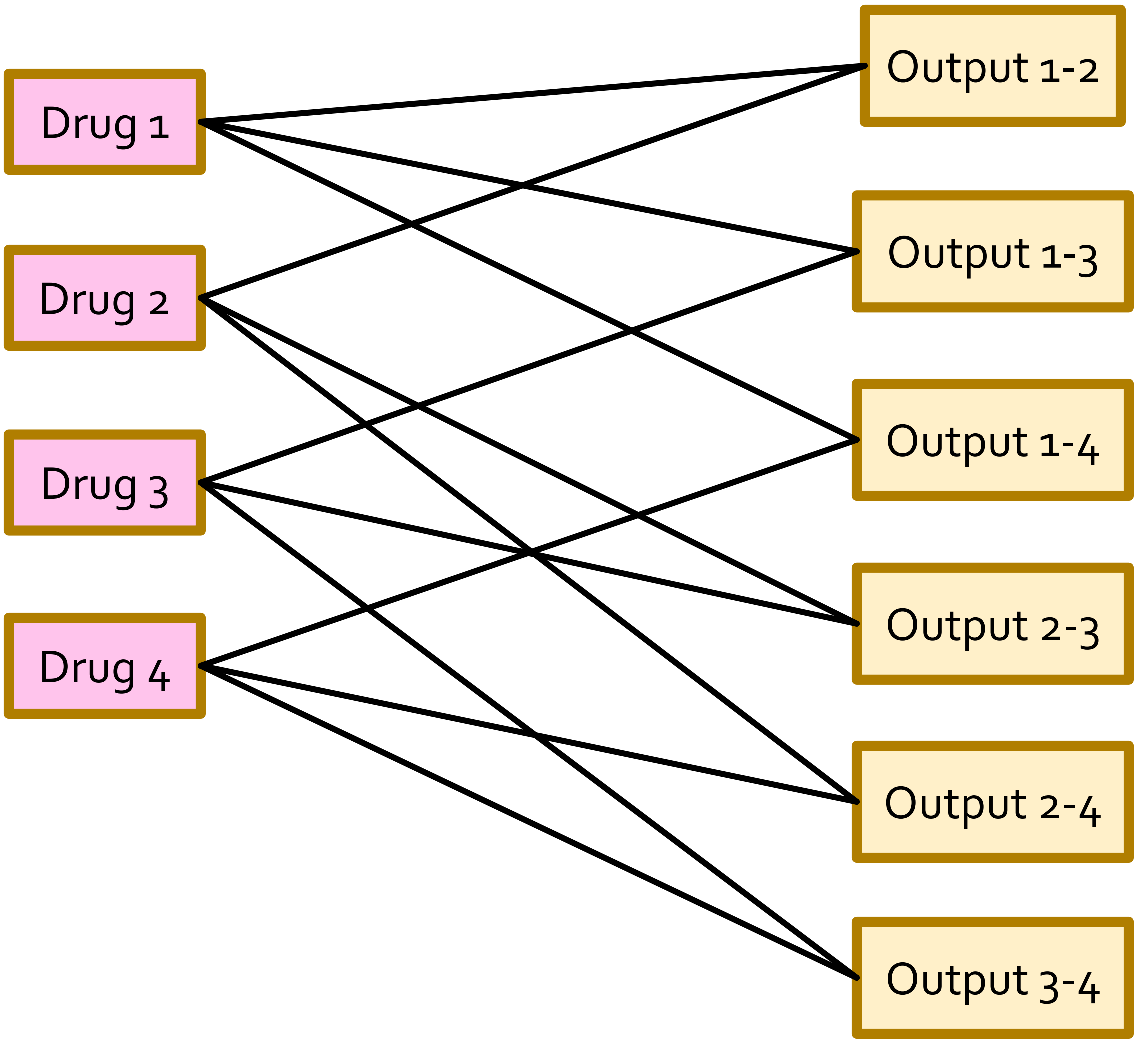
Theory of Map-Reduce Algorithms

Reducer Size
Replication Rate
Mapping Schemas
Lower Bounds

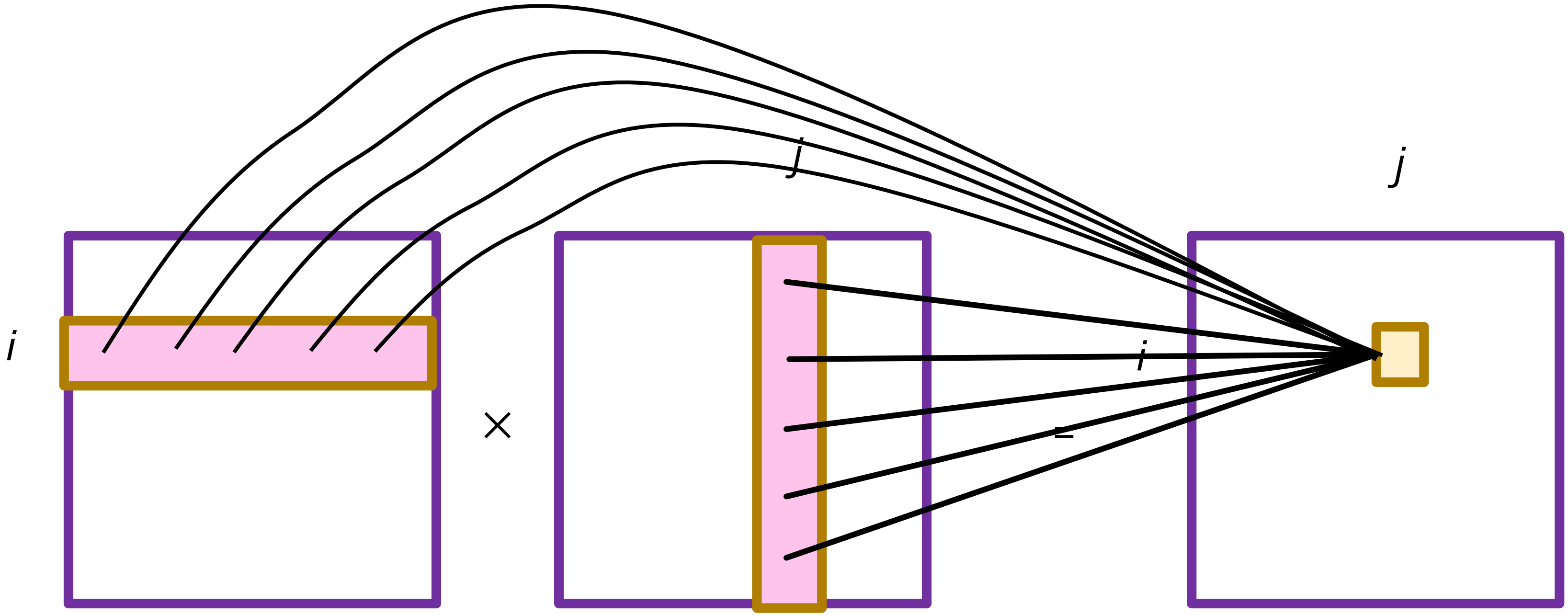
A Model for Map-Reduce Algorithms

1. A set of *inputs*.
   - Example: the drug records.
2. A set of *outputs*.
   - Example: One output for each pair of drugs.
3. A many-many relationship between each output and the inputs needed to compute it.
   - Example: The output for the pair of drugs $\{i, j\}$ is related to inputs i and j.

Example: Drug Inputs/Outputs



Example: Matrix Multiplication



Reducer Size

- *Reducer size*, denoted q, is the maximum number of inputs that a given reducer can have.
  - I.e., the length of the value list.
- Limit might be based on how many inputs can be handled in main memory.
- Or: make q low to force lots of parallelism.

Replication Rate

- The average number of key-value pairs created by each mapper is the *replication rate*.
  - Denoted r.
- Represents the communication cost per input.

Example: Drug Interaction

- Suppose we use g groups and d drugs.
- A reducer needs two groups, so $q = 2d/g$.
- Each of the d inputs is sent to g-1 reducers, or approximately $r = g$.
- Replace g by r in $q = 2d/g$ to get $r = 2d/q$.

Tradeoff!
The bigger the reducers,
the less communication.

Upper and Lower Bounds on r

- What we did gives an upper bound on r as a function of q.
- A solid investigation of map-reduce algorithms for a problem includes lower bounds.
  - Proofs that you cannot have lower r for a given q.

Proofs Need Mapping Schemas

- A *mapping schema* for a problem and a reducer size q is an assignment of inputs to sets of reducers, with two conditions:
  1. No reducer is assigned more than q inputs.
  2. For every output, there is some reducer that receives all of the inputs associated with that output.
     - Say the reducer *covers* the output.

Mapping Schemas – (2)

- Every map-reduce algorithm has a mapping schema.
- The requirement that there be a mapping schema is what distinguishes map-reduce algorithms from general parallel algorithms.

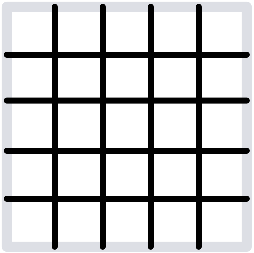
Example: Drug Interactions

- d drugs, reducer size q.
- Each drug has to meet each of the d-1 other drugs at some reducer.
- If a drug is sent to a reducer, then at most q-1 other drugs are there.
- Thus, each drug is sent to at least $(d-1)/(q-1)$ reducers, and $r \geq \lceil (d-1)/(q-1) \rceil$.
- Half the r from the algorithm we described.
- Better algorithm gives $r = d/q + 1$, so lower bound is actually tight.

The Better Algorithm

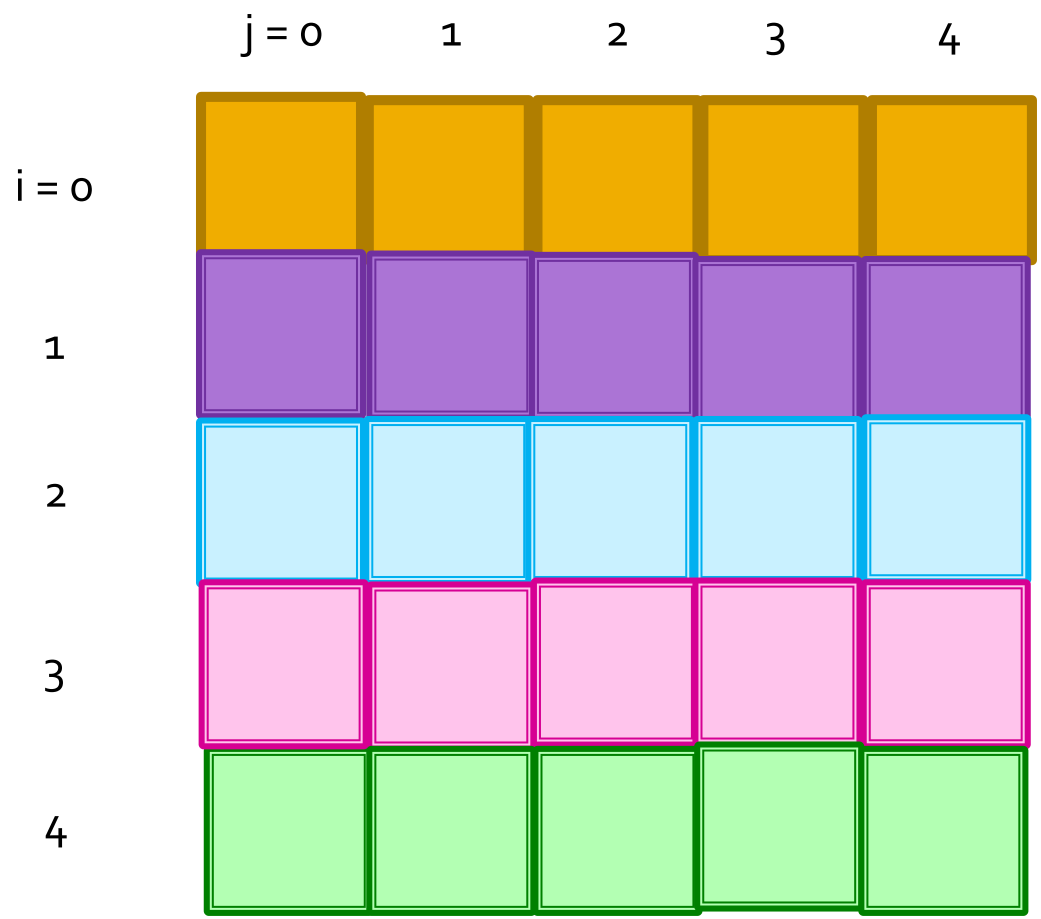
- The problem with the algorithm dividing inputs into g groups is that members of a group appear together at many reducers.
  - Thus, each reducer can only productively compare about half the pairs it has available to it.
- Better: use smaller groups, with each reducer getting many little groups.
  - Eliminates almost all the redundancy.

Optimal Algorithm for All-Pairs



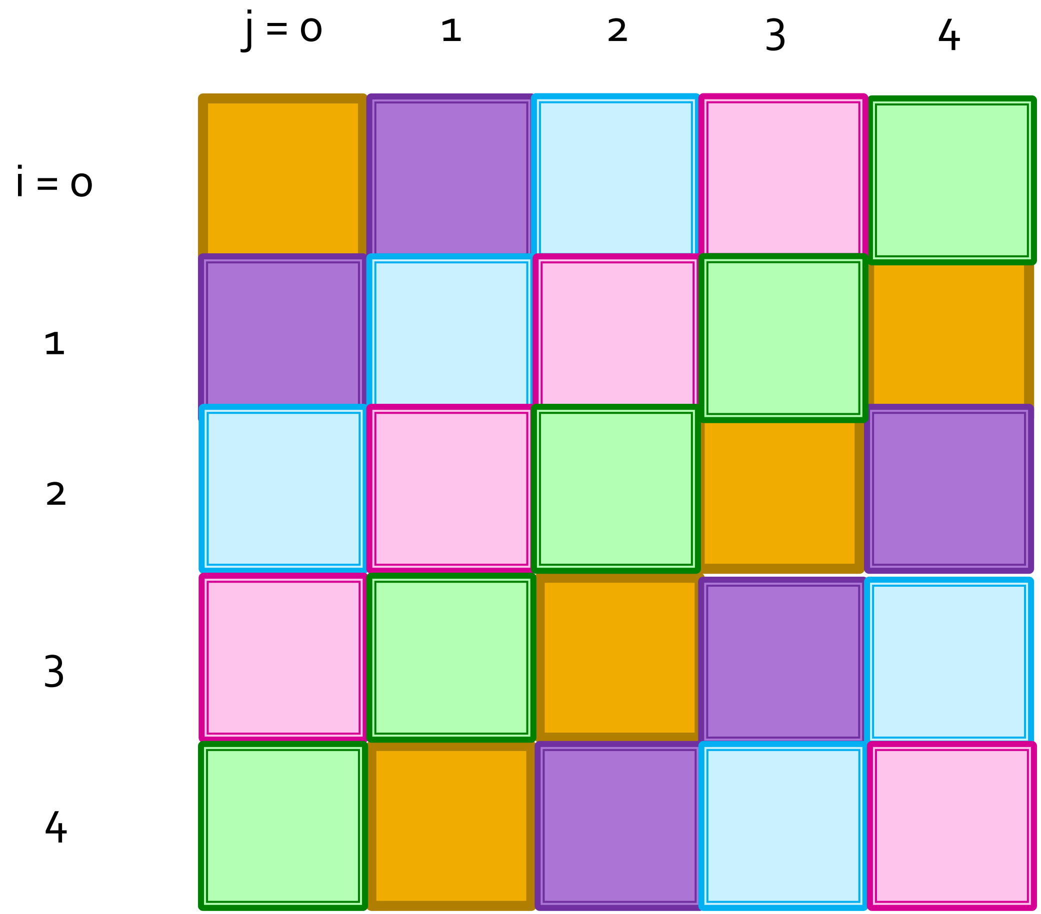
- Assume d inputs.
- Let p be a prime, where p^2 divides d.
- Divide inputs into p^2 groups of d/p^2 inputs each.
- Name the groups (i, j), where $0 \leq i, j < p$.
- Use p(p+1) reducers, organized into p+1 *teams* of p reducers each.
- For $0 \leq k < p$, group (i, j) is sent to the reducer i+kj (mod p) in group k.
- In the last team (p), group (i, j) is sent to reducer j.

Example: Teams for p = 5



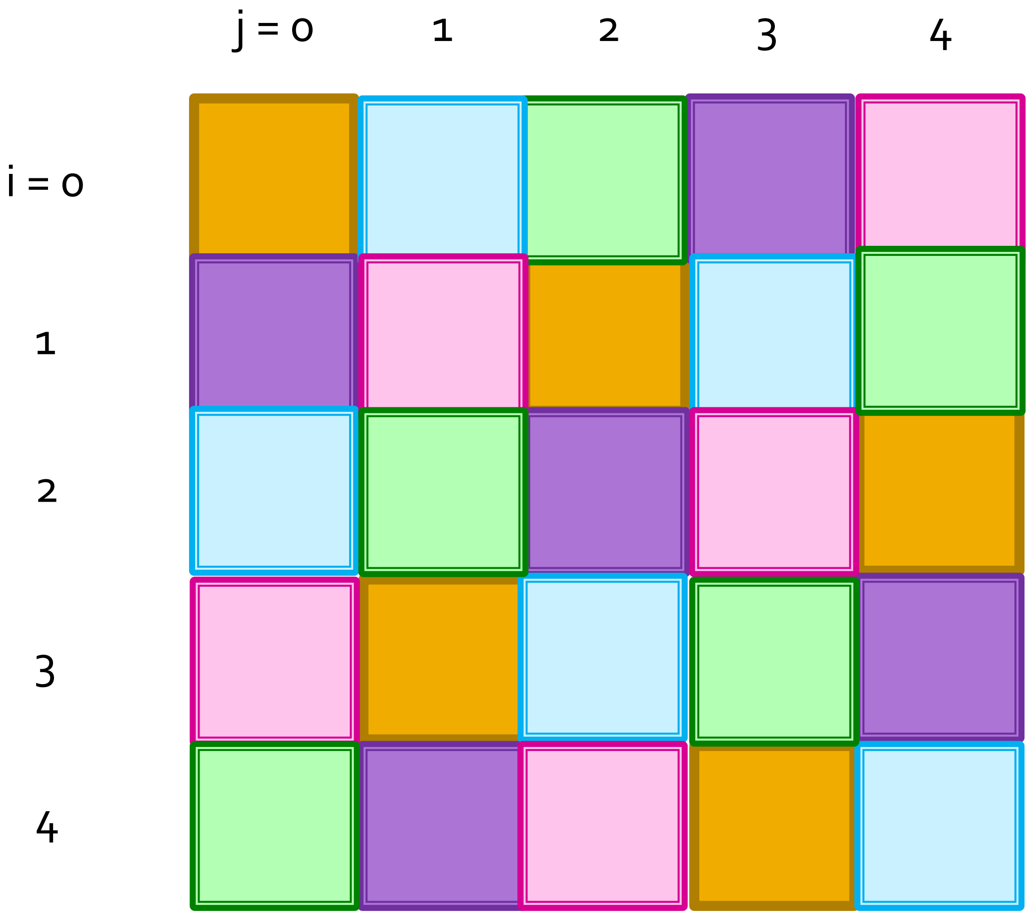
Team 0

Example: Teams for p = 5



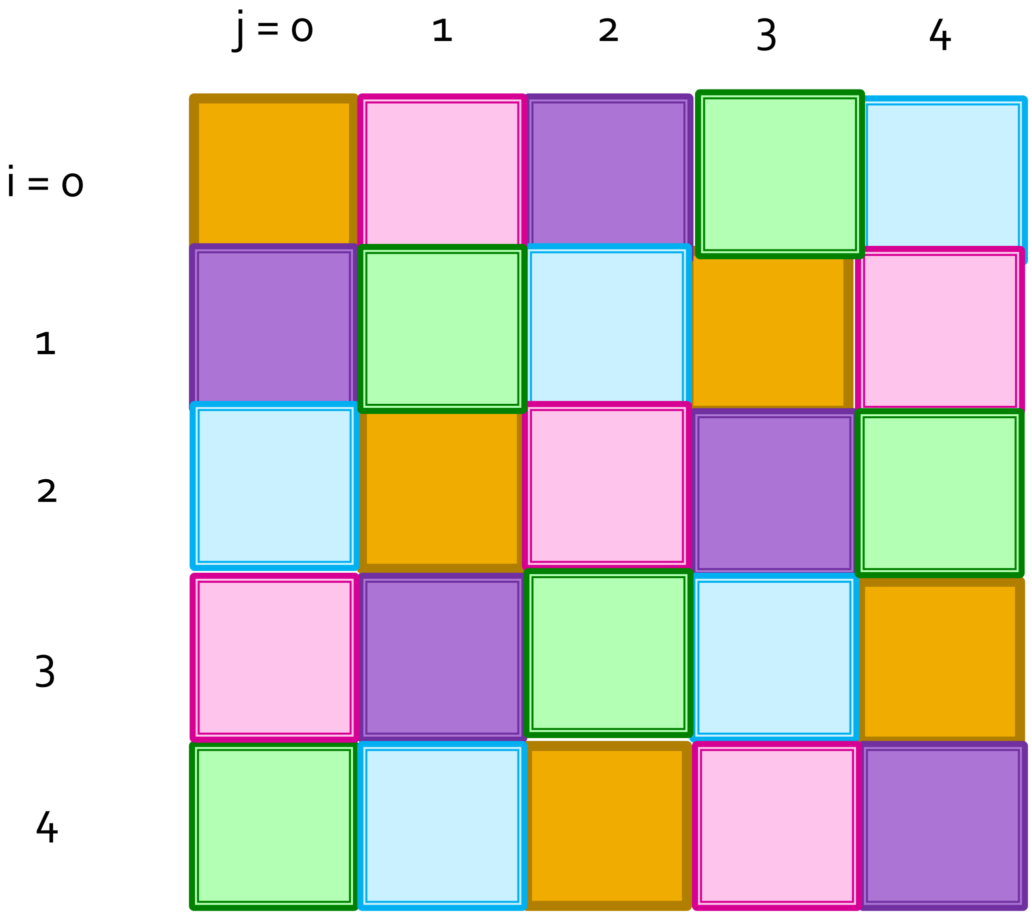
Team 1

Example: Teams for p = 5



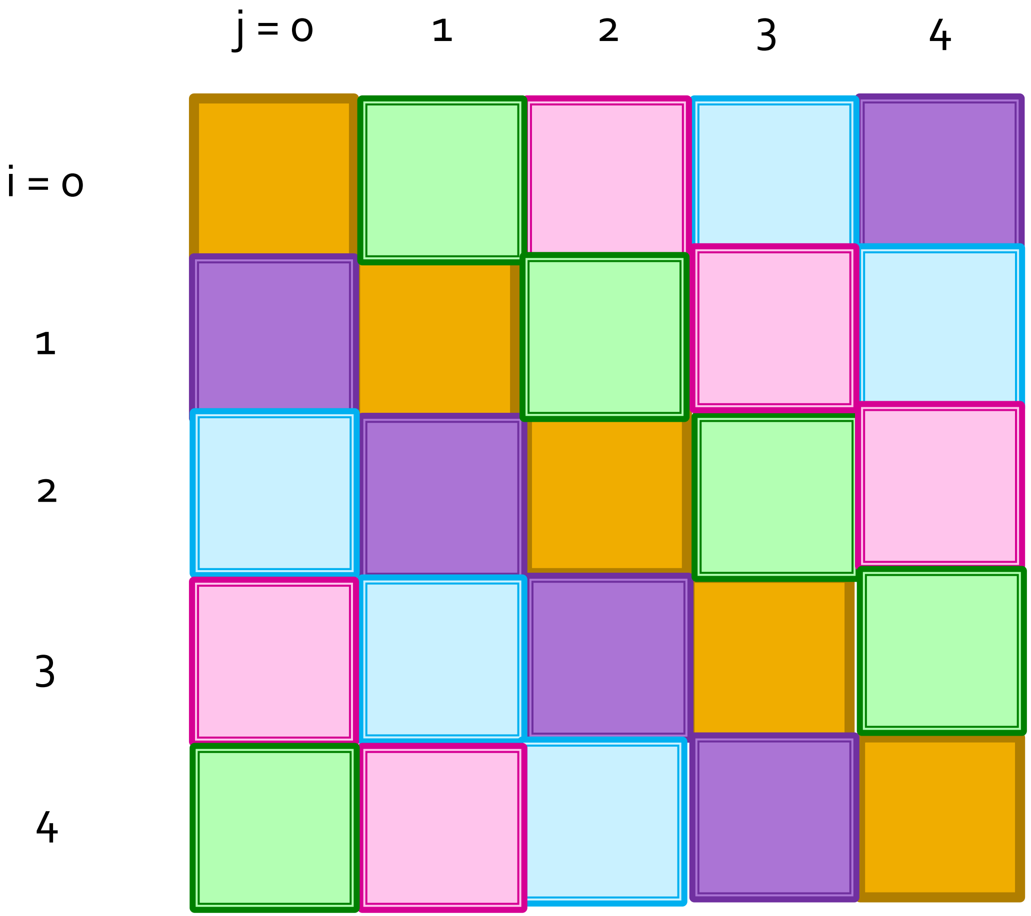
Team 2

Example: Teams for p = 5



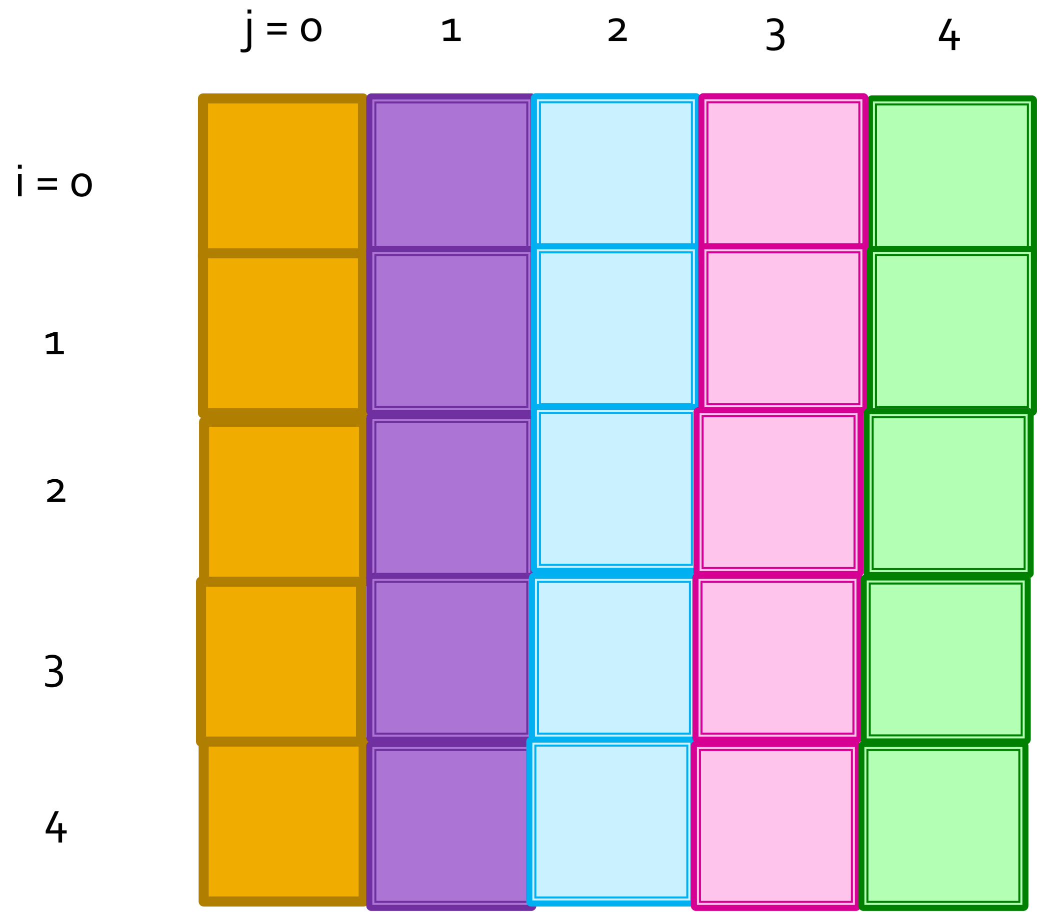
Team 3

Example: Teams for p = 5



Team 4

Example: Teams for p = 5



Team 5

Why It Works

- Let two inputs be in groups (i, j) and (i', j').
- If the same group, these inputs obviously share a reducer.
- If $j = j'$, then they share a reducer in group p.
- If $j \neq j'$, then they share a reducer in team k provided $i + kj = i' + kj'$.
- Equivalently, $(i-i') = k(j-j')$.
- But since $j \neq j'$, $(j-j')$ has an inverse modulo p.
- Thus, team $k = (i-i')(j-j')^{-1}$ has a reducer for which $i + kj = i' + kj'$.

Why It Is Optimal

- The replication rate r is p+1, since every input is sent to one reducer in each team.
- The reducer size q is $pd/p^2 = d/p$, since each reducer gets p groups of size d/p^2.
- Thus, $r = d/q + 1$.
- $(d/q + 1) - (d-1)/(q-1) < 1$ provided $q < d$.
  - But if $q \geq d$, we can do everything in one reducer, and $r = 1$.
- The upper bound $r \leq d/q + 1$ and the lower bound $r \geq \lceil (d-1)/(q-1) \rceil$ differ by less than 1, and are integers, so they are equal.

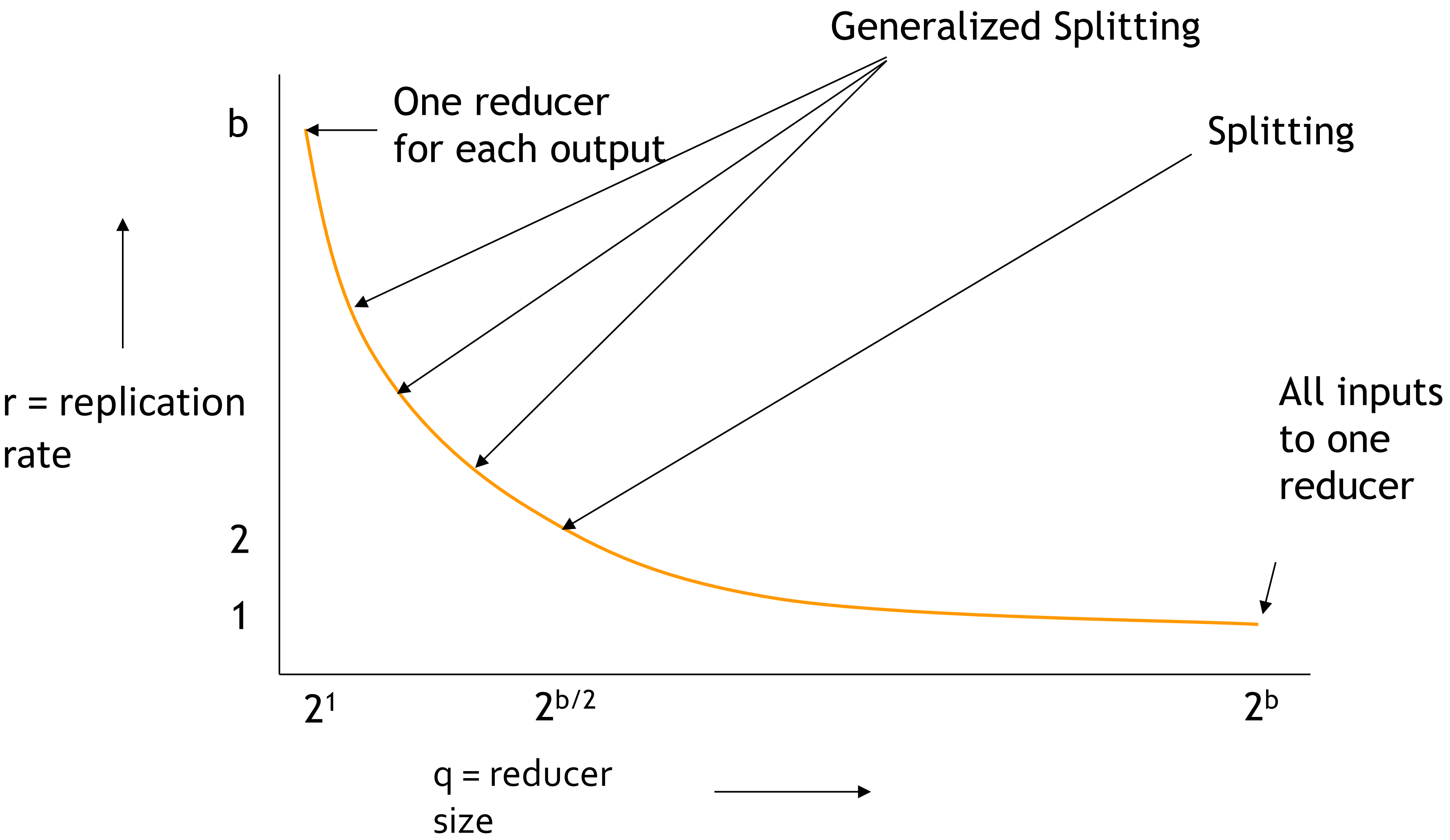
Specific Problems

Hamming Distance 1
Matrix Multiplication

Definition of HD1 Problem

- Given a set of bit strings of length b, find all those that differ in exactly one bit.
- Theorem: $r \geq b/\log_2 q$.

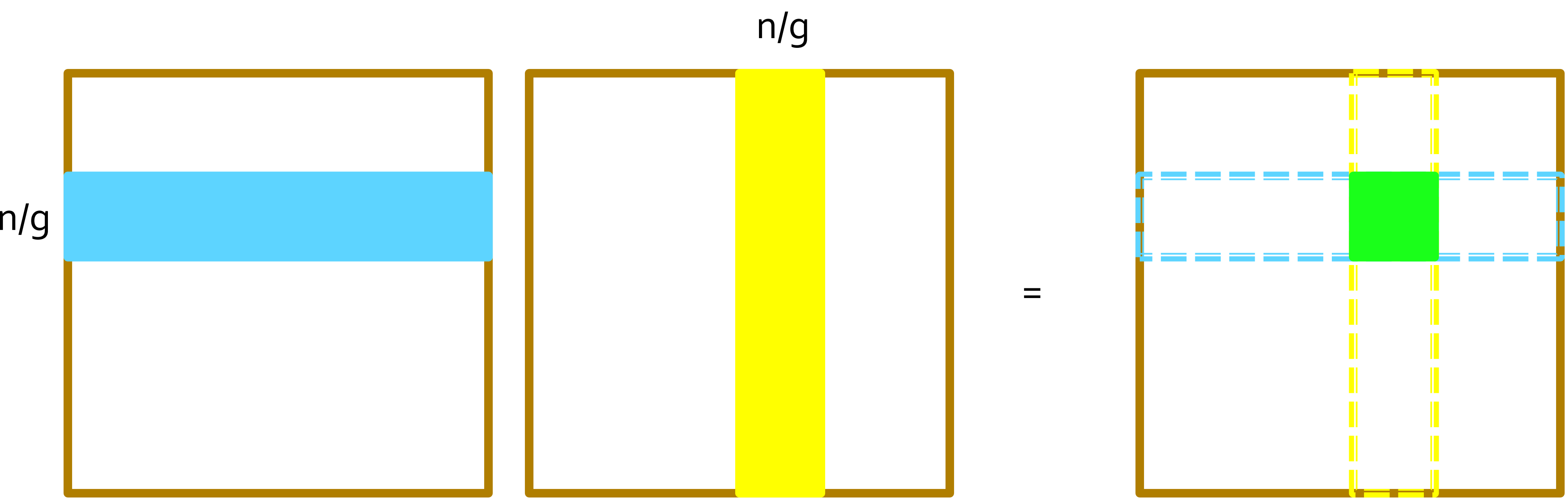
Algorithms Matching Lower Bound



Matrix Multiplication

- Assume $n \times n$ matrices $AB = C$.
- Theorem: For matrix multiplication, $r \geq 2n^2/q$.

Matching Algorithm



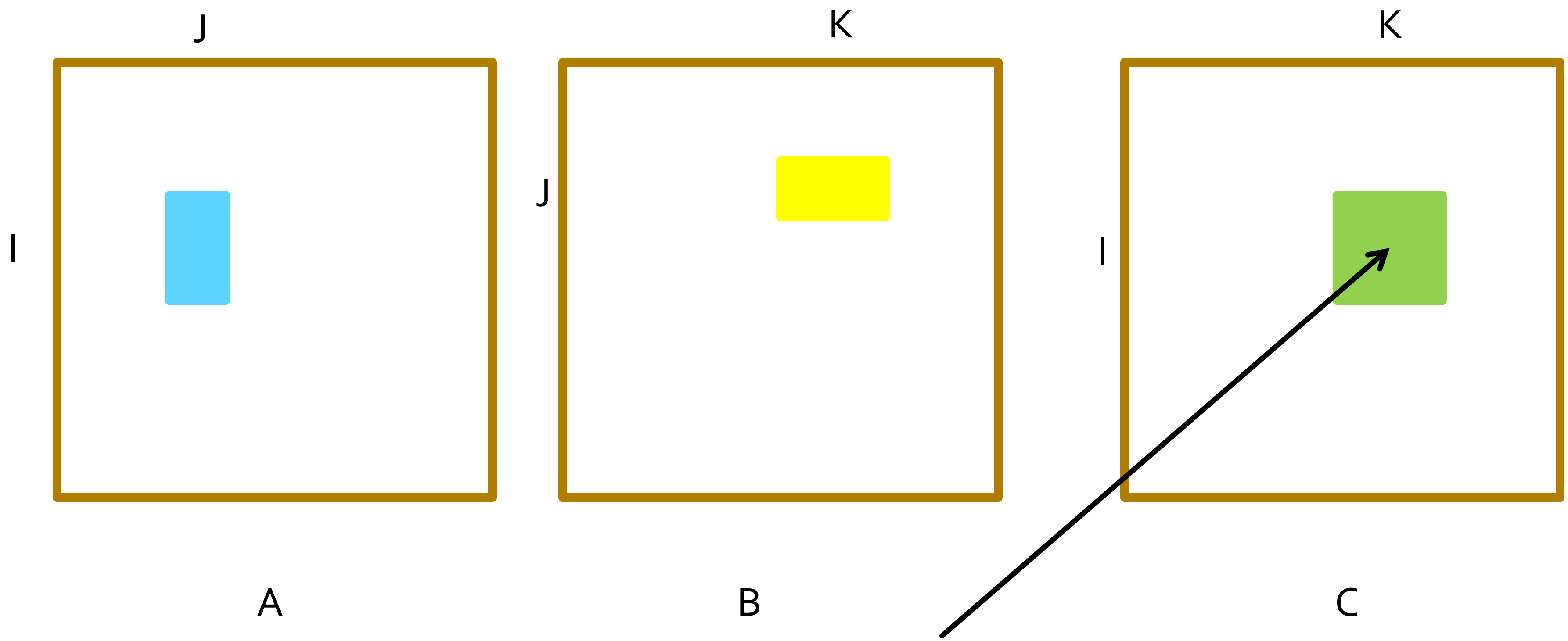
Divide rows of A and columns of B into g groups gives

$r = g = 2n^2/q$

Two-Job Map-Reduce Algorithm

- A better way: use two map-reduce jobs.
- Job 1: Divide both input matrices into rectangles.
  - Reducer takes two rectangles and produces partial sums of certain outputs.
- Job 2: Sum the partial sums.

Picture of First Job



For *i* in I and *k* in K, contribution is $\Sigma_{j \text{ in J}} A_{ij} \times B_{jk}$

Comparison: Communication Cost

- One-job method: Total communication = $4n^4/q$.
- Two-job method Total communication = $4n^3/\sqrt{q}$.
  - Since $q < n^2$ (or we really have a serial implementation), two jobs wins!

Summary

- Represent problems as input-output mappings.
- MapReduce algorithm is described by a mapping schema – yields lower bounds on replication rate as a function of reducer size.
- For “drug interaction”: exact match between upper and lower bounds.
- For HD = 1 problem: exact match.
- 1-job matrix multiplication analyzed exactly.
- But 2-job MM yields better total communication.