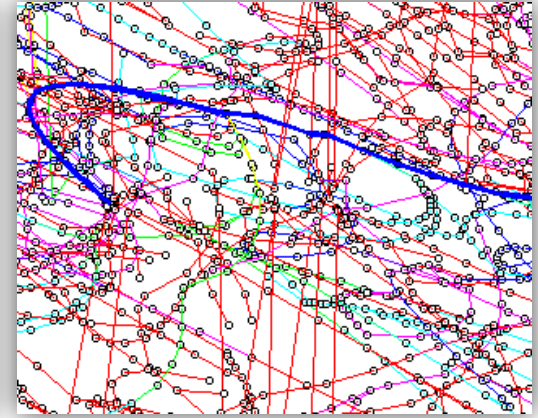
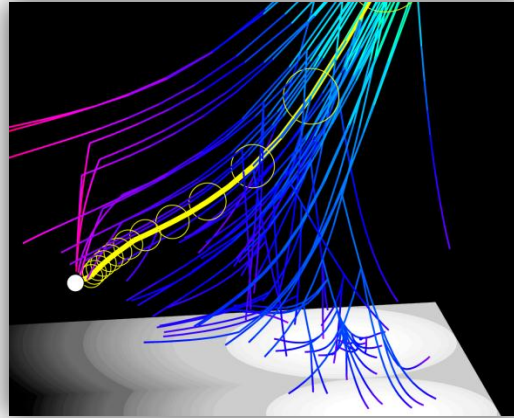
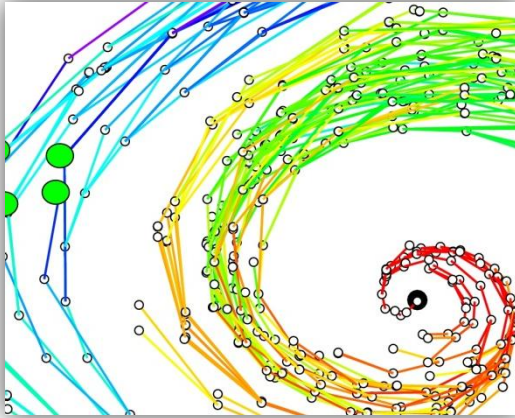


LQR-RRT*: Optimal Sampling-Based Motion Planning with Automatically Derived Extension Heuristics



Alejandro Perez

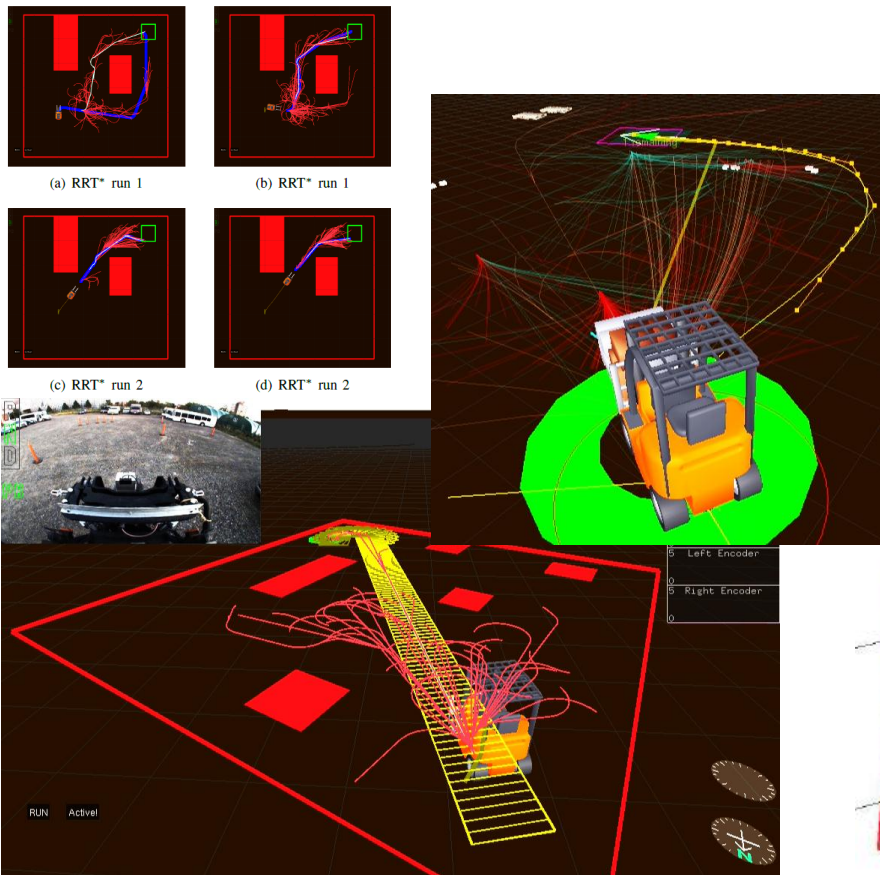
Robert Platt Jr., George Konidaris,

Leslie Kaelbling and Tomas Lozano-Perez

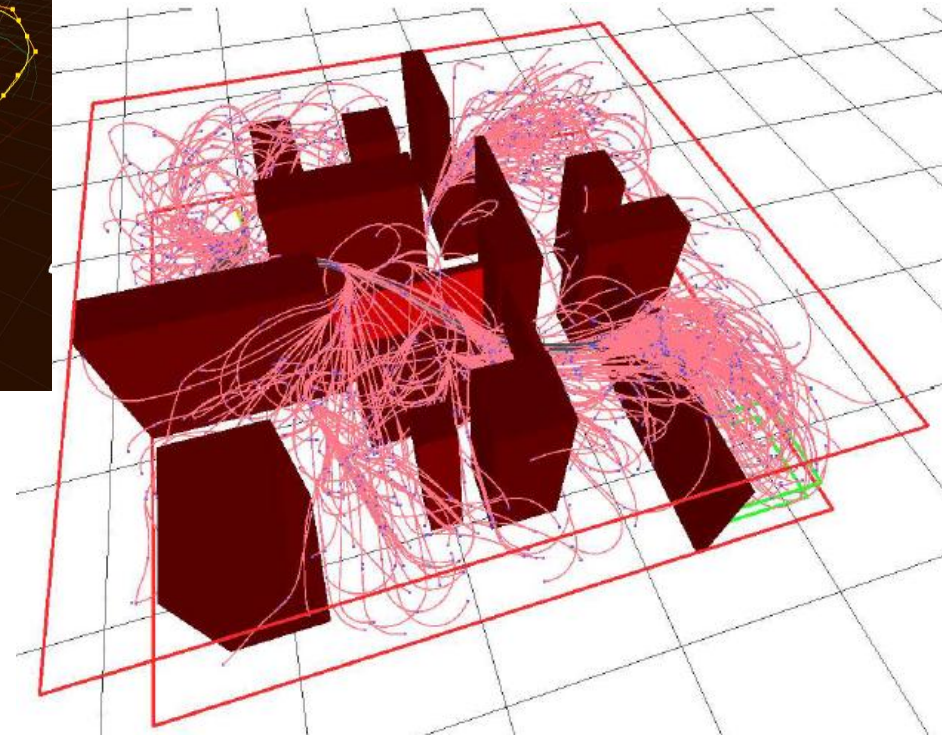
MIT/CSAIL



Motion Planning with Dynamics



Karaman, Walter, Perez, Frazzoli, and Teller, "Anytime Motion Planning using the RRT*," in IEEE International Conference on Robotics and Automation (ICRA), 2011.



Karaman and Frazzoli, "Optimal Kinodynamic Motion Planning using Incremental Sampling-based Methods," in IEEE Conference on Decision and Control (CDC), Atlanta, GA, 2010.

Motion Planning with Dynamics

Torque-limited pendulum

Two-dimensional state space, $(\theta, \dot{\theta})$

$$\ddot{\theta} = u - b\dot{\theta} - g \cos(\theta), \quad g = 9.81, \quad b = 0.1, \quad u \in [-3, 3]$$

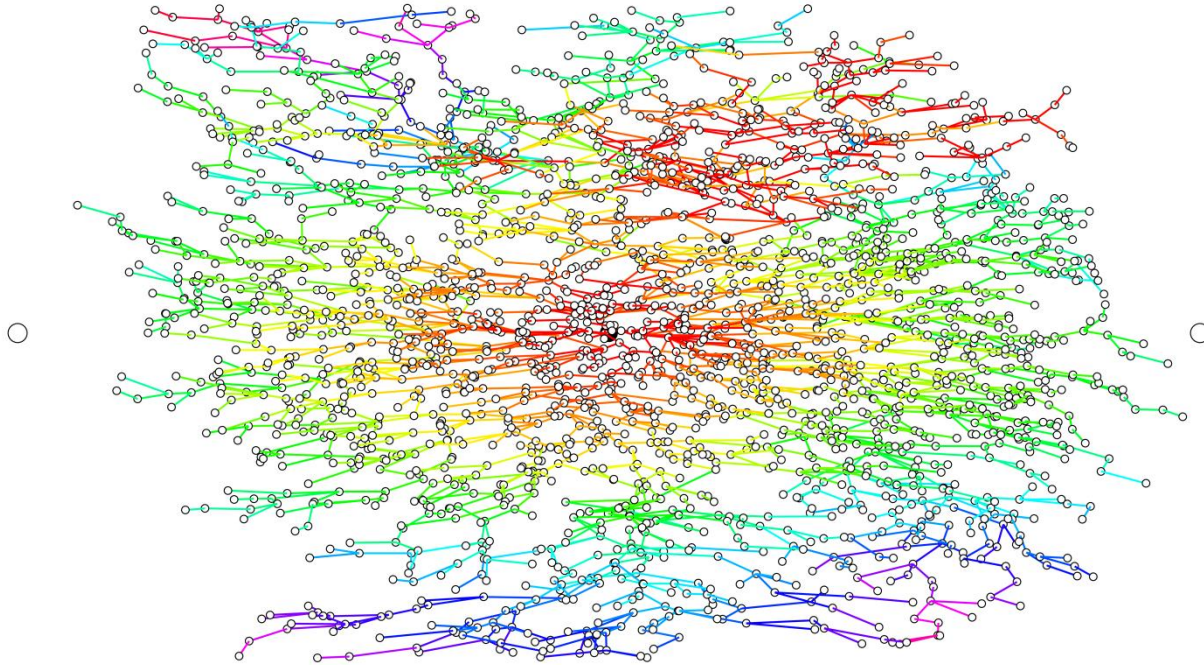


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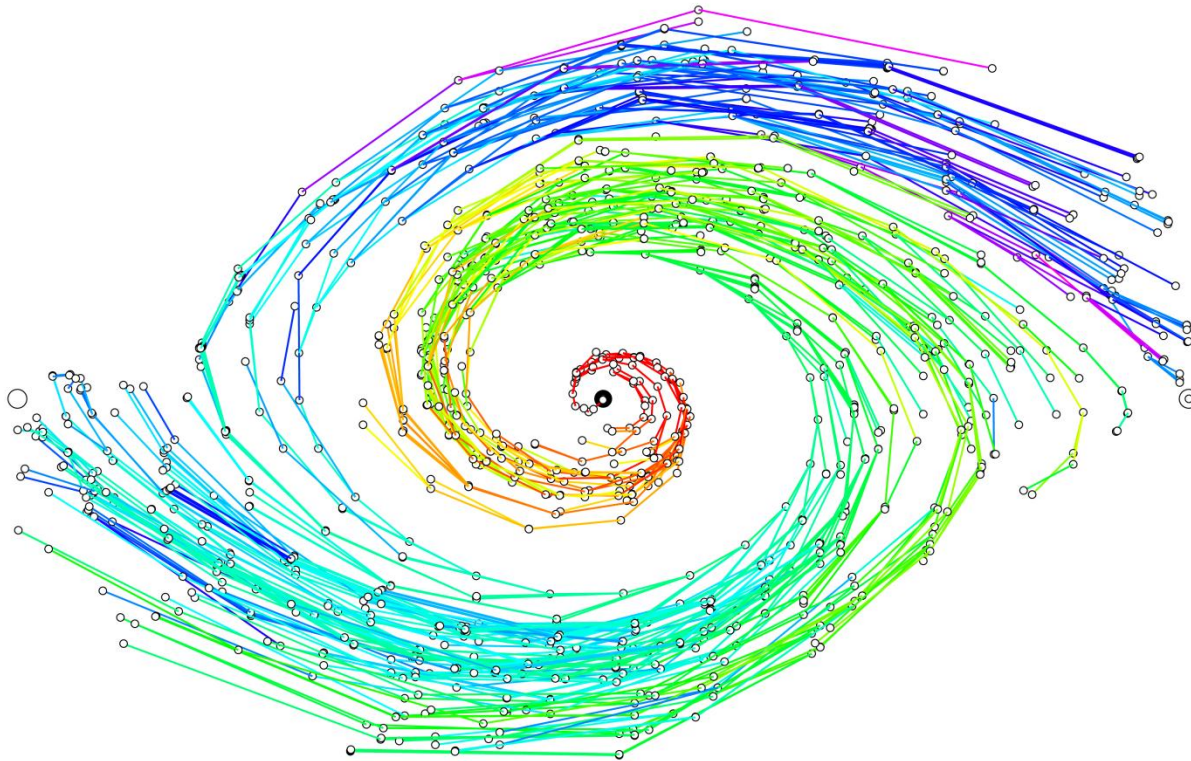


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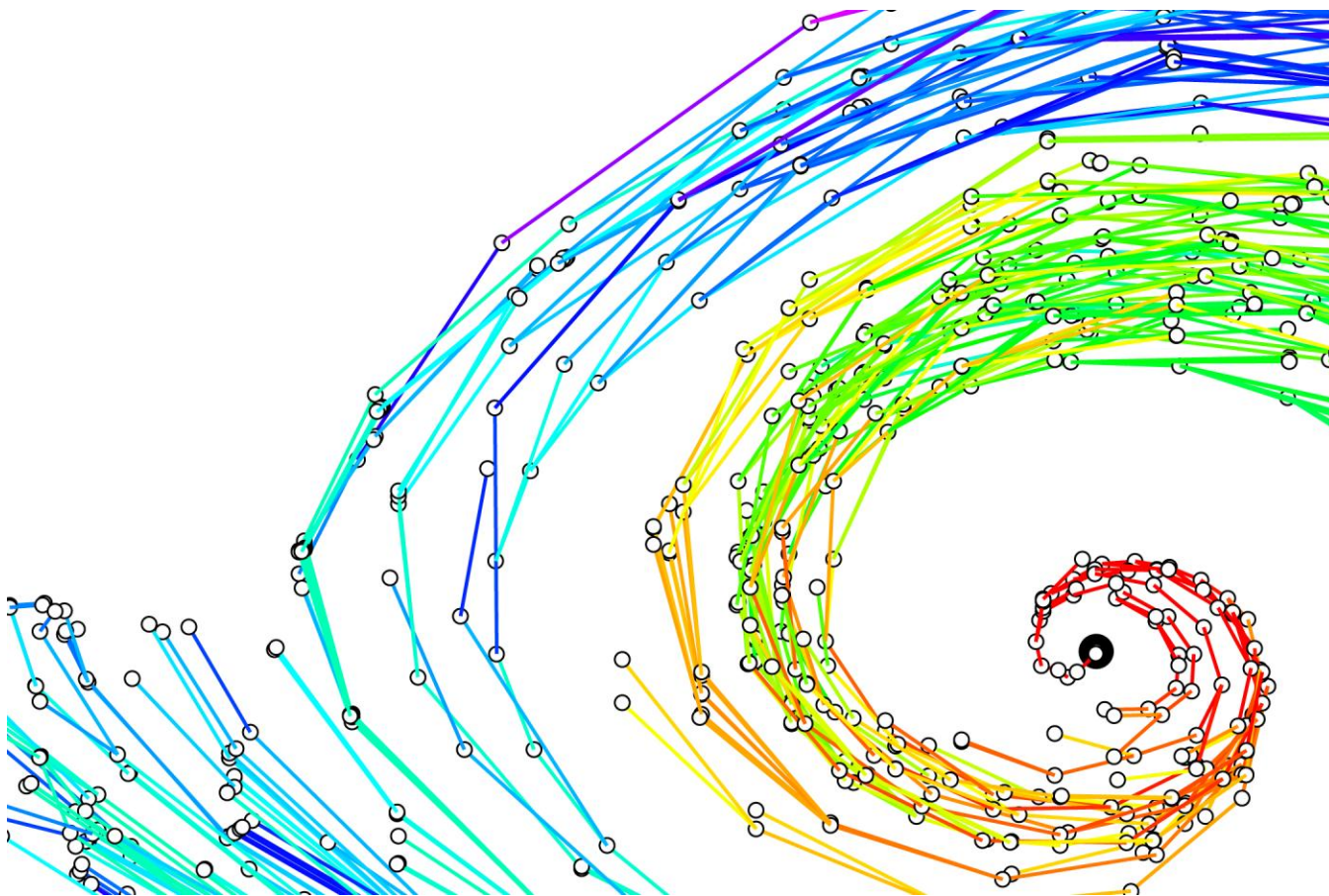
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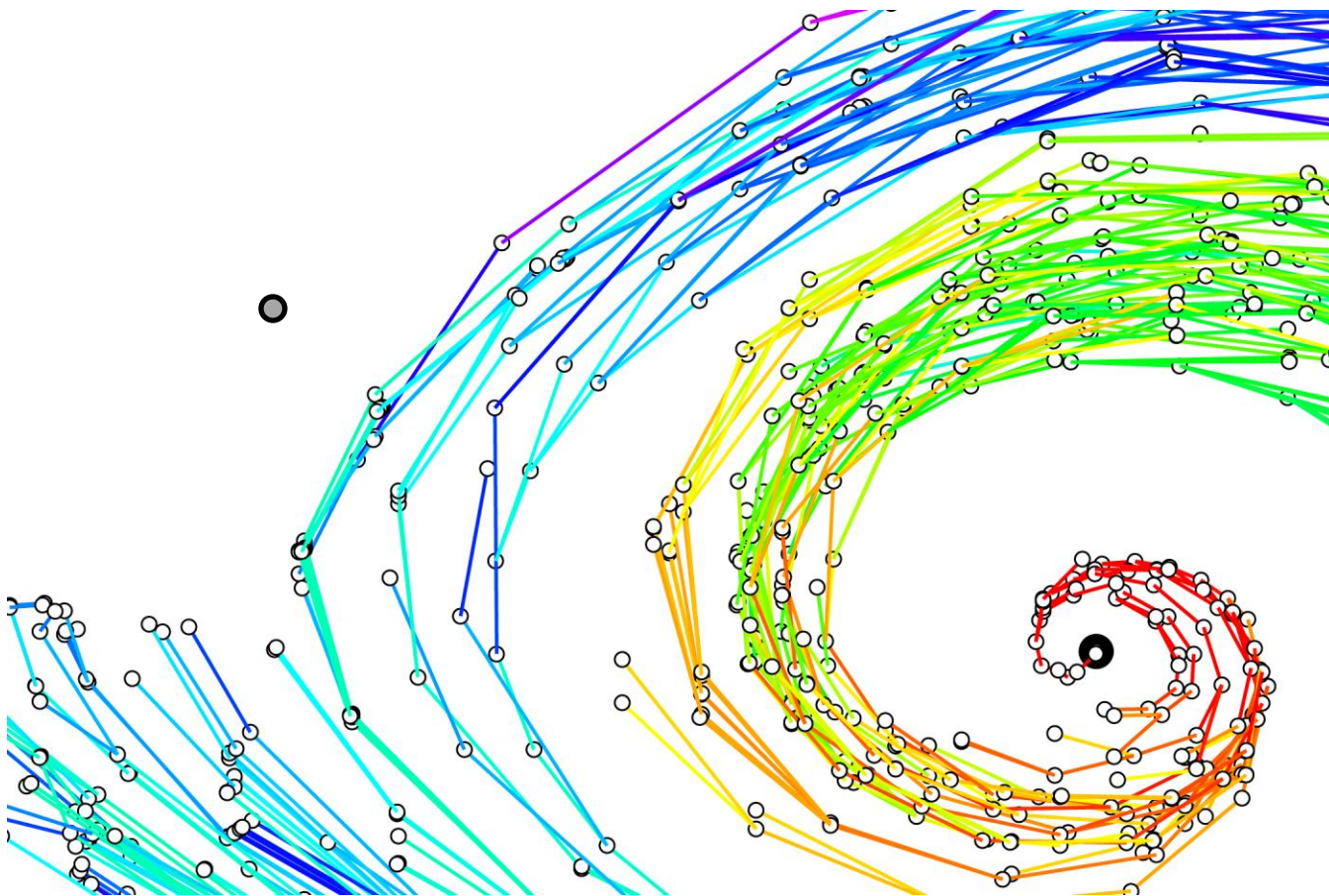
Motion Planning with Dynamics

Cost Metric



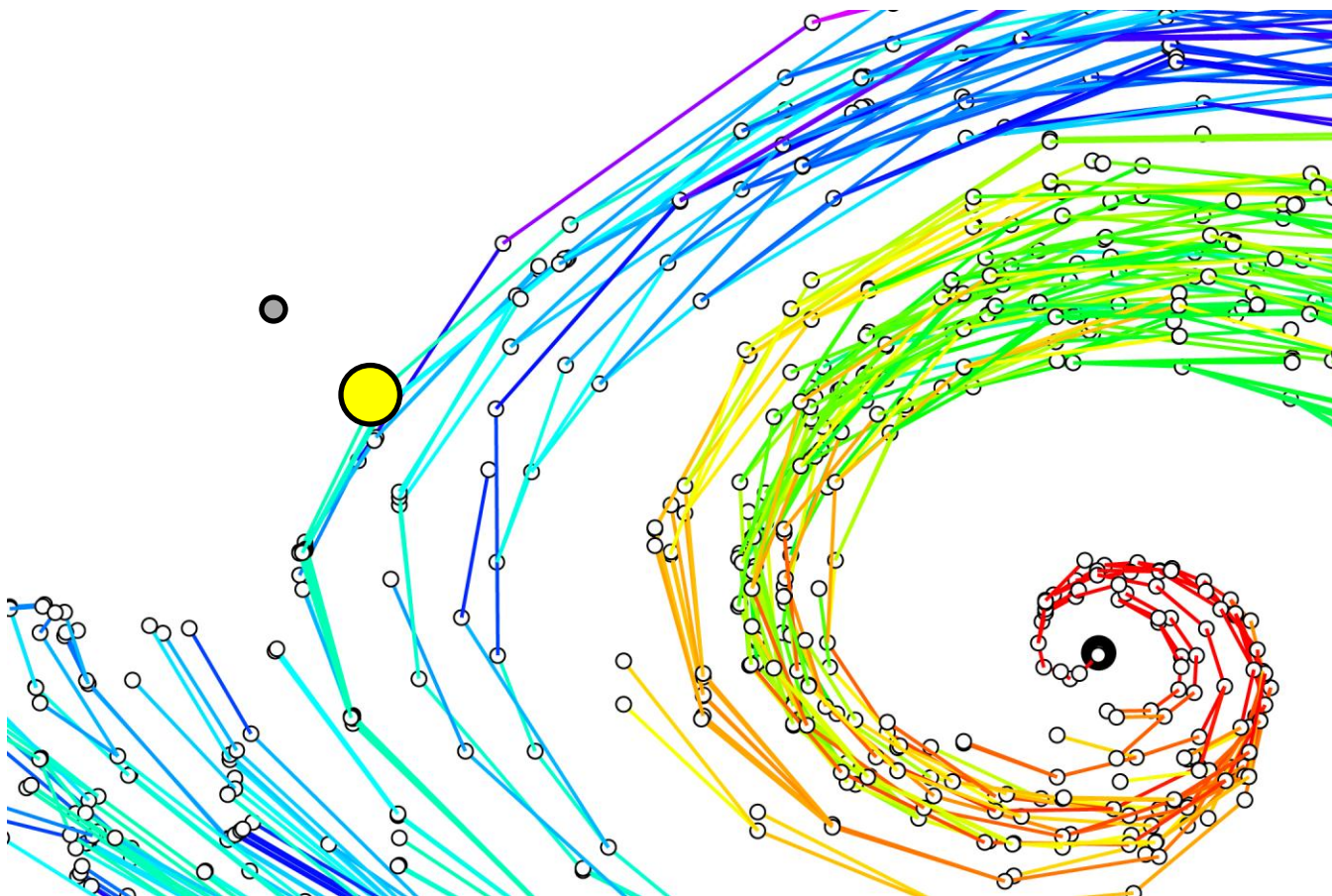
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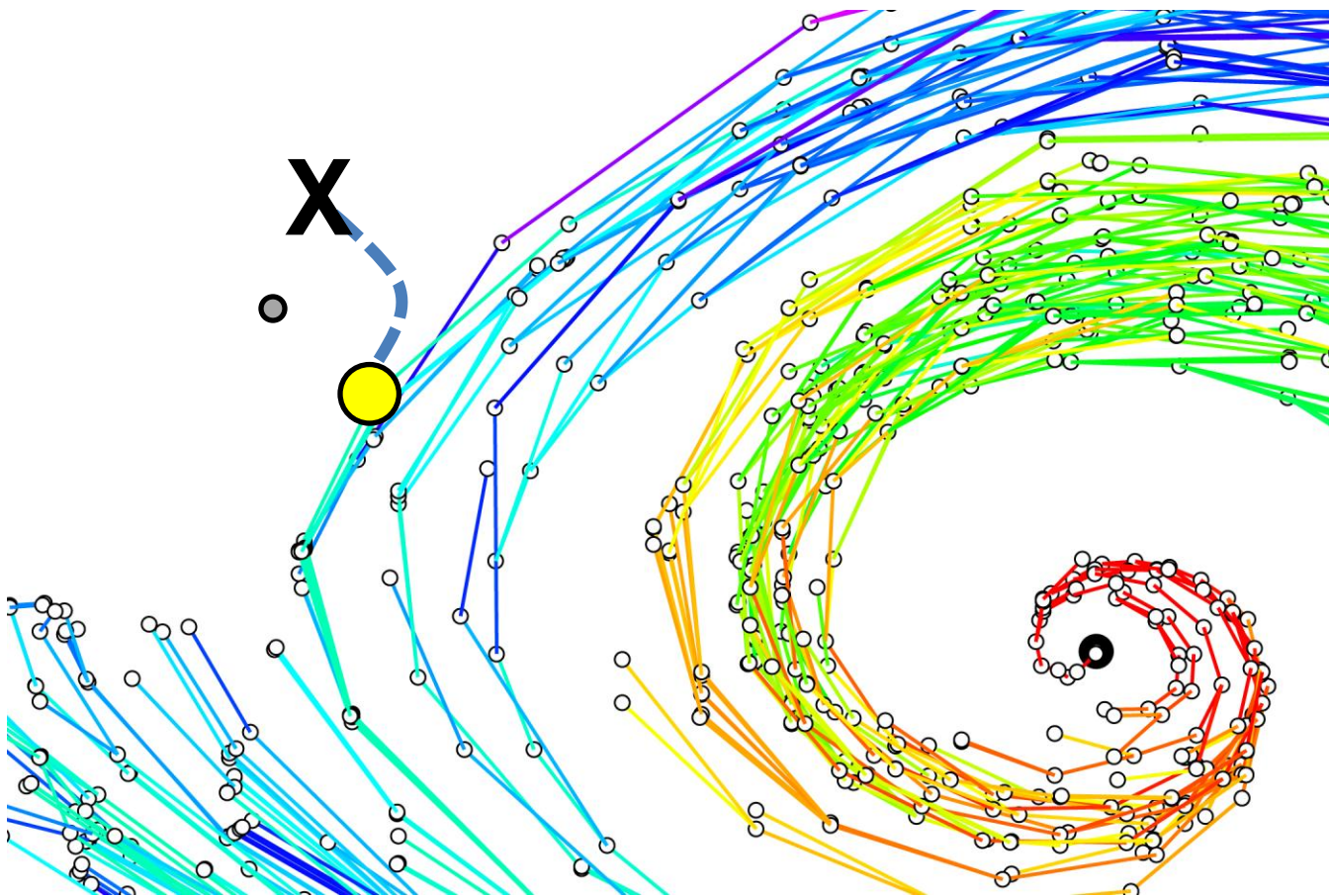
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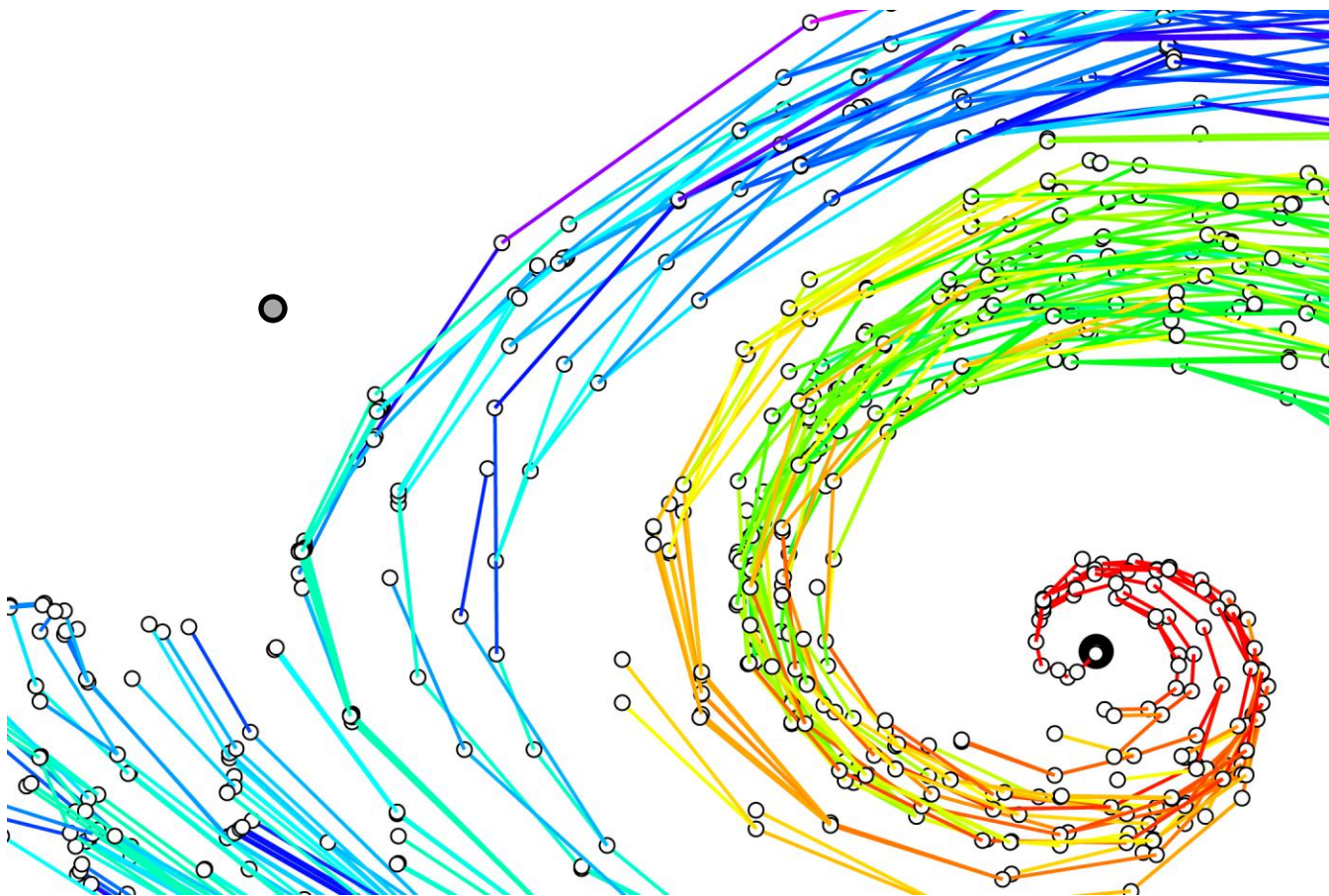
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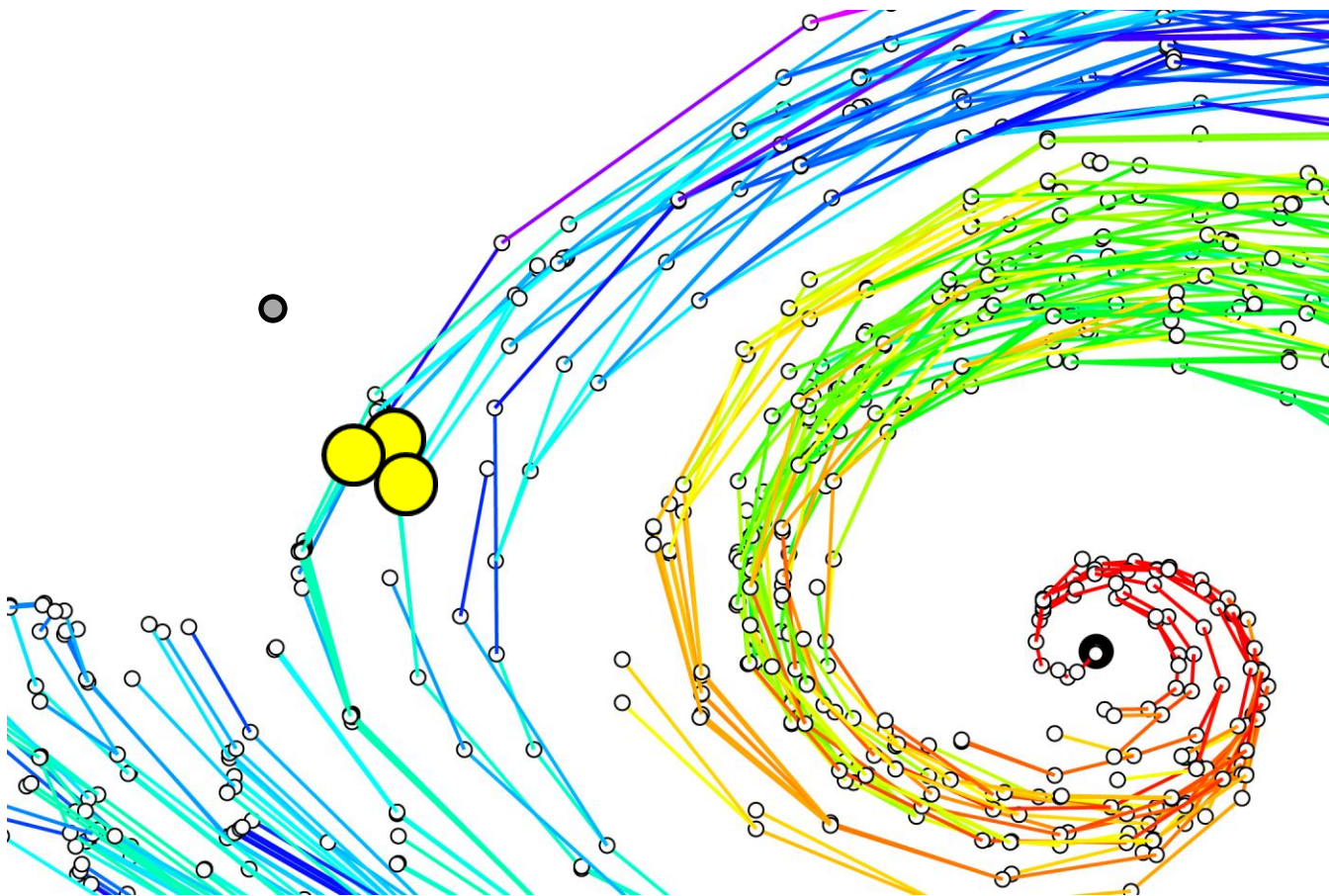
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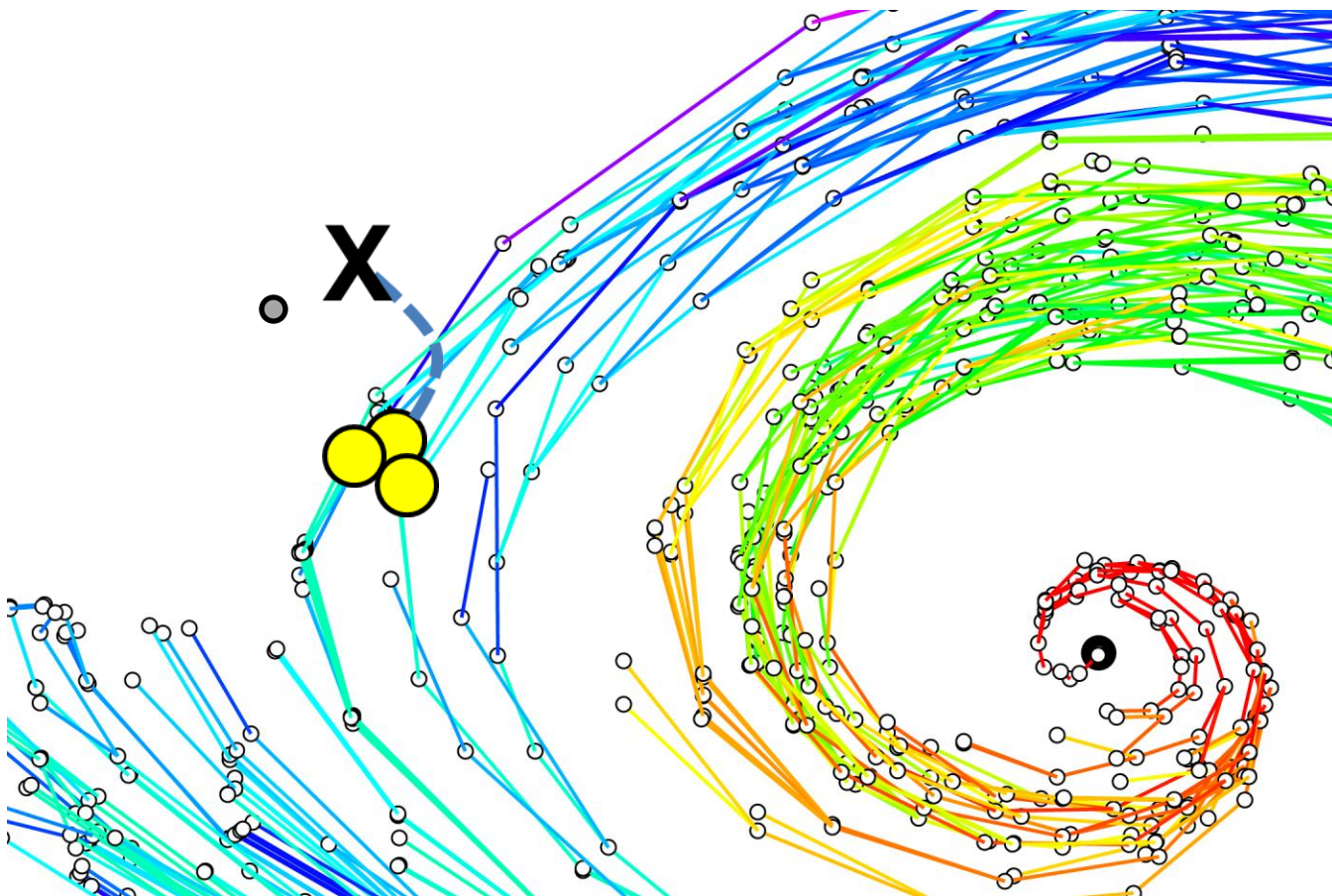
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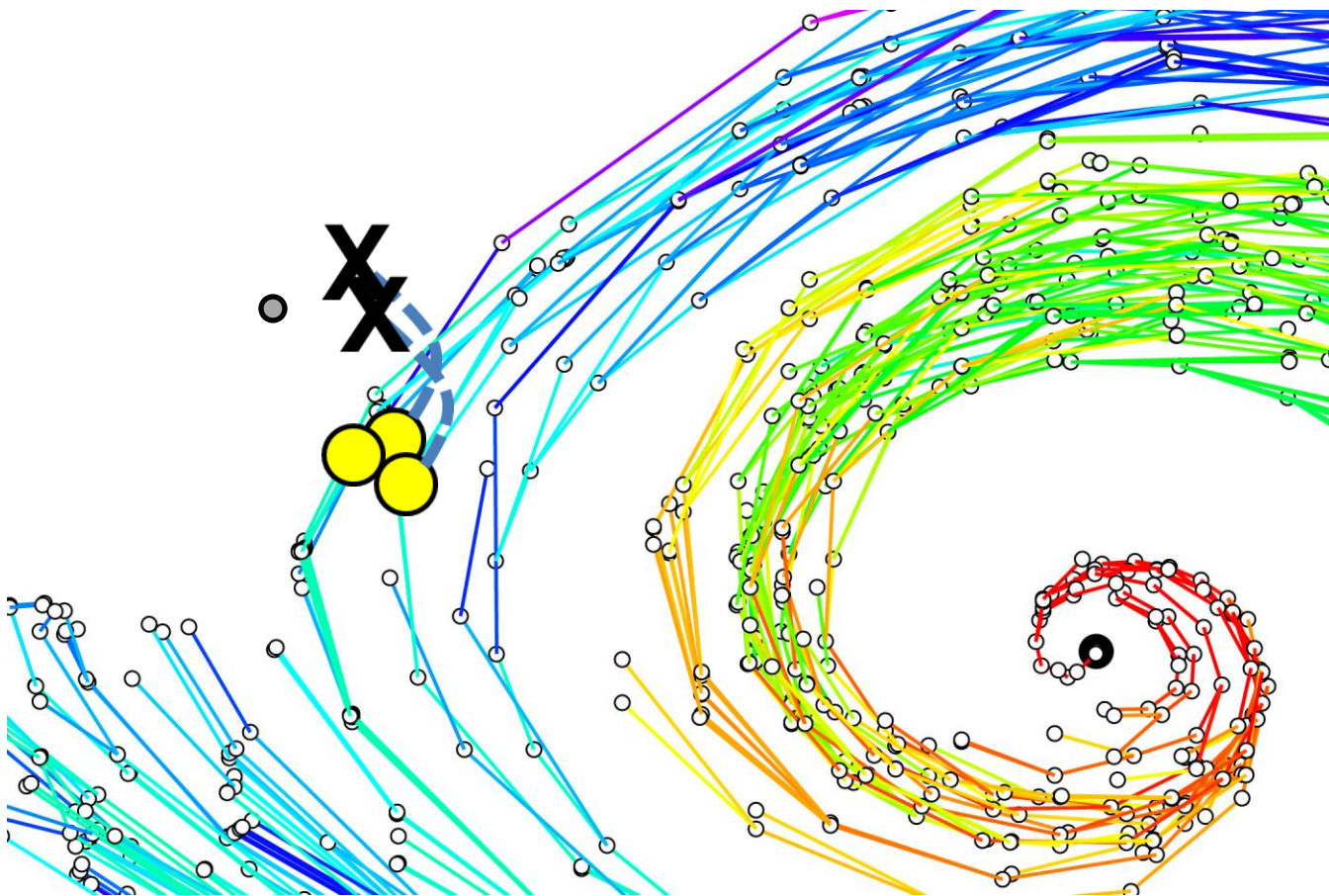
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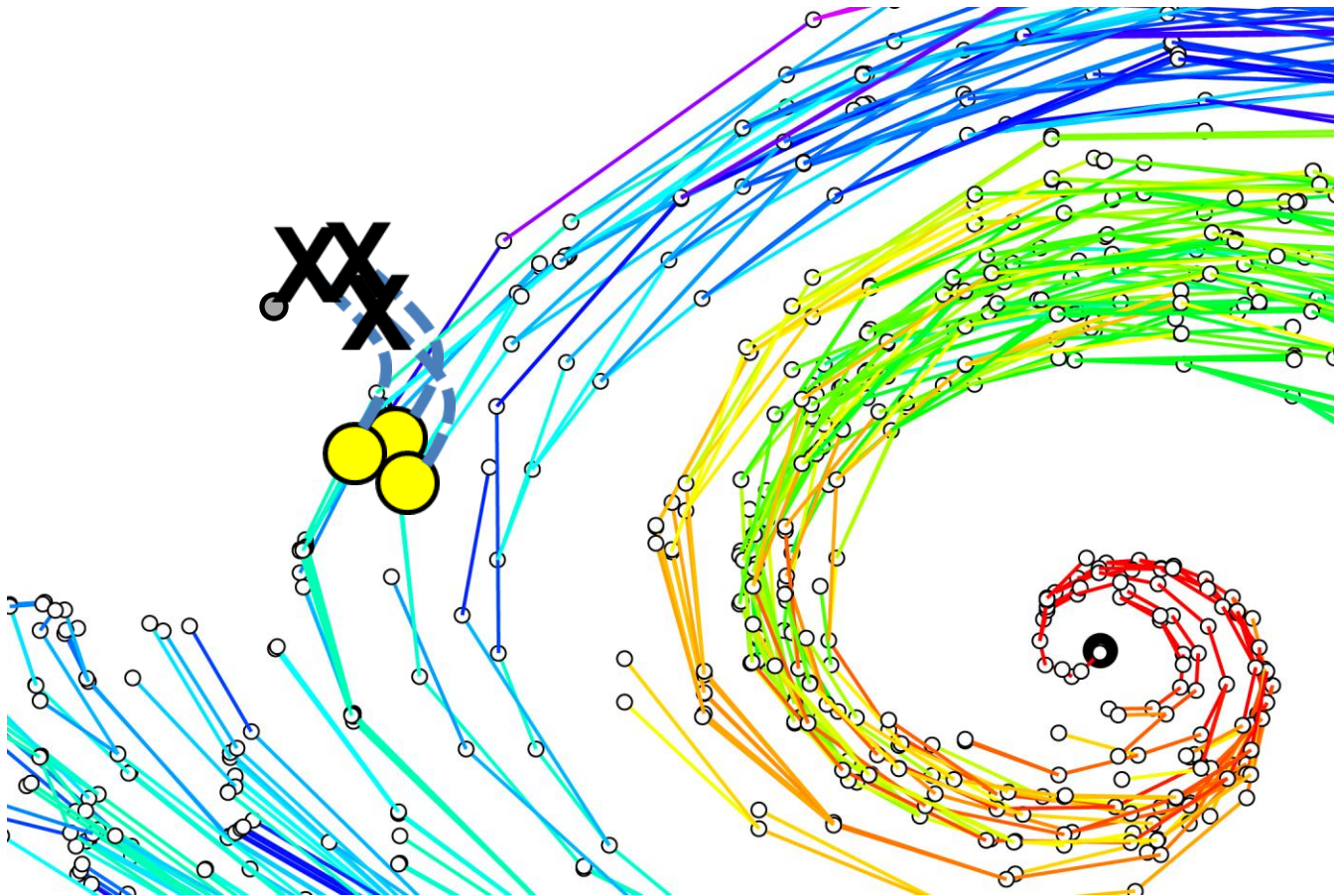
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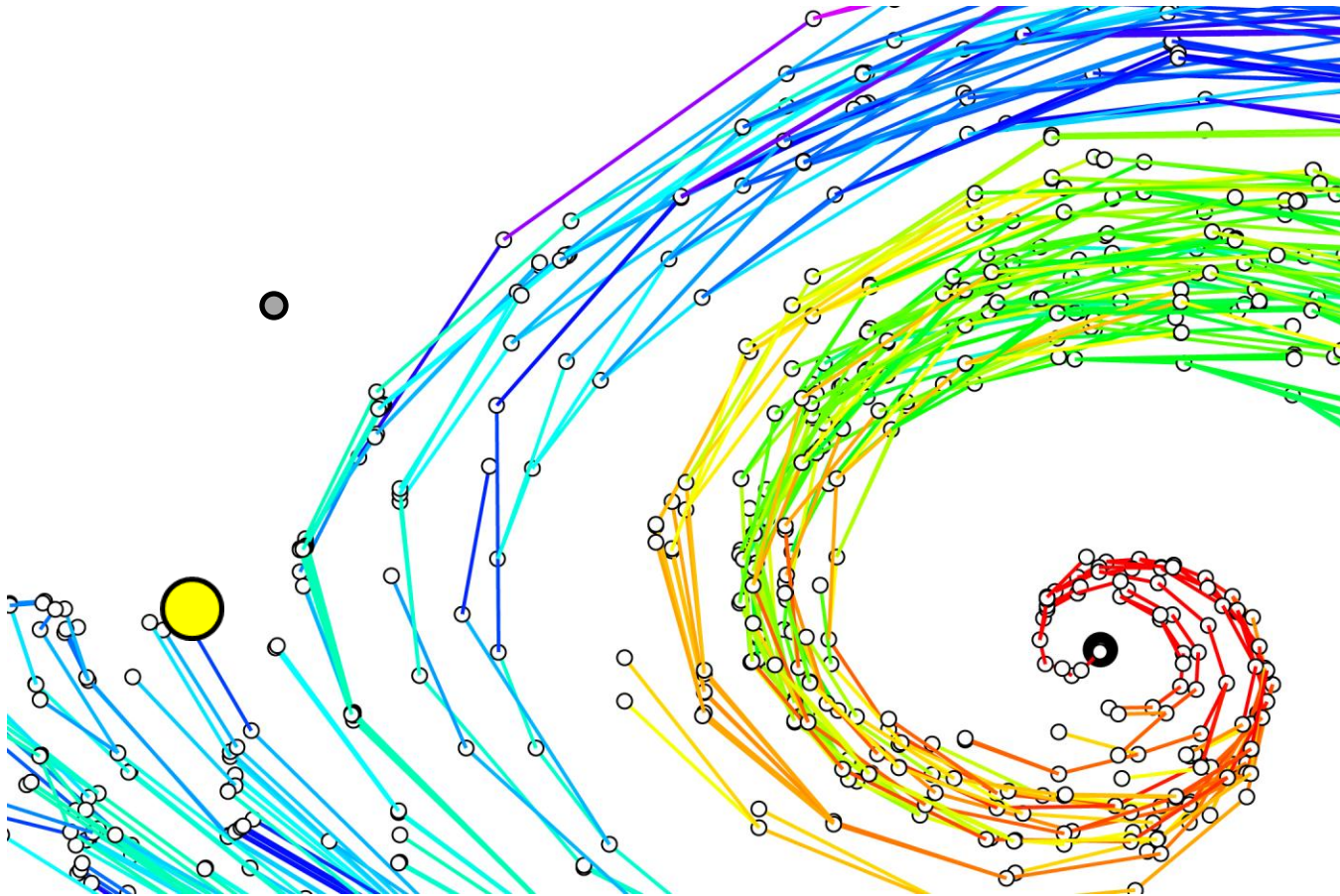
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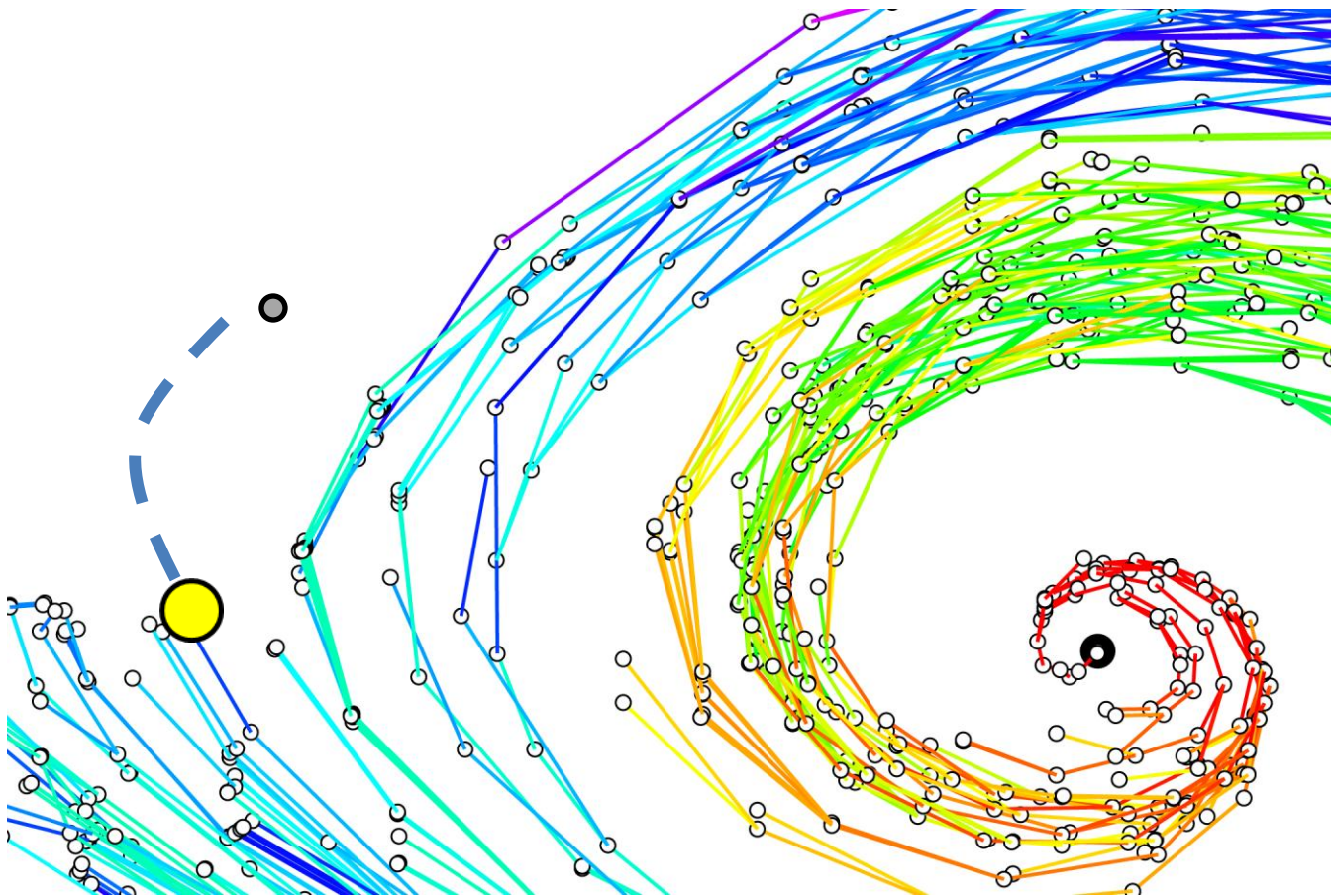
Motion Planning with Dynamics

Steer Function



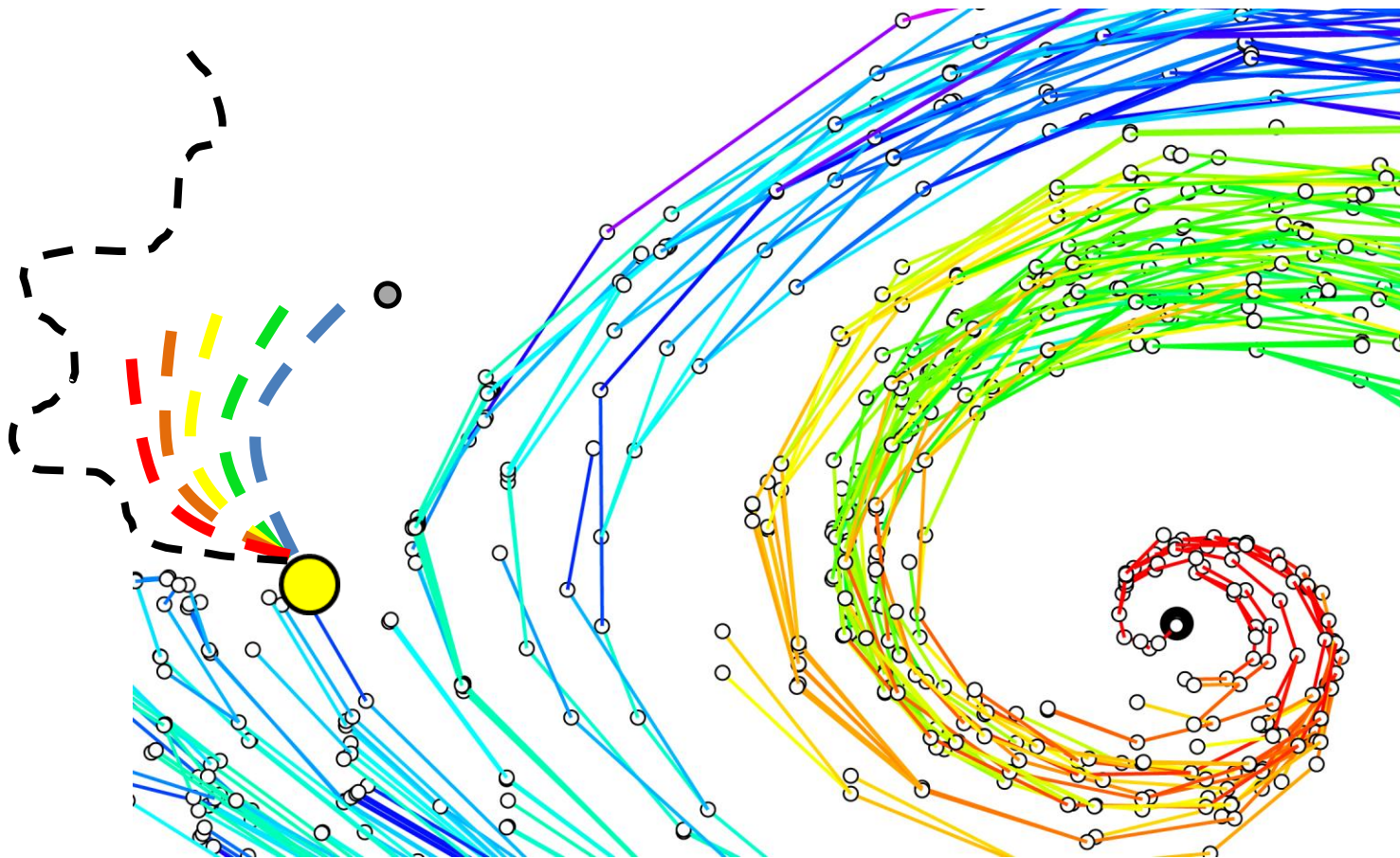
Motion Planning with Dynamics

Steer Function



Motion Planning with Dynamics

Steer Function



Motion Planning with Dynamics

Relevant Literature

Performance of RRT-based algorithms is sensitive to the distance metric used

S. M. LaValle, "From dynamic programming to RRTs: Algorithmic design of feasible trajectories," in *Control Problems in Robotics*. Springer-Verlag, 2002.

Asymptotic optimality of RRT* is shown

S. Karaman and E. Frazzoli, "Sampling-based algorithms for optimal motion planning," *International Journal of Robotics Research*, June 2011.

RRTs efficiently explore the state space only when this metric reflects the true cost-to-go

P. Cheng and S. M. LaValle, "Reducing metric sensitivity in randomized trajectory design," in *IEEE International Conference on Intelligent Robots and Systems*, 2001, pp. 43–48.

Cost-to-go functions can be based on optimal control for linearized systems

S. M. LaValle and J. J. Kuffner, "Randomized kinodynamic planning," *International Journal of Robotics Research*, vol. 20, no. 5, pp. 378–400, May 2001.

Cost-to-go pseudometric based on LQR is derived

E. Glassman and R. Tedrake, "A quadratic regulator-based heuristic for rapidly exploring state space," in *Proceedings of the IEEE International Conference on Robotics and Automation*, May 2010.

Sufficient conditions for optimality of RRT* for kinodynamics are presented

Karaman and Frazzoli, "Optimal Kinodynamic Motion Planning using Incremental Sampling-based Methods," in *IEEE Conference on Decision and Control (CDC)*, Atlanta, GA, 2010. .



Approach Overview

- Asymptotically optimal incremental sampling-based approach
- Linearize process dynamics about sampled state
- Quadratic cost-to-go metric
- Steer with approximated locally optimal policy

Linear Quadratic Regulation (LQR)

We will use the notation

$$[\mathbf{K}, \mathbf{S}] = \mathbf{LQR}(\mathbf{A}, \mathbf{B}, \mathbf{Q}, \mathbf{R}),$$

to denote the function that calculates the LQR gain matrix, \mathbf{K} , and the cost matrix, \mathbf{S} .

LQR-RRT*

Algorithm 4: LQR – RRT* $((V, E), N)$

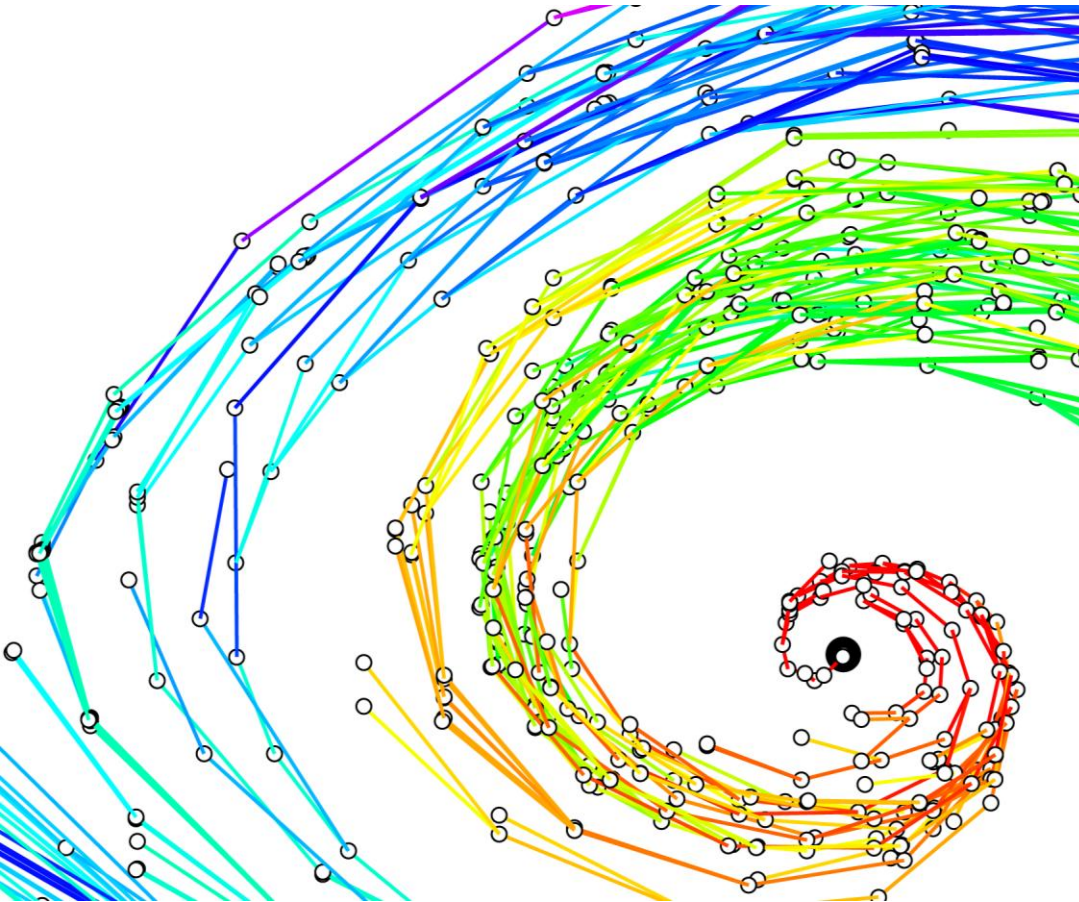
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2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
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5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if CollisionFree( $\sigma$ ) then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
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10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;
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Algorithm 5: ChooseParent($X_{\text{near}}, x_{\text{new}}$)


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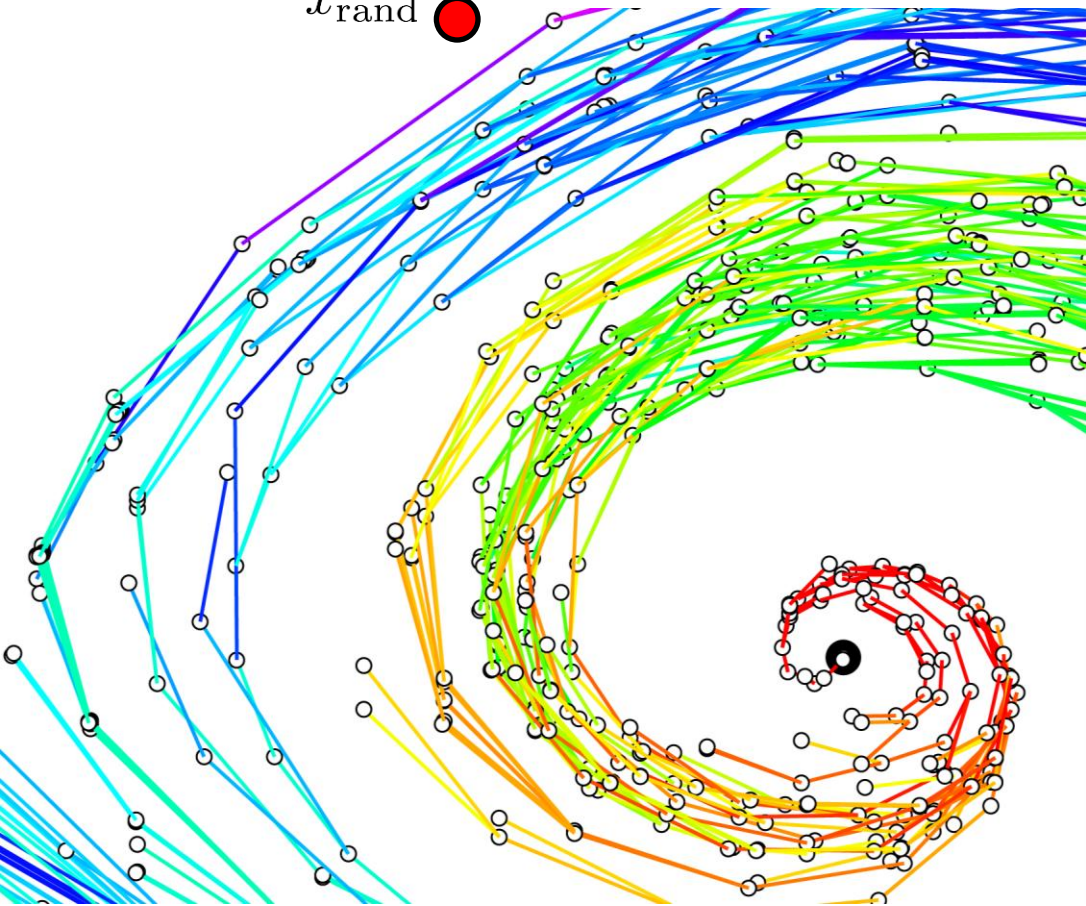
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LQR-RRT*

x_{rand} 



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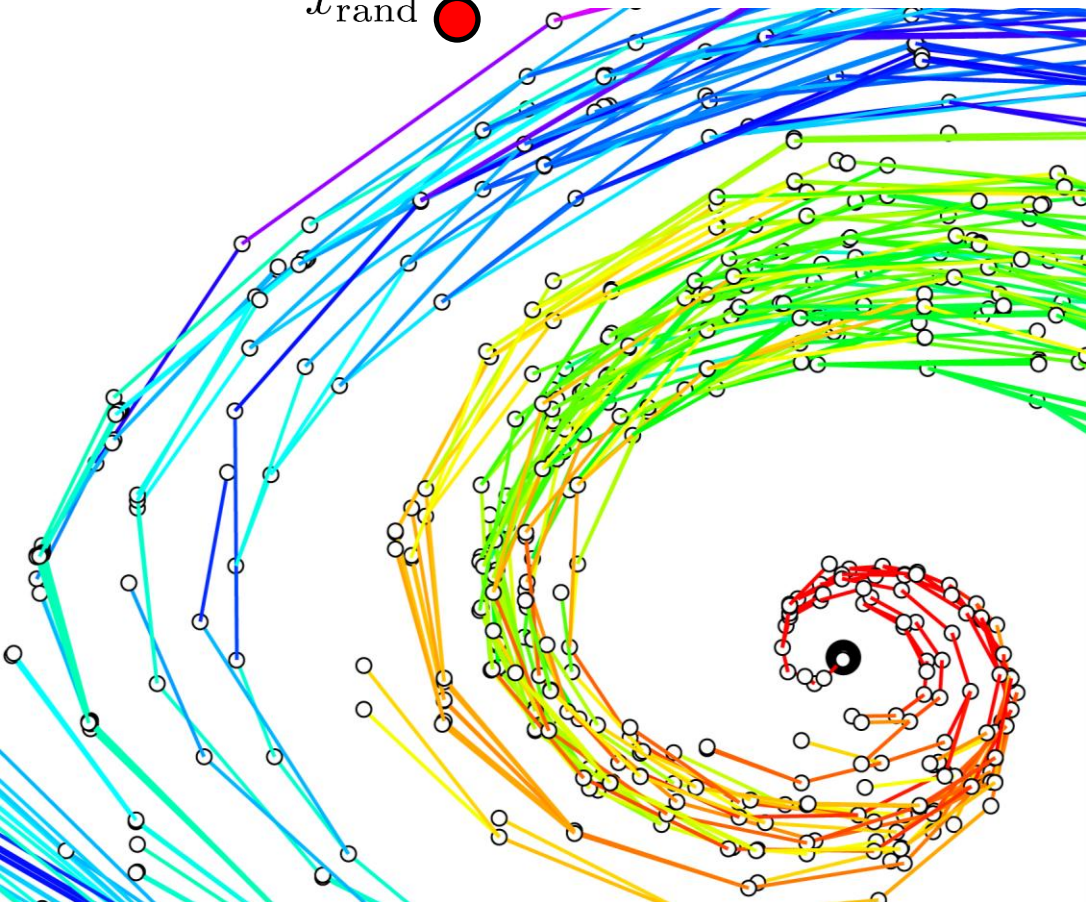
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x_{rand} 



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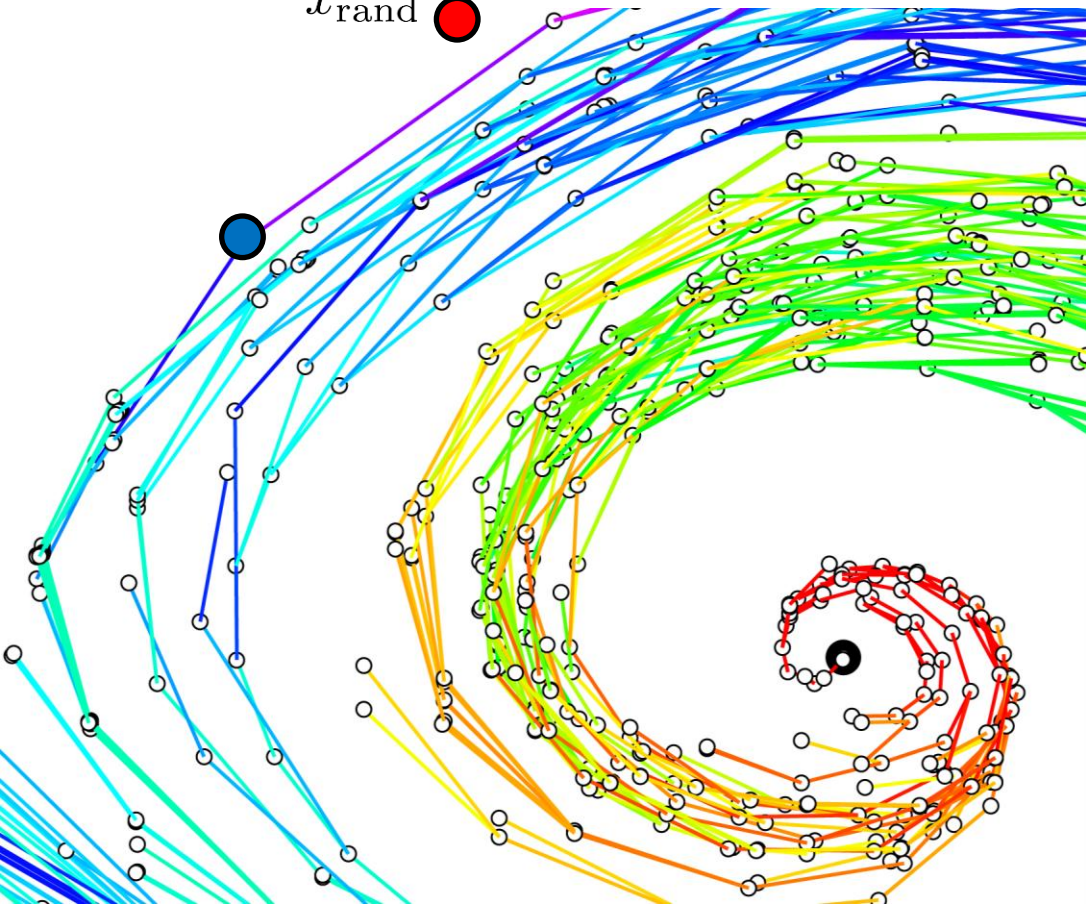
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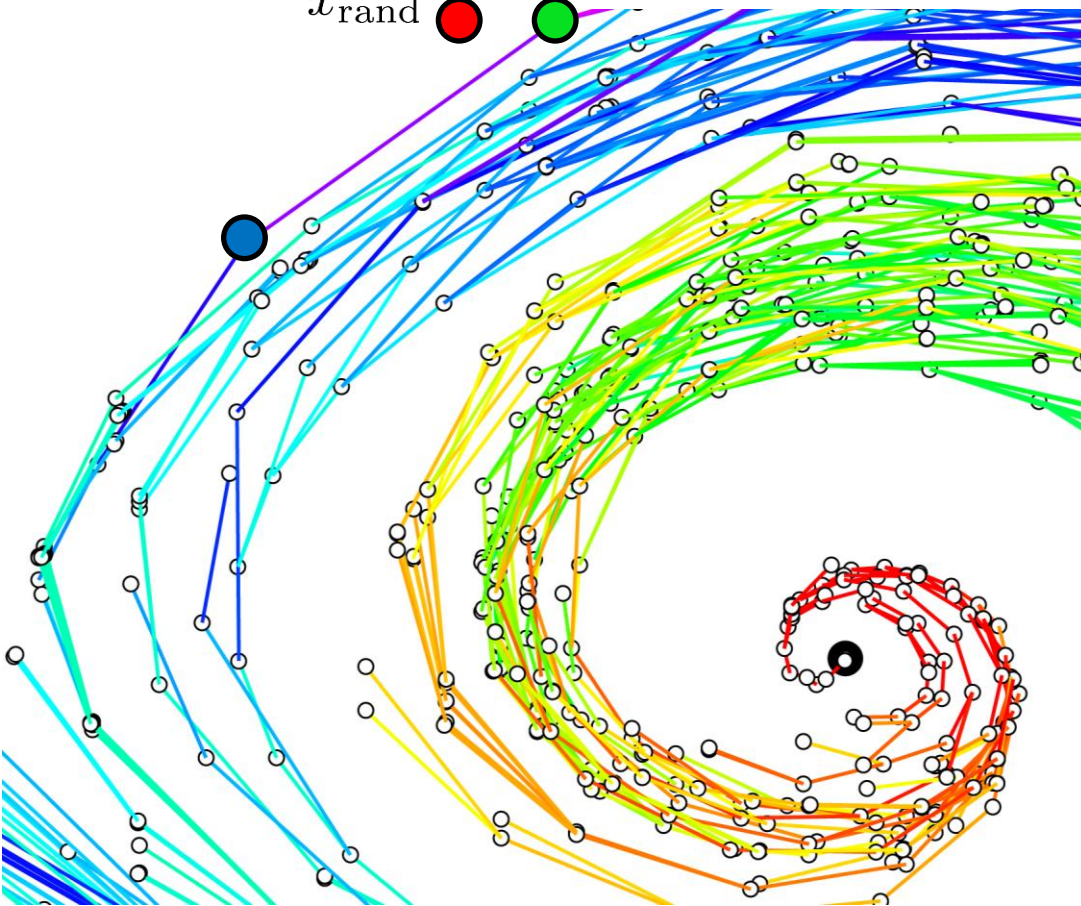
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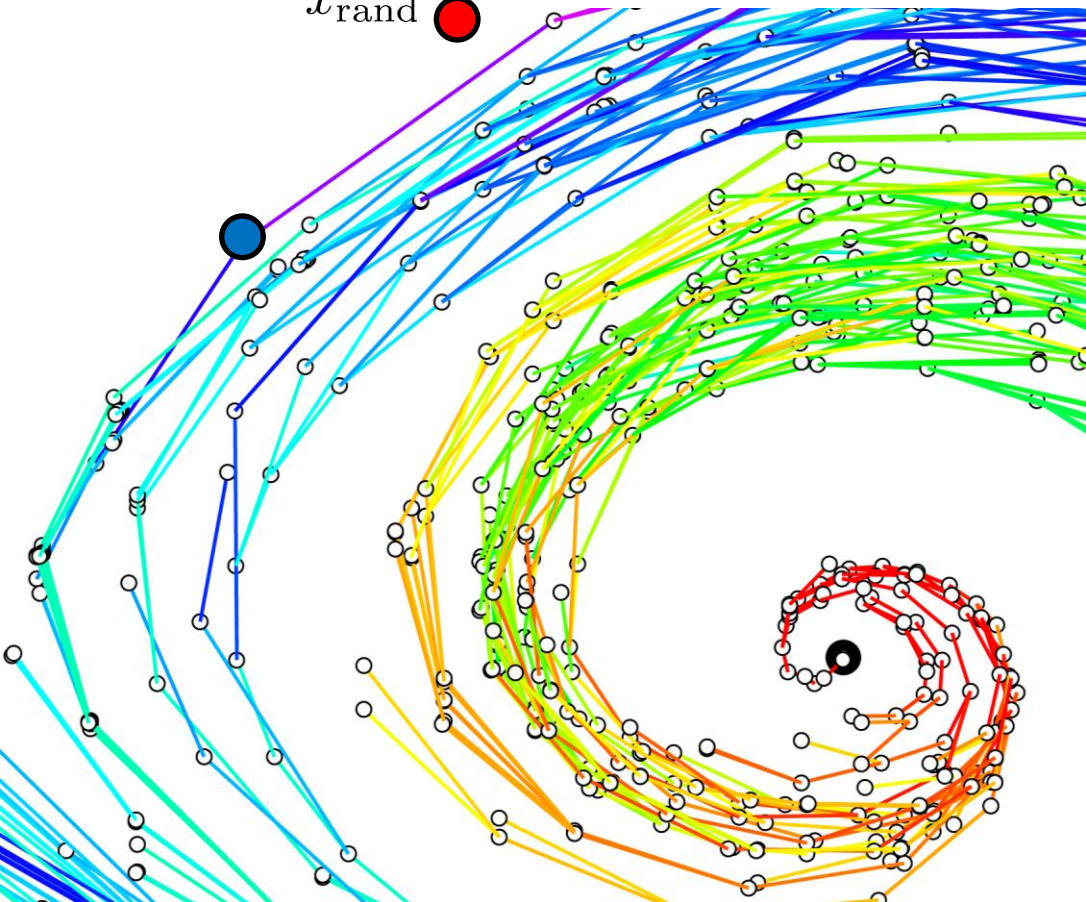
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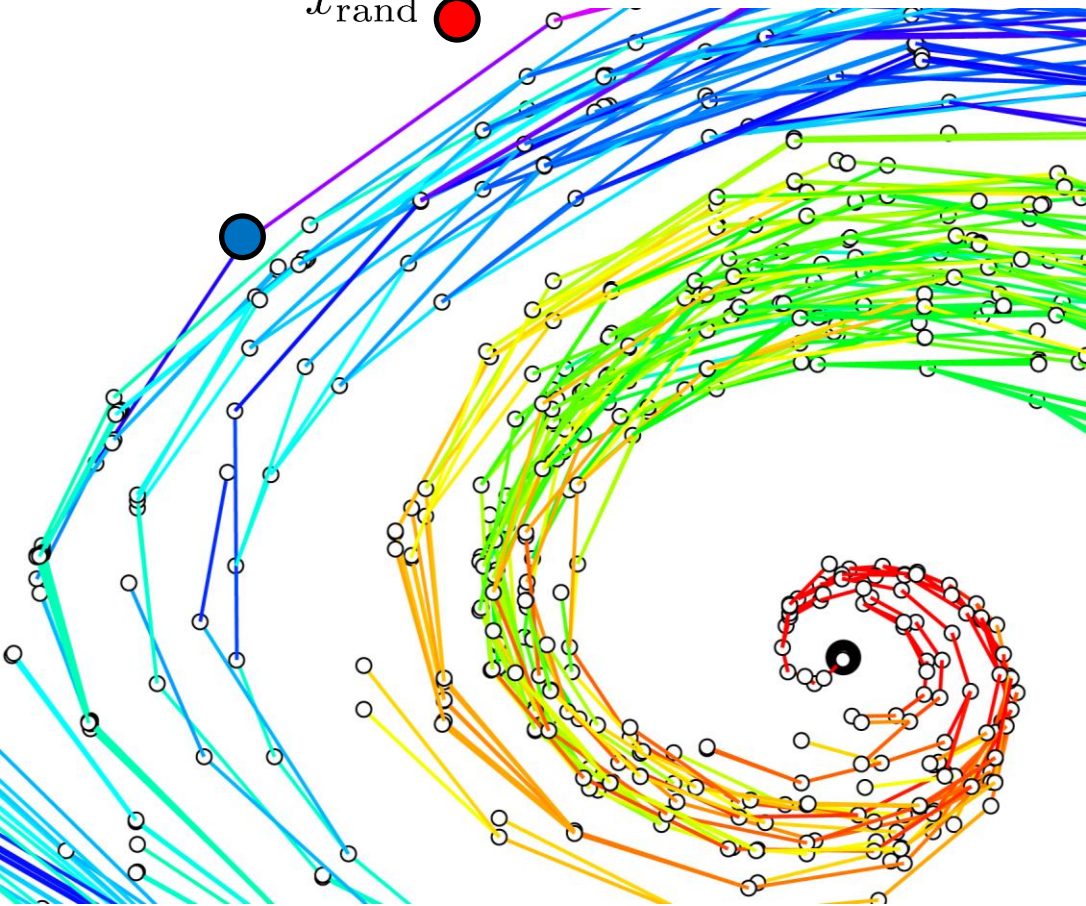
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3   if Cost( $x_{\text{new}}$ ) + Cost( $\sigma$ ) < Cost( $x_{\text{near}}$ ) then
4     if CollisionFree( $\sigma$ ) then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*

x_{rand}



Algorithm 4: LQR - RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if CollisionFree $(\sigma)$  then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent $(X_{\text{near}}, x_{\text{new}})$

```

1 minCost  $\leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if Cost $(x_{\text{near}}) + \text{Cost}(\sigma) < \text{minCost}$  then
5     minCost  $\leftarrow \text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma)$ ;
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

Algorithm 6: Rewire $((V, E), X_{\text{near}}, x_{\text{new}})$

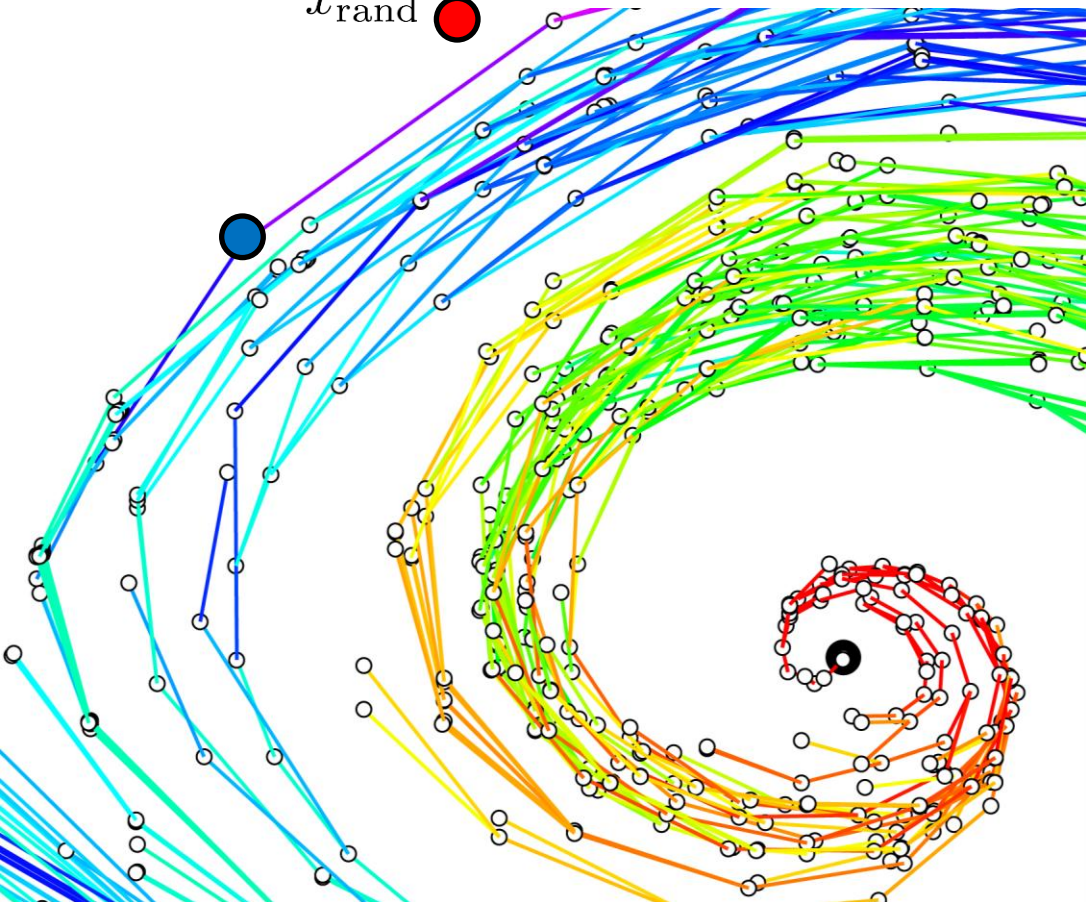
```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if Cost $(x_{\text{new}}) + \text{Cost}(\sigma) < \text{Cost}(x_{\text{near}})$  then
4     if CollisionFree $(\sigma)$  then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*

x_{rand}



Algorithm 4: LQR - RRT* $((V, E), N)$

```
1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if CollisionFree( $\sigma$ ) then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;
```

Algorithm 5: ChooseParent($X_{\text{near}}, x_{\text{new}}$)

```
1 minCost  $\leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ ) < minCost then
5     minCost  $\leftarrow$  Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ );
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;
```

Algorithm 6: Rewire($(V, E), X_{\text{near}}, x_{\text{new}}$)

```
1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if Cost( $x_{\text{new}}$ ) + Cost( $\sigma$ ) < Cost( $x_{\text{near}}$ ) then
4     if CollisionFree( $\sigma$ ) then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;
```

LQR-RRT*

Algorithm 4: LQR - RRT* $((V, E), N)$

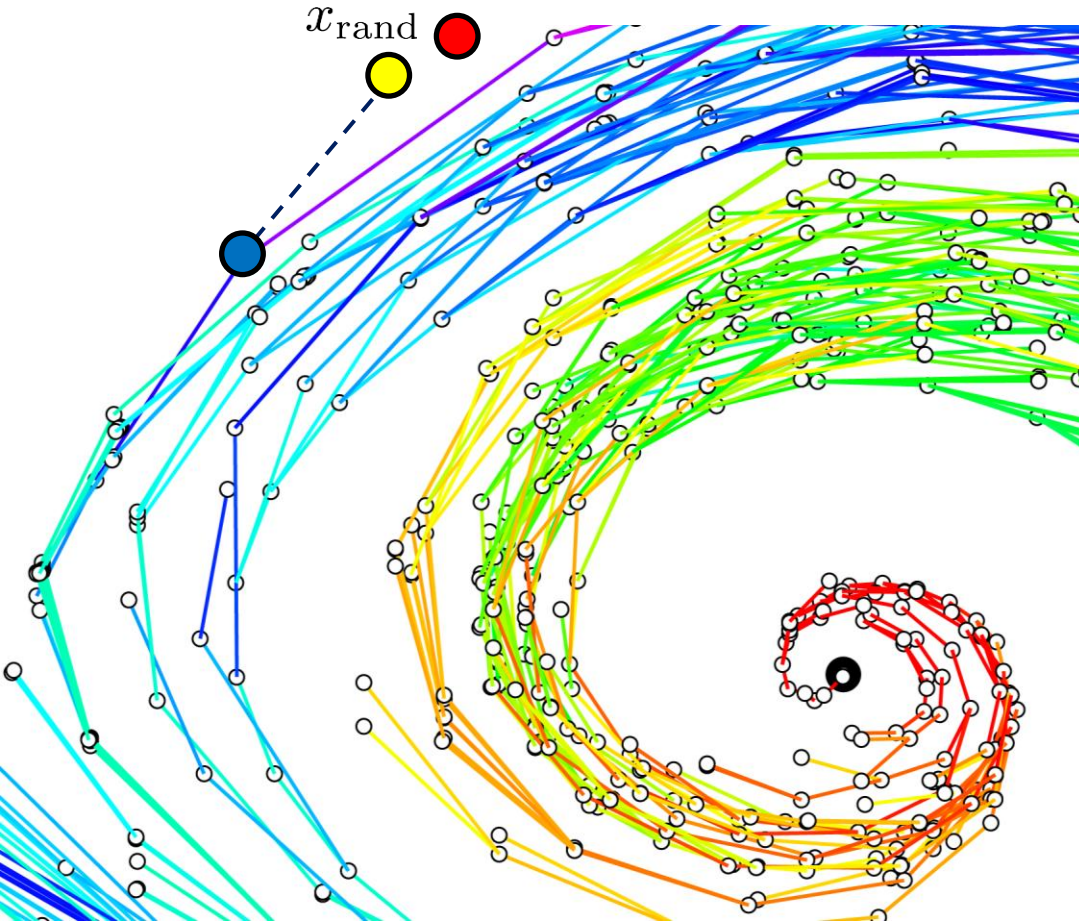
```
1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if CollisionFree( $\sigma$ ) then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;
```

Algorithm 5: ChooseParent($X_{\text{near}}, x_{\text{new}}$)

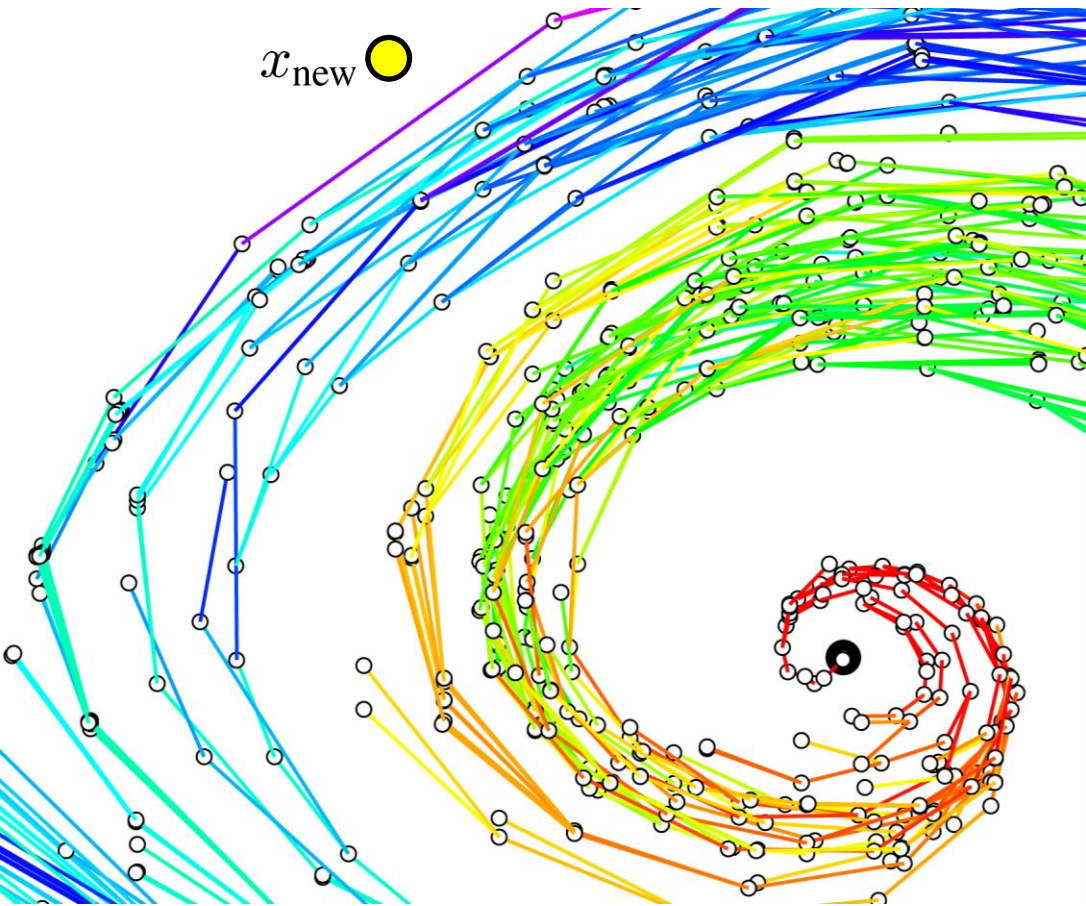
```
1 minCost  $\leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ ) < minCost then
5     minCost  $\leftarrow$  Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ );
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;
```

Algorithm 6: Rewire($(V, E), X_{\text{near}}, x_{\text{new}}$)

```
1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if Cost( $x_{\text{new}}$ ) + Cost( $\sigma$ ) < Cost( $x_{\text{near}}$ ) then
4     if CollisionFree( $\sigma$ ) then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;
```



LQR-RRT*



Algorithm 4: LQR - RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if CollisionFree $(\sigma)$  then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent $(X_{\text{near}}, x_{\text{new}})$

```

1 minCost  $\leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if Cost $(x_{\text{near}}) + \text{Cost}(\sigma) < \text{minCost}$  then
5     minCost  $\leftarrow \text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma)$ ;
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

Algorithm 6: Rewire $((V, E), X_{\text{near}}, x_{\text{new}})$

```

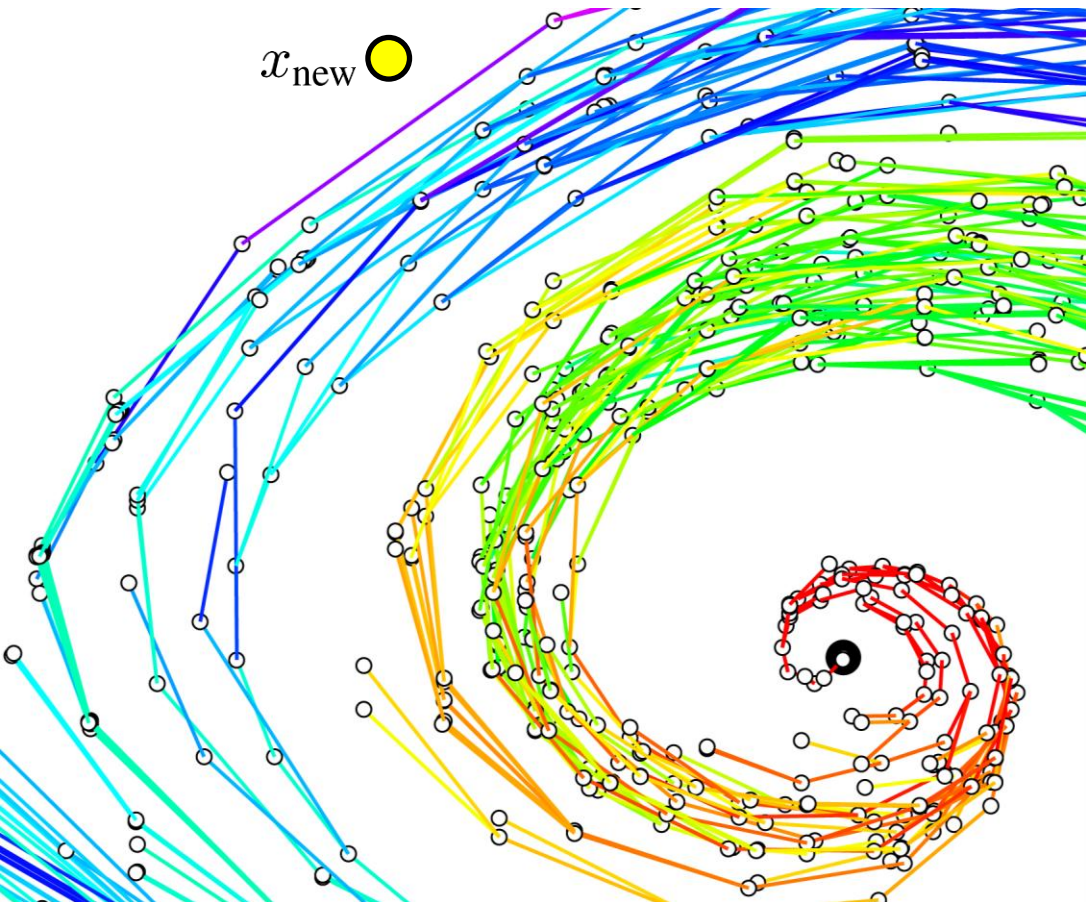
1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if Cost $(x_{\text{new}}) + \text{Cost}(\sigma) < \text{Cost}(x_{\text{near}})$  then
4     if CollisionFree $(\sigma)$  then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*

$$\text{LQRNear}(V, x) = \left\{ v \in V : (v - x)^T S (v - x) \leq \gamma \left(\frac{\log n}{n} \right)^{\frac{1}{d}} \right\}$$

x_{new} 



Algorithm 4: LQR - RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if CollisionFree( $\sigma$ ) then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent $(X_{\text{near}}, x_{\text{new}})$

```

1 minCost  $\leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ ) < minCost then
5     minCost  $\leftarrow$  Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ );
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

Algorithm 6: Rewire $((V, E), X_{\text{near}}, x_{\text{new}})$

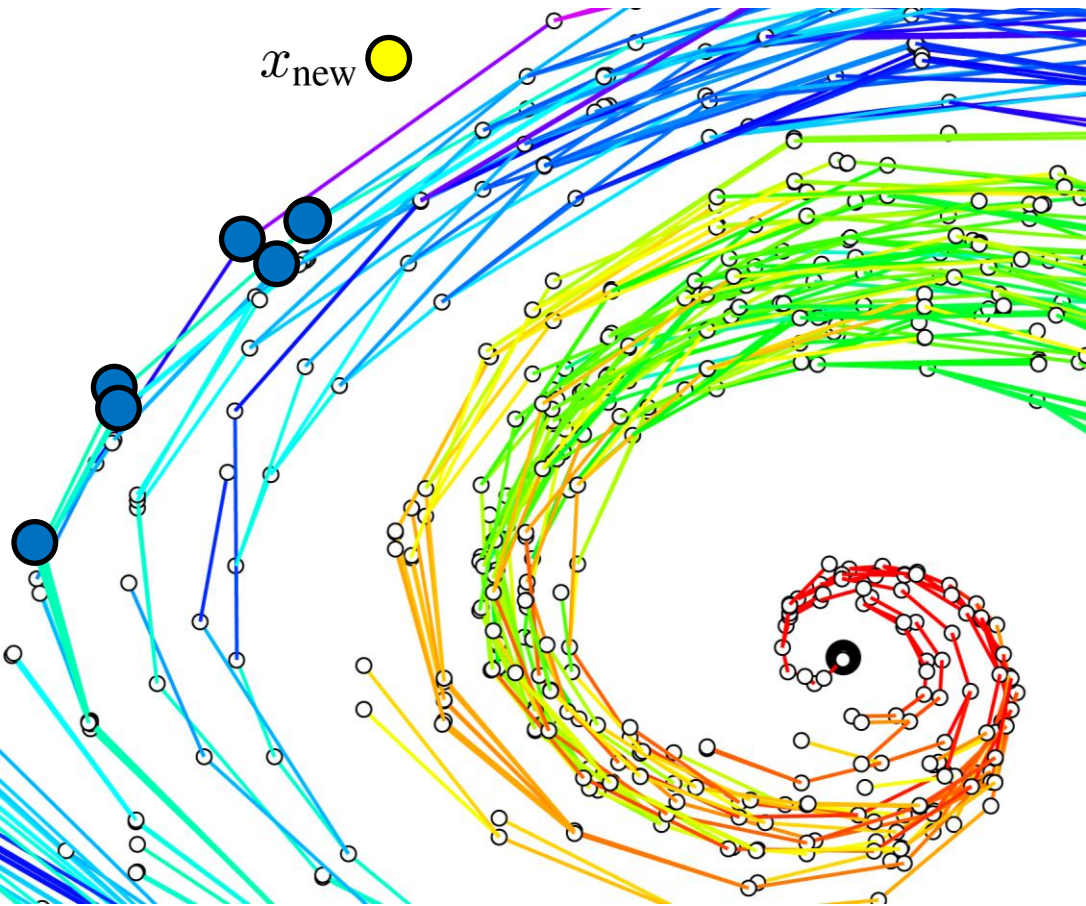
```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if Cost( $x_{\text{new}}$ ) + Cost( $\sigma$ ) < Cost( $x_{\text{near}}$ ) then
4     if CollisionFree( $\sigma$ ) then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*

$$\text{LQRNear}(V, x) = \left\{ v \in V : (v - x)^T S (v - x) \leq \gamma \left(\frac{\log n}{n} \right)^{\frac{1}{d}} \right\}$$



Algorithm 4: LQR - RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if CollisionFree( $\sigma$ ) then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent($X_{\text{near}}, x_{\text{new}}$)

```

1 minCost  $\leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ ) < minCost then
5     minCost  $\leftarrow$  Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ );
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

Algorithm 6: Rewire($(V, E), X_{\text{near}}, x_{\text{new}}$)

```

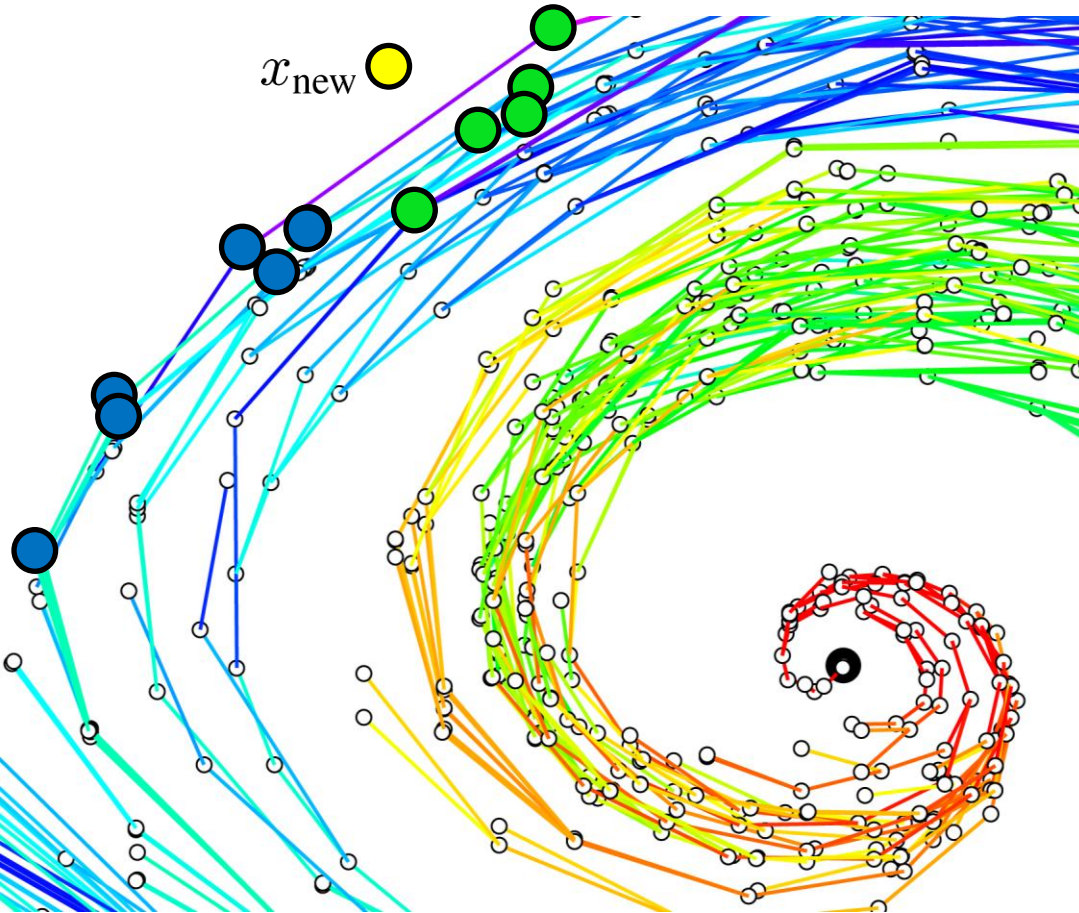
1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if Cost( $x_{\text{new}}$ ) + Cost( $\sigma$ ) < Cost( $x_{\text{near}}$ ) then
4     if CollisionFree( $\sigma$ ) then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*

LQRNear(V, x)

$$= \left\{ v \in V : (v - x)^T S (v - x) \leq \gamma \left(\frac{\log n}{n} \right)^{\frac{1}{d}} \right\}$$



Algorithm 4: LQR - RRT* ($(V, E), N$)

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if CollisionFree( $\sigma$ ) then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent($X_{\text{near}}, x_{\text{new}}$)

```

1 minCost  $\leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ ) < minCost then
5     minCost  $\leftarrow$  Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ );
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

Algorithm 6: Rewire($(V, E), X_{\text{near}}, x_{\text{new}}$)

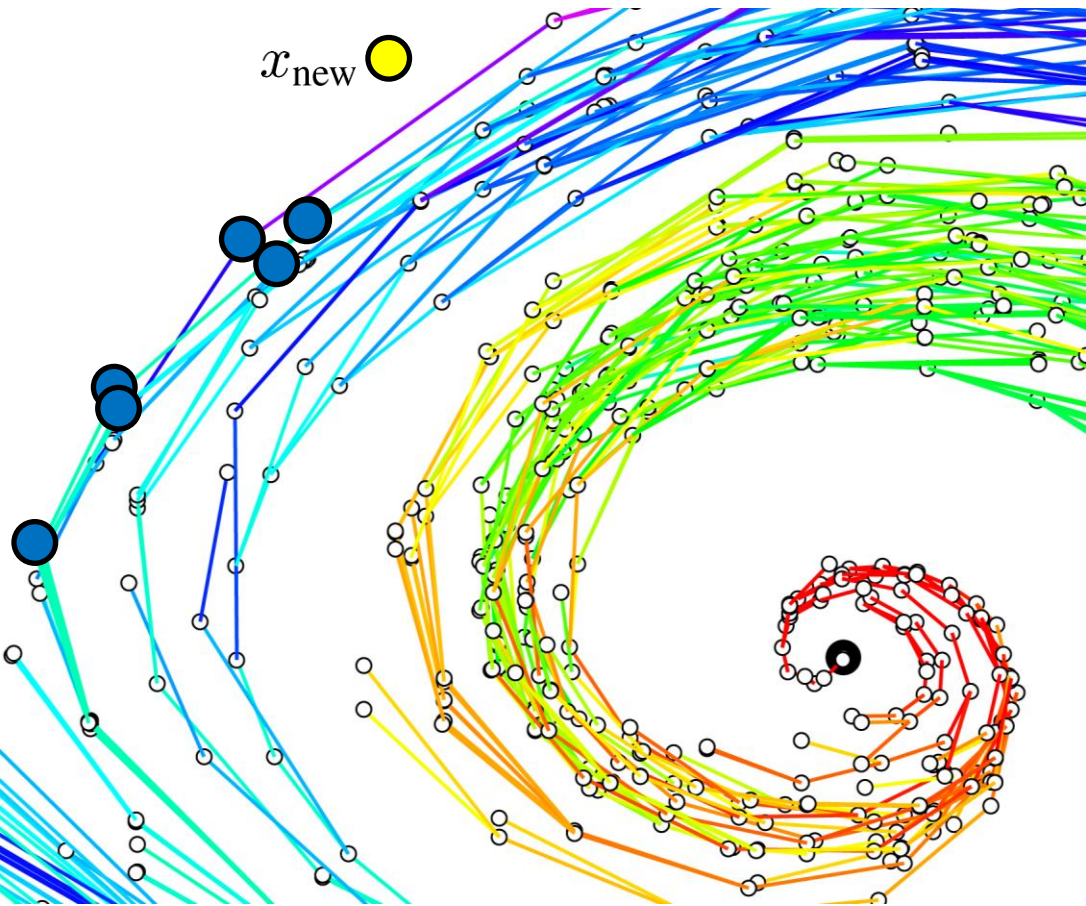
```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if Cost( $x_{\text{new}}$ ) + Cost( $\sigma$ ) < Cost( $x_{\text{near}}$ ) then
4     if CollisionFree( $\sigma$ ) then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*

$$\text{LQRNear}(V, x) = \left\{ v \in V : (v - x)^T S (v - x) \leq \gamma \left(\frac{\log n}{n} \right)^{\frac{1}{d}} \right\}$$



Algorithm 4: LQR - RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if CollisionFree( $\sigma$ ) then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent($X_{\text{near}}, x_{\text{new}}$)

```

1 minCost  $\leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ ) < minCost then
5     minCost  $\leftarrow$  Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ );
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

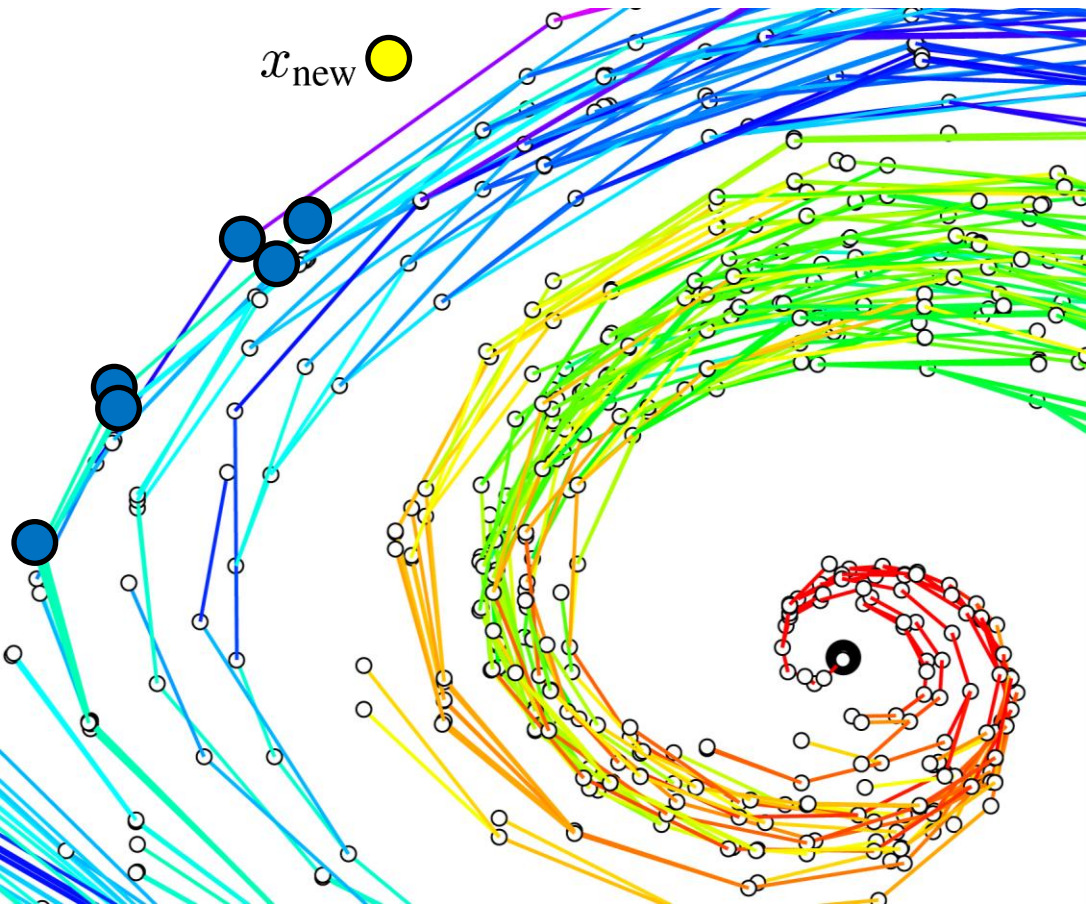
Algorithm 6: Rewire($(V, E), X_{\text{near}}, x_{\text{new}}$)

```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if Cost( $x_{\text{new}}$ ) + Cost( $\sigma$ ) < Cost( $x_{\text{near}}$ ) then
4     if CollisionFree( $\sigma$ ) then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*



Algorithm 4: LQR – RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if CollisionFree( $\sigma$ ) then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent($X_{\text{near}}, x_{\text{new}}$)

```

1 minCost  $\leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ ) < minCost then
5     minCost  $\leftarrow$  Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ );
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

Algorithm 6: Rewire($(V, E), X_{\text{near}}, x_{\text{new}}$)

```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if Cost( $x_{\text{new}}$ ) + Cost( $\sigma$ ) < Cost( $x_{\text{near}}$ ) then
4     if CollisionFree( $\sigma$ ) then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*

Algorithm 4: LQR - RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if CollisionFree( $\sigma$ ) then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent($X_{\text{near}}, x_{\text{new}}$)

```

1 minCost  $\leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ ) < minCost then
5     minCost  $\leftarrow$  Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ );
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

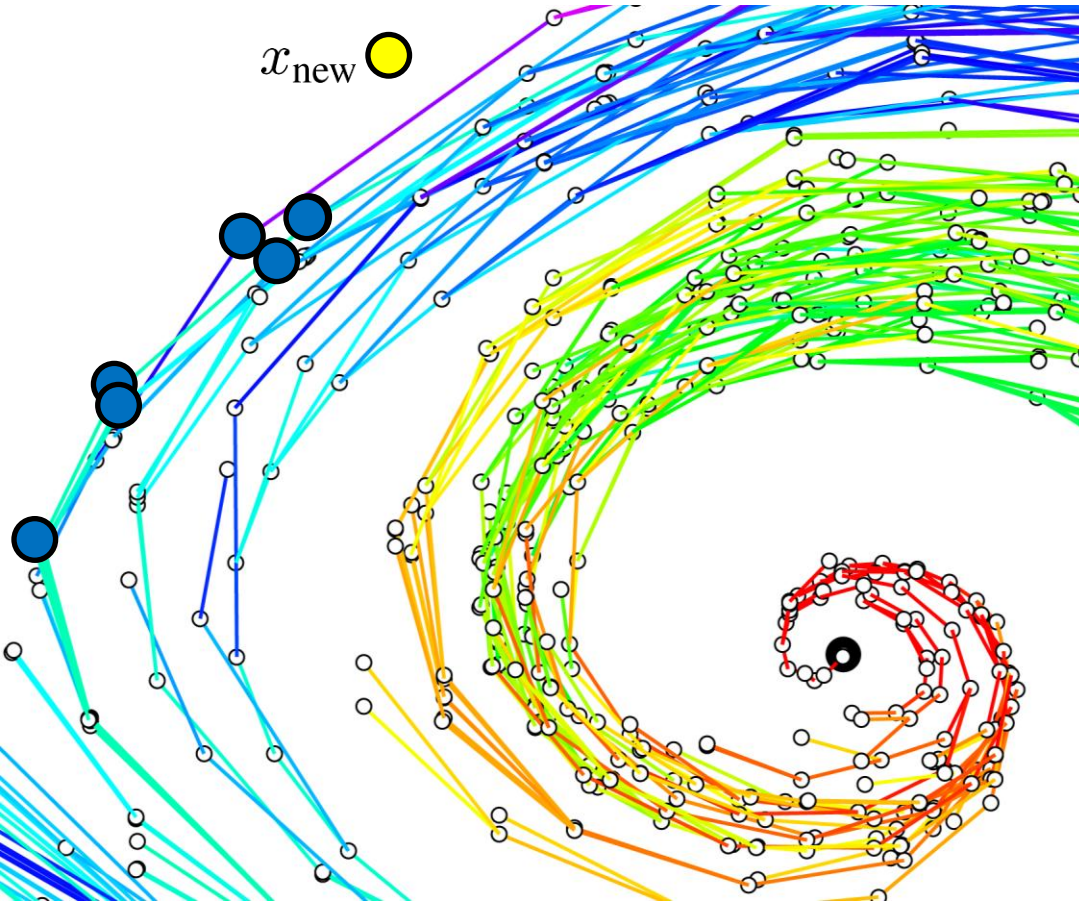
```

Algorithm 6: Rewire($(V, E), X_{\text{near}}, x_{\text{new}}$)

```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if Cost( $x_{\text{new}}$ ) + Cost( $\sigma$ ) < Cost( $x_{\text{near}}$ ) then
4     if CollisionFree( $\sigma$ ) then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```



LQR-RRT*

Algorithm 4: LQR – RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if CollisionFree( $\sigma$ ) then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}(V, E, X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent($X_{\text{near}}, x_{\text{new}}$)

```

1 minCost  $\leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ ) < minCost then
5     minCost  $\leftarrow$  Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ );
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

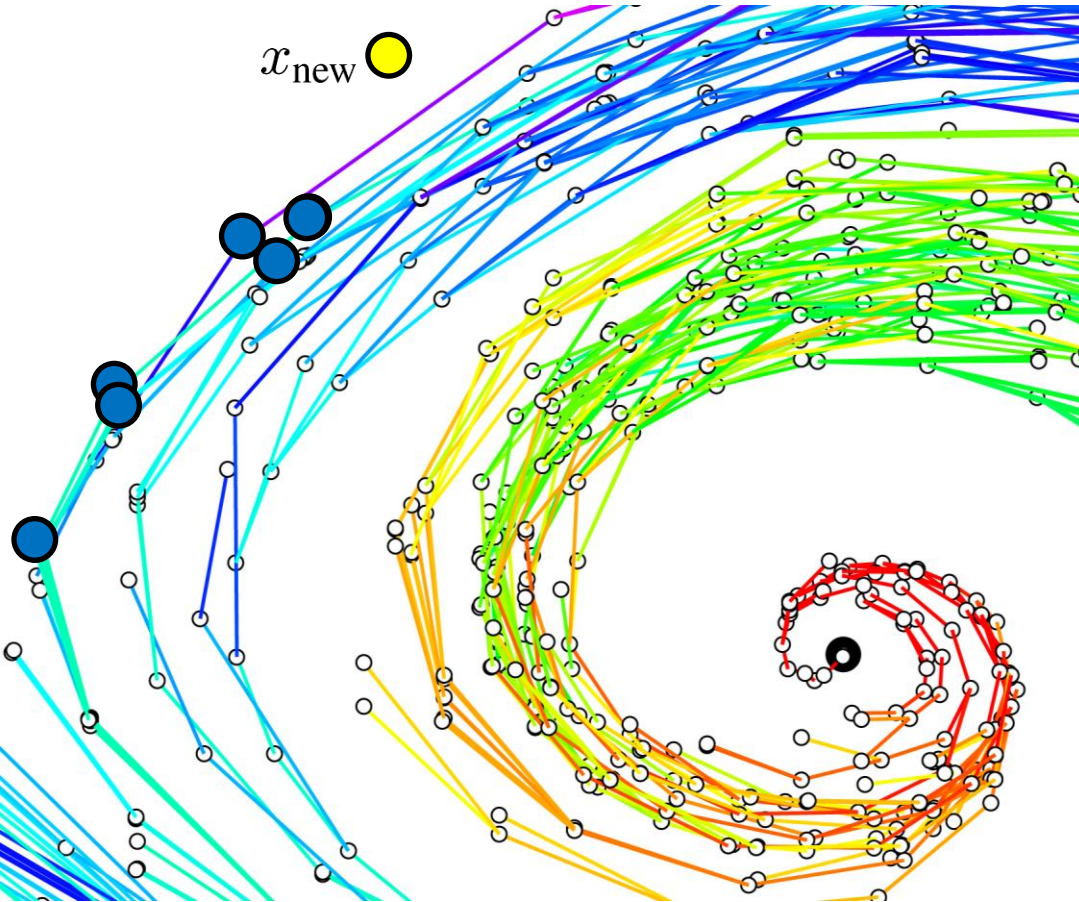
```

Algorithm 6: Rewire($(V, E), X_{\text{near}}, x_{\text{new}}$)

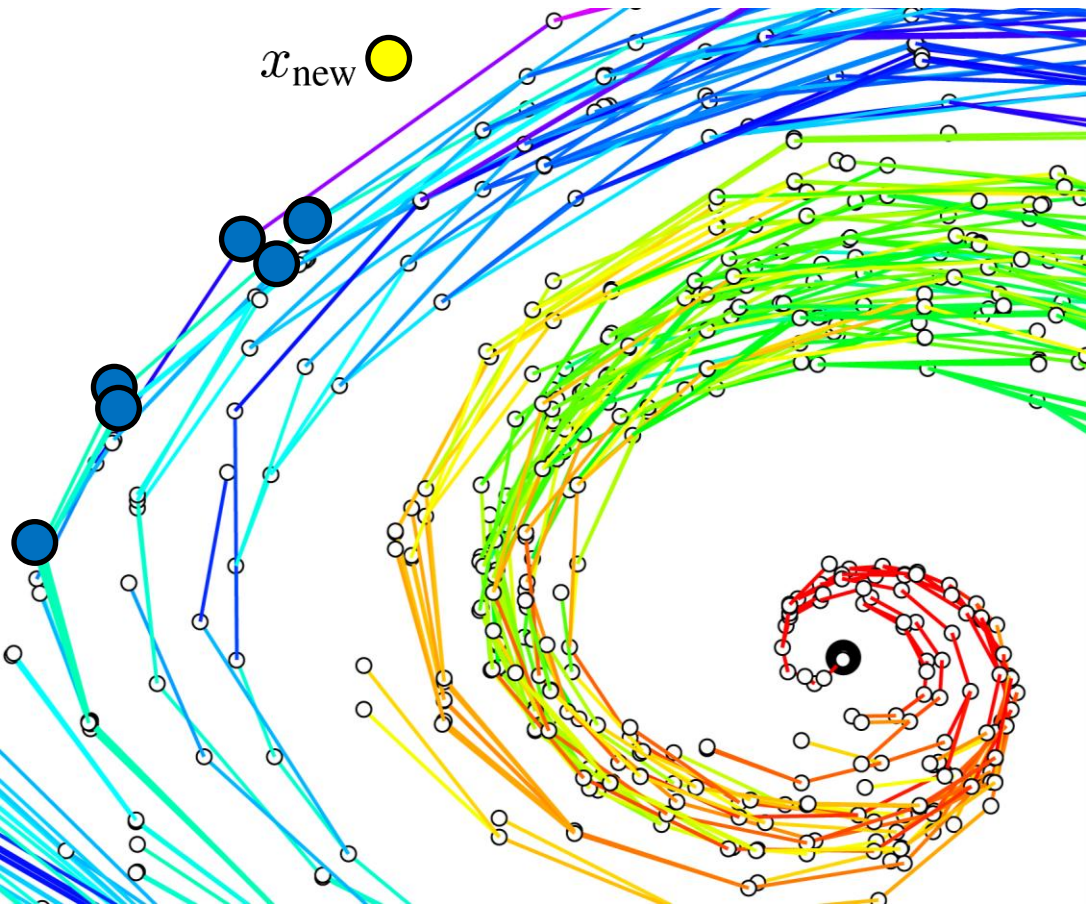
```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if Cost( $x_{\text{new}}$ ) + Cost( $\sigma$ ) < Cost( $x_{\text{near}}$ ) then
4     if CollisionFree( $\sigma$ ) then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```



LQR-RRT*



Algorithm 4: LQR – RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if  $\text{CollisionFree}(\sigma)$  then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: $\text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$

```

1  $\text{minCost} \leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if  $\text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma) < \text{minCost}$  then
5      $\text{minCost} \leftarrow \text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma)$ ;
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

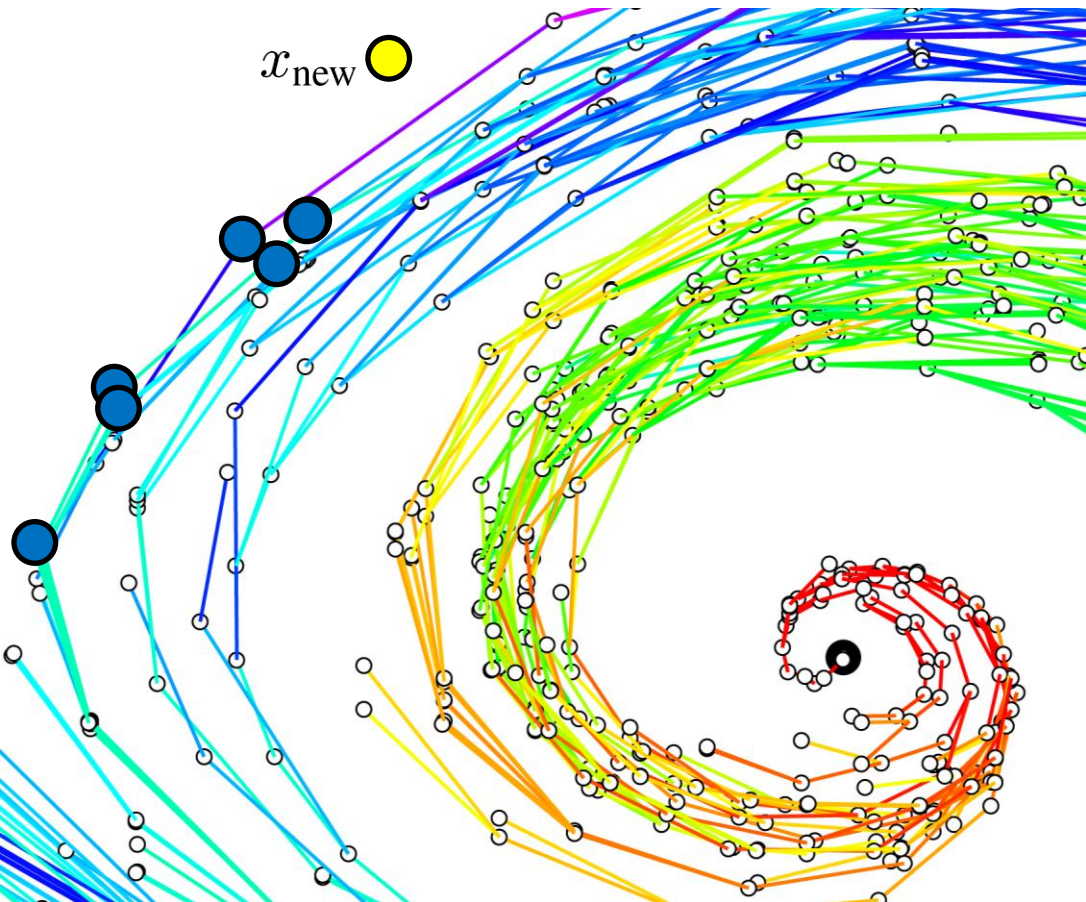
Algorithm 6: $\text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$

```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if  $\text{Cost}(x_{\text{new}}) + \text{Cost}(\sigma) < \text{Cost}(x_{\text{near}})$  then
4     if  $\text{CollisionFree}(\sigma)$  then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*



Algorithm 4: LQR – RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if CollisionFree( $\sigma$ ) then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent($X_{\text{near}}, x_{\text{new}}$)

```

1 minCost  $\leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ ) < minCost then
5     minCost  $\leftarrow$  Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ );
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

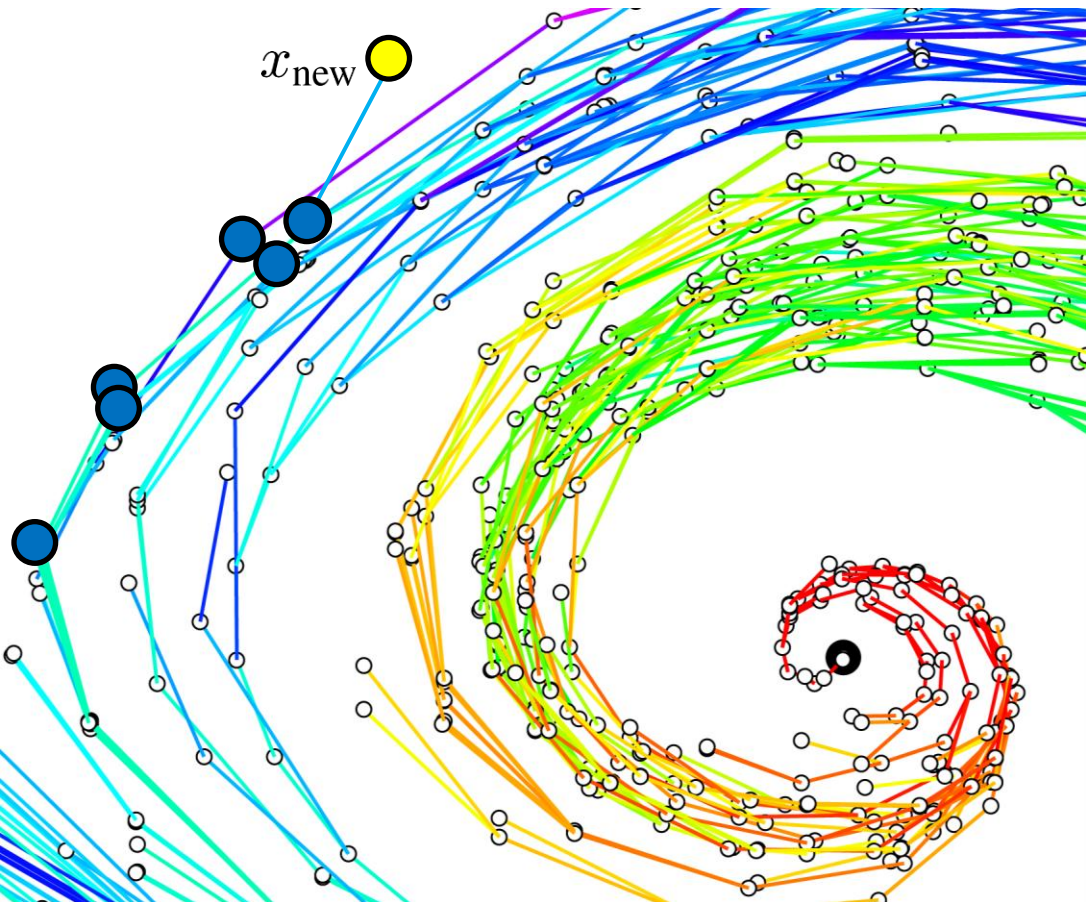
Algorithm 6: Rewire($(V, E), X_{\text{near}}, x_{\text{new}}$)

```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if Cost( $x_{\text{new}}$ ) + Cost( $\sigma$ ) < Cost( $x_{\text{near}}$ ) then
4     if CollisionFree( $\sigma$ ) then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*



Algorithm 4: LQR – RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if CollisionFree( $\sigma$ ) then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent($X_{\text{near}}, x_{\text{new}}$)

```

1 minCost  $\leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ ) < minCost then
5     minCost  $\leftarrow$  Cost( $x_{\text{near}}$ ) + Cost( $\sigma$ );
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

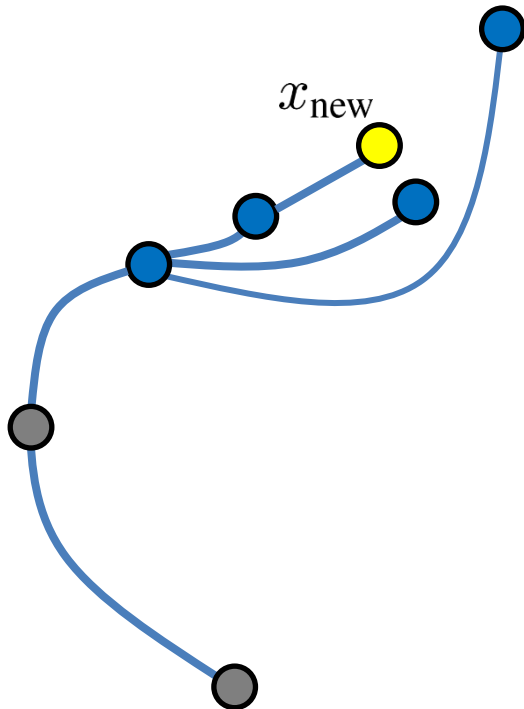
Algorithm 6: Rewire($(V, E), X_{\text{near}}, x_{\text{new}}$)

```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if Cost( $x_{\text{new}}$ ) + Cost( $\sigma$ ) < Cost( $x_{\text{near}}$ ) then
4     if CollisionFree( $\sigma$ ) then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*



Algorithm 4: LQR – RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if  $\text{CollisionFree}(\sigma)$  then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent $(X_{\text{near}}, x_{\text{new}})$

```

1  $\text{minCost} \leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if  $\text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma) < \text{minCost}$  then
5      $\text{minCost} \leftarrow \text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma)$ ;
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

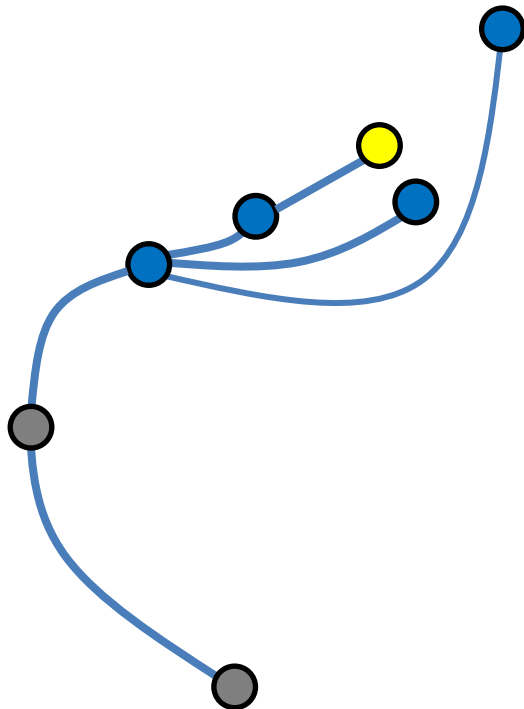
Algorithm 6: Rewire $((V, E), X_{\text{near}}, x_{\text{new}})$

```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if  $\text{Cost}(x_{\text{new}}) + \text{Cost}(\sigma) < \text{Cost}(x_{\text{near}})$  then
4     if  $\text{CollisionFree}(\sigma)$  then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*



Algorithm 4: LQR – RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if  $\text{CollisionFree}(\sigma)$  then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: $\text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$

```

1  $\text{minCost} \leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if  $\text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma) < \text{minCost}$  then
5      $\text{minCost} \leftarrow \text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma)$ ;
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

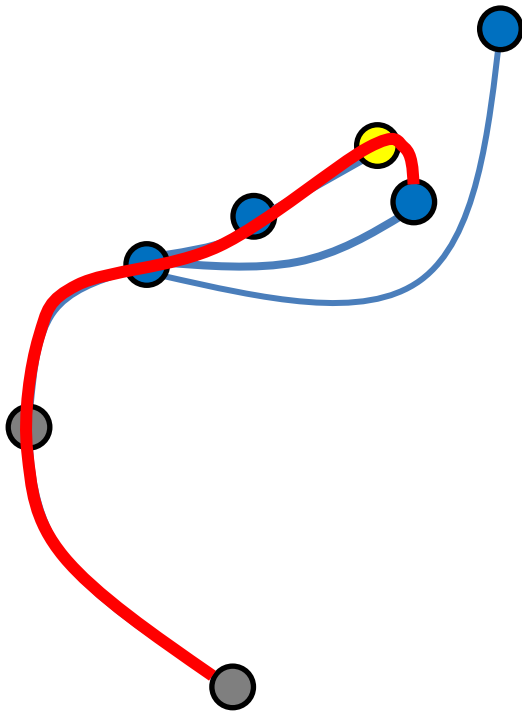
Algorithm 6: $\text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$

```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if  $\text{Cost}(x_{\text{new}}) + \text{Cost}(\sigma) < \text{Cost}(x_{\text{near}})$  then
4     if  $\text{CollisionFree}(\sigma)$  then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*



Algorithm 4: LQR – RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if  $\text{CollisionFree}(\sigma)$  then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent $(X_{\text{near}}, x_{\text{new}})$

```

1  $\text{minCost} \leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if  $\text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma) < \text{minCost}$  then
5      $\text{minCost} \leftarrow \text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma)$ ;
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

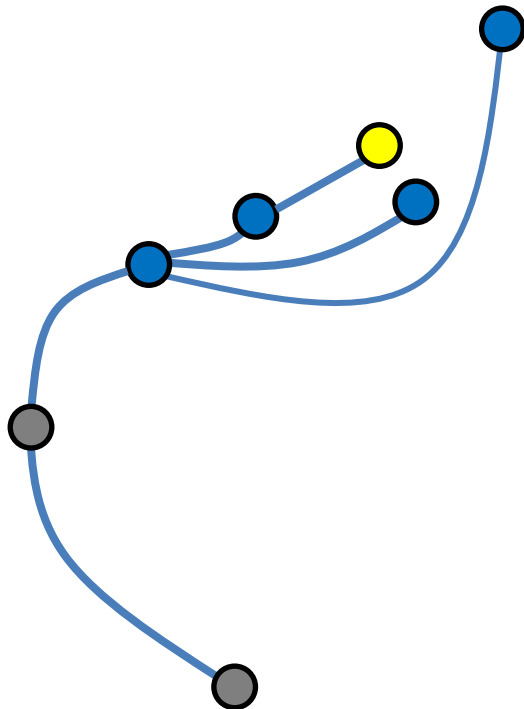
Algorithm 6: Rewire $((V, E), X_{\text{near}}, x_{\text{new}})$

```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if  $\text{Cost}(x_{\text{new}}) + \text{Cost}(\sigma) < \text{Cost}(x_{\text{near}})$  then
4     if  $\text{CollisionFree}(\sigma)$  then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*



Algorithm 4: LQR – RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if  $\text{CollisionFree}(\sigma)$  then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent $(X_{\text{near}}, x_{\text{new}})$

```

1  $\text{minCost} \leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if  $\text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma) < \text{minCost}$  then
5      $\text{minCost} \leftarrow \text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma)$ ;
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

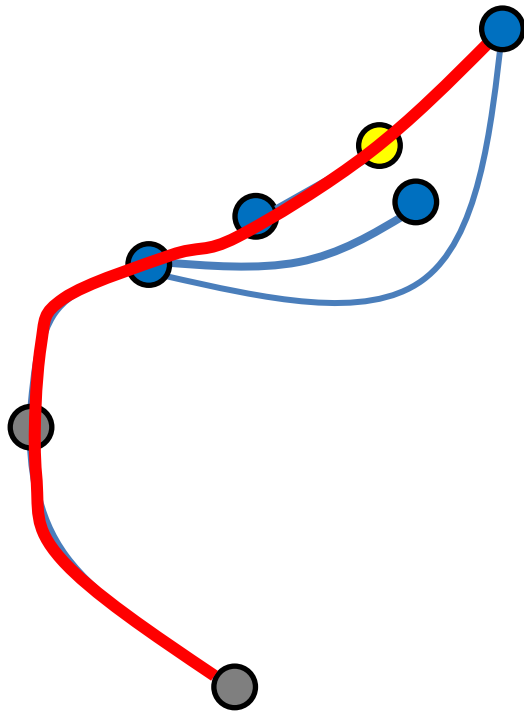
Algorithm 6: Rewire $((V, E), X_{\text{near}}, x_{\text{new}})$

```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if  $\text{Cost}(x_{\text{new}}) + \text{Cost}(\sigma) < \text{Cost}(x_{\text{near}})$  then
4     if  $\text{CollisionFree}(\sigma)$  then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*



Algorithm 4: LQR – RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if  $\text{CollisionFree}(\sigma)$  then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent $(X_{\text{near}}, x_{\text{new}})$

```

1  $\text{minCost} \leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if  $\text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma) < \text{minCost}$  then
5      $\text{minCost} \leftarrow \text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma)$ ;
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

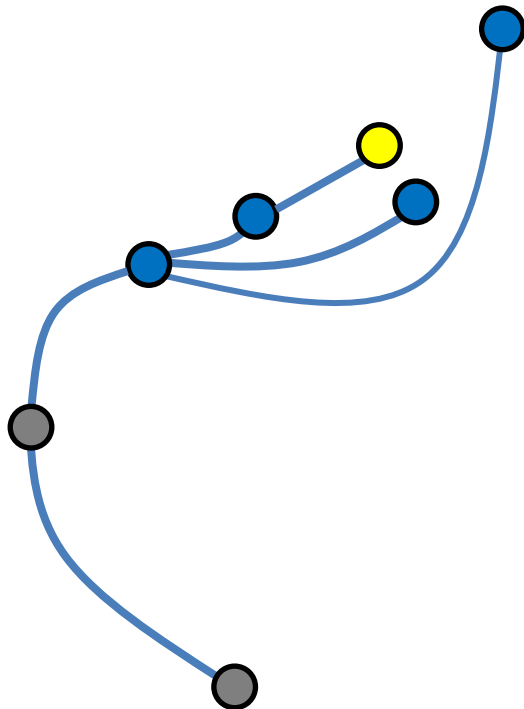
Algorithm 6: Rewire $((V, E), X_{\text{near}}, x_{\text{new}})$

```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
2    $\sigma \leftarrow \text{LQRSteer}(x_{\text{new}}, x_{\text{near}})$ ;
3   if  $\text{Cost}(x_{\text{new}}) + \text{Cost}(\sigma) < \text{Cost}(x_{\text{near}})$  then
4     if  $\text{CollisionFree}(\sigma)$  then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*



Algorithm 4: LQR – RRT* $((V, E), N)$

```

1 for  $i = 1, \dots, N$  do
2    $x_{\text{rand}} \leftarrow \text{Sample}$ ;
3    $x_{\text{nearest}} \leftarrow \text{LQRNearest}(V, x_{\text{rand}})$ ;
4    $x_{\text{new}} \leftarrow \text{LQRSteer}(x_{\text{nearest}}, x_{\text{rand}})$ ;
5    $X_{\text{near}} \leftarrow \text{LQRNear}(V, x_{\text{new}})$ ;
6    $(x_{\text{min}}, \sigma_{\text{min}}) \leftarrow \text{ChooseParent}(X_{\text{near}}, x_{\text{new}})$ ;
7   if  $\text{CollisionFree}(\sigma)$  then
8      $X \leftarrow X \cup \{x_{\text{new}}\}$ ;
9      $E \leftarrow E \cup \{(x_{\text{min}}, x_{\text{new}})\}$ ;
10     $(V, E) \leftarrow \text{Rewire}((V, E), X_{\text{near}}, x_{\text{new}})$ ;
11 return  $G = (V, E)$ ;

```

Algorithm 5: ChooseParent $(X_{\text{near}}, x_{\text{new}})$

```

1  $\text{minCost} \leftarrow \infty$ ;  $x_{\text{min}} \leftarrow \text{NULL}$ ;  $\sigma_{\text{min}} \leftarrow \text{NULL}$ ;
2 for  $x_{\text{near}} \in X_{\text{near}}$  do
3    $\sigma \leftarrow \text{LQRSteer}(x_{\text{near}}, x_{\text{new}})$ ;
4   if  $\text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma) < \text{minCost}$  then
5      $\text{minCost} \leftarrow \text{Cost}(x_{\text{near}}) + \text{Cost}(\sigma)$ ;
6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

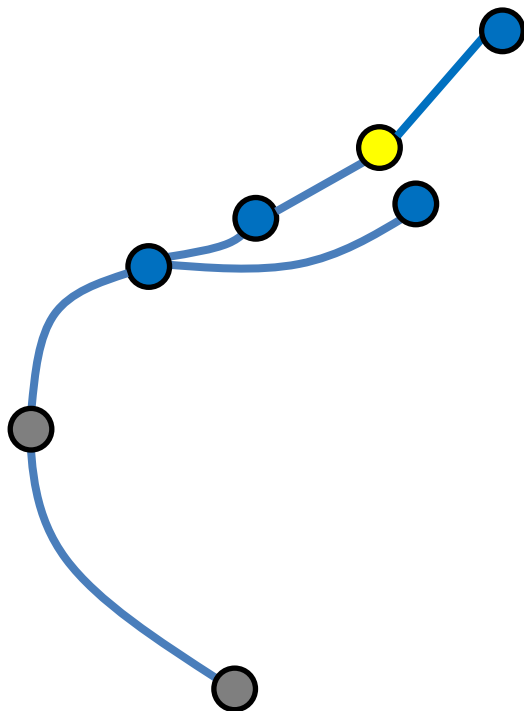
Algorithm 6: Rewire $((V, E), X_{\text{near}}, x_{\text{new}})$

```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
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3   if  $\text{Cost}(x_{\text{new}}) + \text{Cost}(\sigma) < \text{Cost}(x_{\text{near}})$  then
4     if  $\text{CollisionFree}(\sigma)$  then
5        $x_{\text{parent}} \leftarrow \text{Parent}(x_{\text{near}})$ ;
6        $E \leftarrow E \setminus \{x_{\text{parent}}, x_{\text{near}}\}$ ;
7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
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```

LQR-RRT*



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```

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6      $x_{\text{min}} \leftarrow x_{\text{near}}$ ;  $\sigma_{\text{min}} \leftarrow \sigma$ ;
7 return  $(x_{\text{min}}, \sigma_{\text{min}})$ ;

```

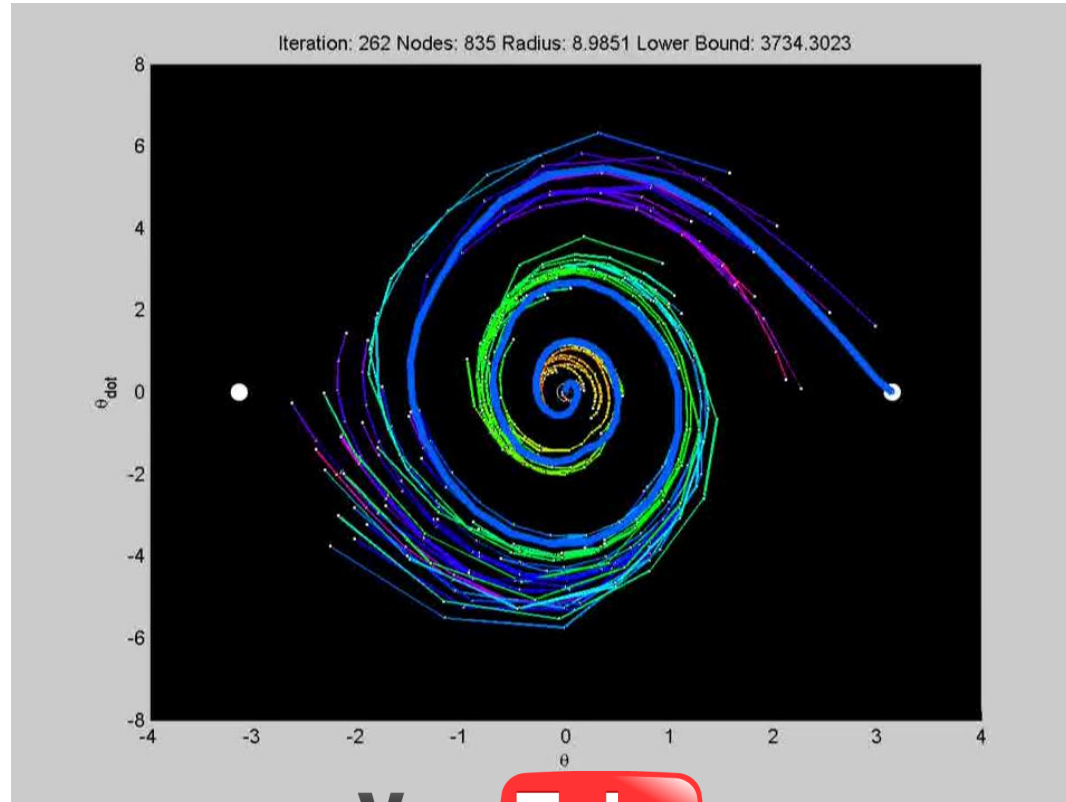
Algorithm 6: Rewire $((V, E), X_{\text{near}}, x_{\text{new}})$

```

1 for  $x_{\text{near}} \in X_{\text{near}}$  do
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7        $E \leftarrow E \cup \{x_{\text{new}}, x_{\text{near}}\}$ ;
8 return  $(V, E)$ ;

```

LQR-RRT*

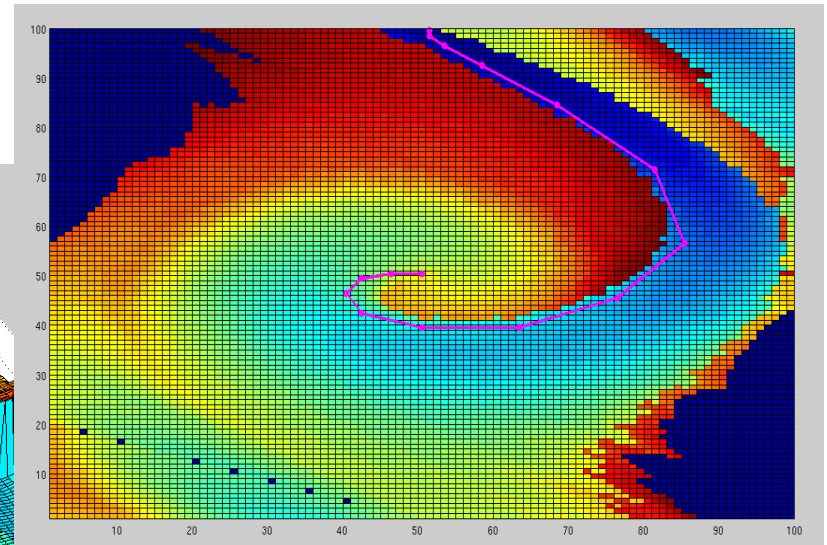
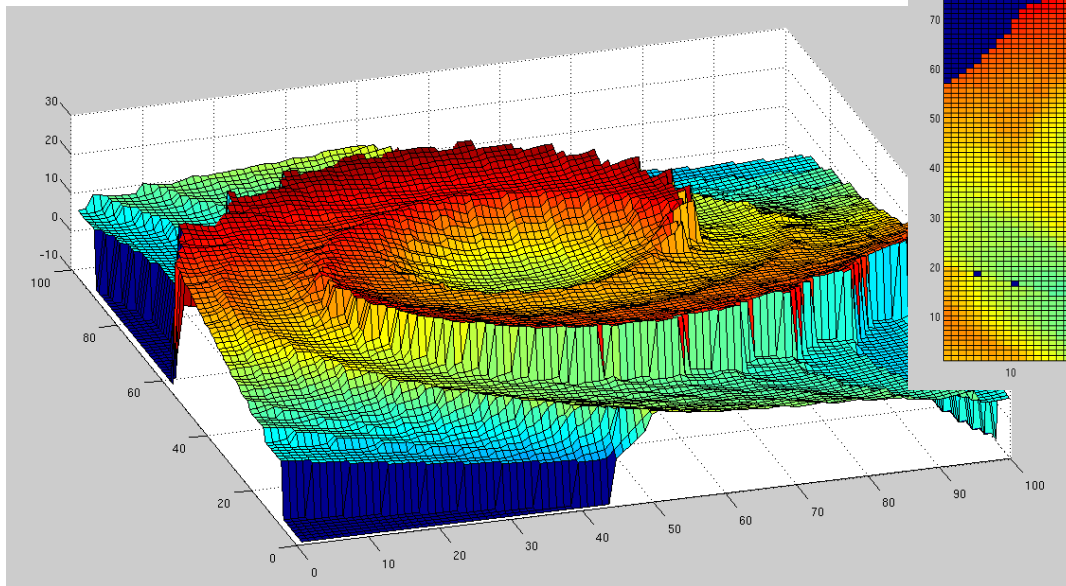


YouTube™

Results: Torque-limited Pendulum (Dynamics with constraints)

Two-dimensional state space, $(\theta, \dot{\theta})$

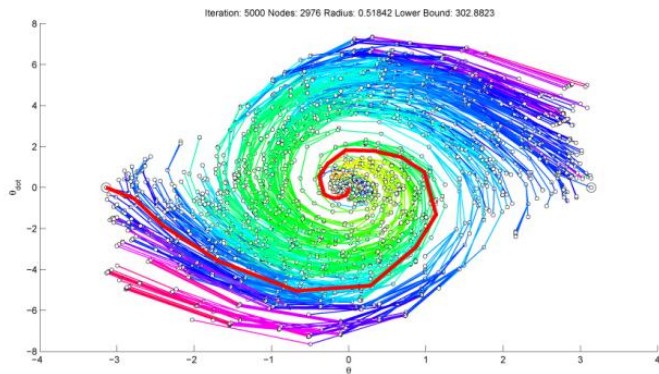
$$\ddot{\theta} = u - b\dot{\theta} - g \cos(\theta), \quad g = 9.81, \quad b = 0.1, \quad u \in [-3, 3]$$



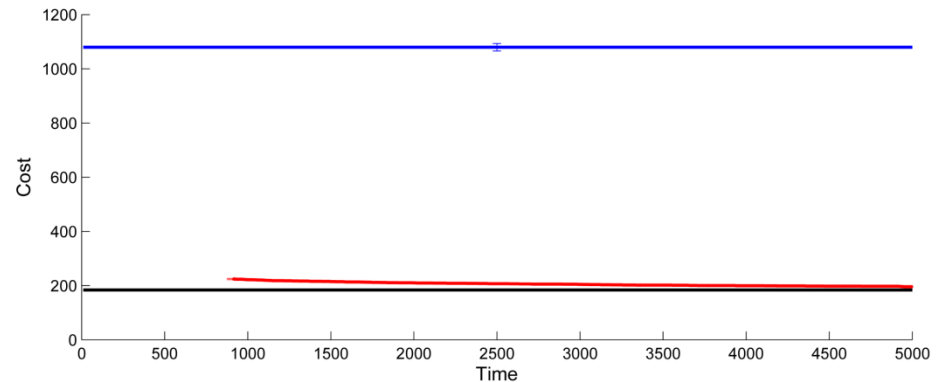
Results: Torque-limited Pendulum (Dynamics with constraints)

Two-dimensional state space, $(\theta, \dot{\theta})$

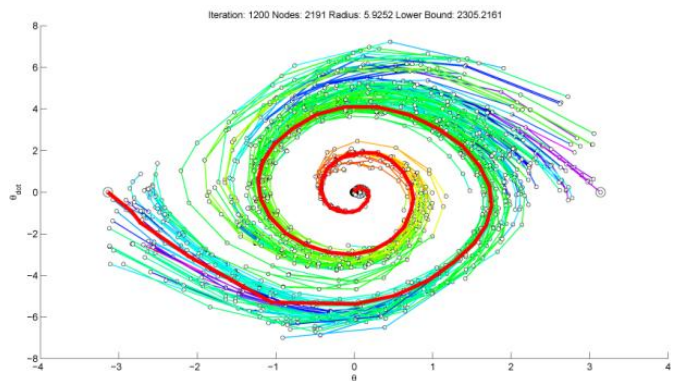
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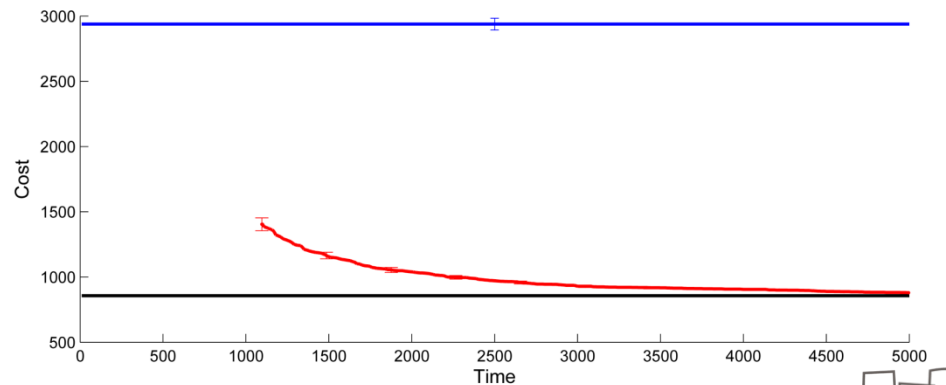
(a) $R = 1$



(a) Torque-limited Pendulum $R = 1$



(b) $R = 50$



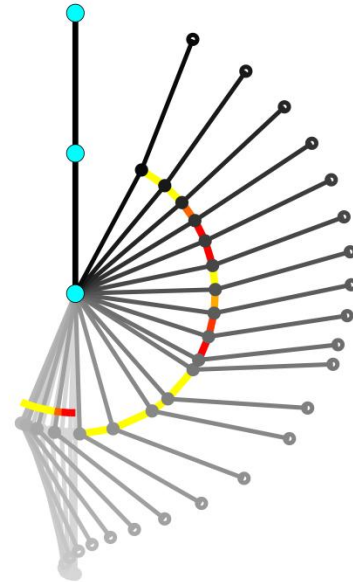
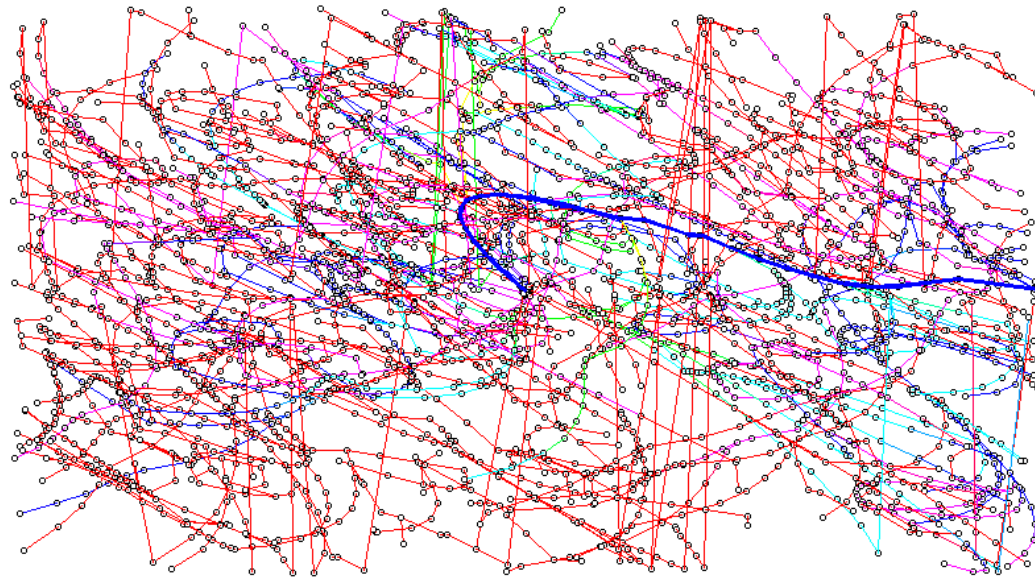
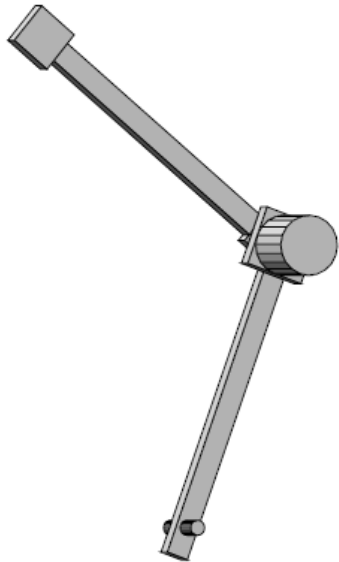
(b) Torque-limited Pendulum $R = 50$



Results: Acrobot (Underactuated Dynamics)

Four-dimensional state space $(q_1, q_2, \dot{q}_1, \dot{q}_2)$

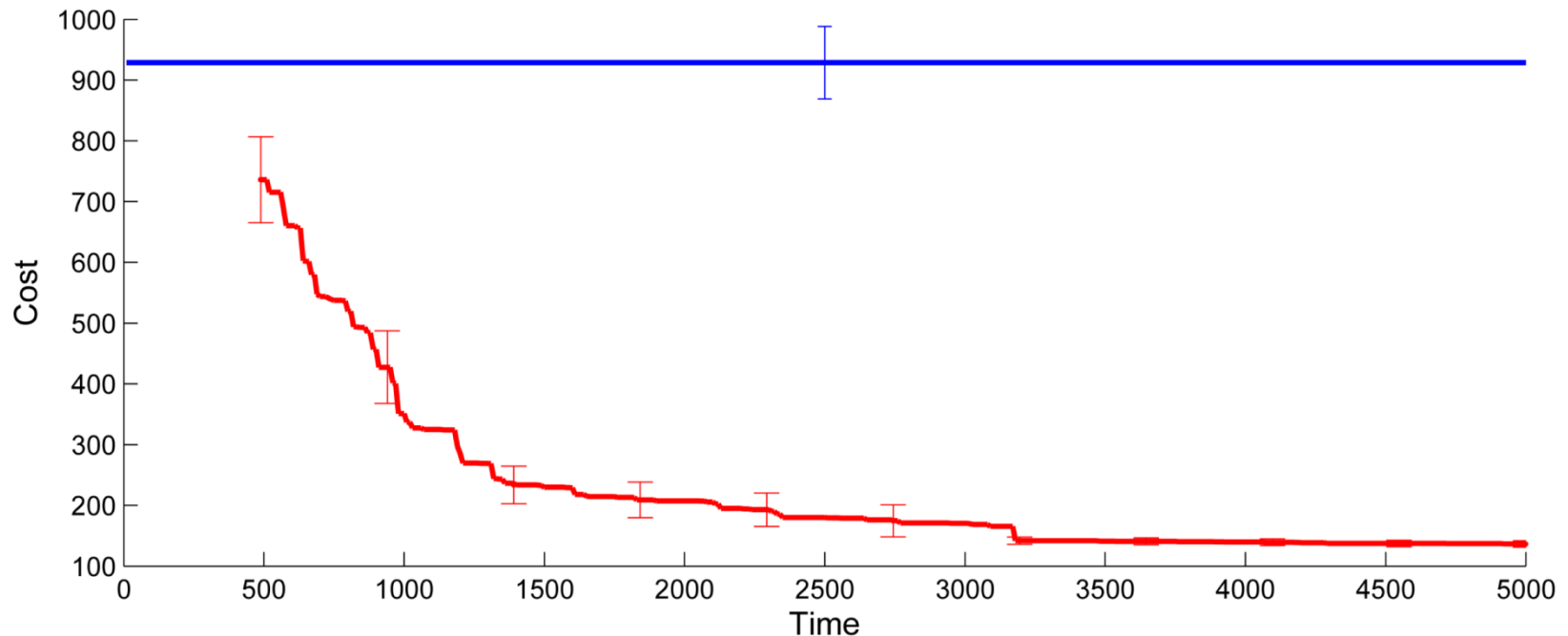
R. Murray and J. Hauser, "A case study in approximate linearization:
The acrobot example," EECS Department, University of California,
Berkeley, Tech. Rep. UCB/ERL M91/46, 1991.



Results: Acrobot (Underactuated Dynamics)

Four-dimensional state space $(q_1, q_2, \dot{q}_1, \dot{q}_2)$

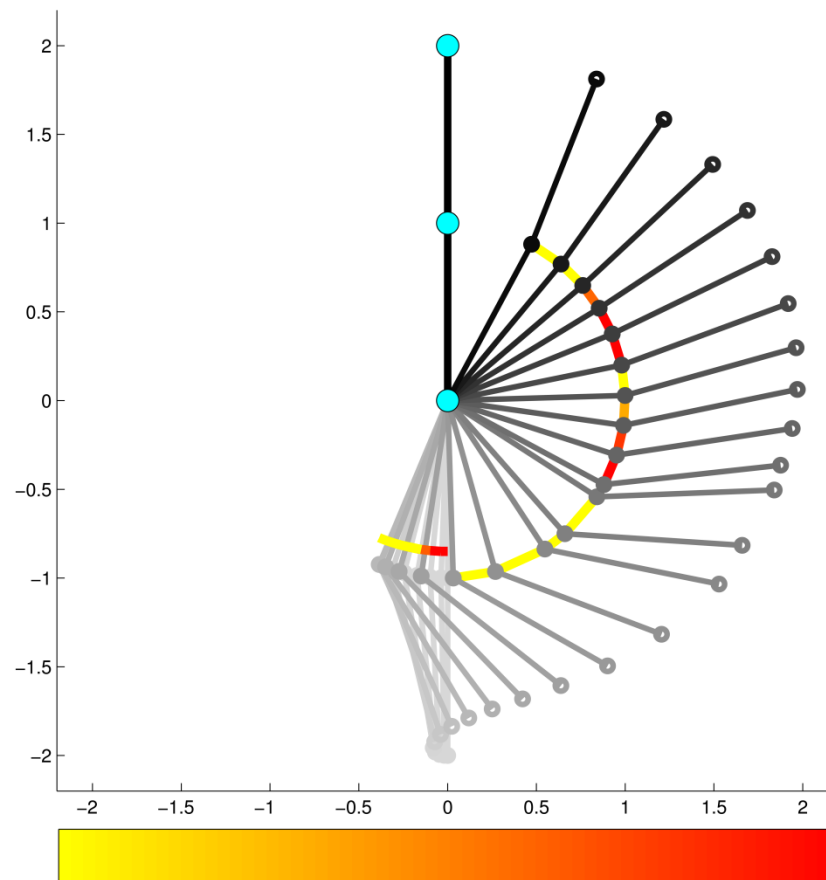
R. Murray and J. Hauser, "A case study in approximate linearization:
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R. Murray and J. Hauser, "A case study in approximate linearization: The acrobot example," EECS Department, University of California, Berkeley, Tech. Rep. UCB/ERL M91/46, 1991.



Results: Light-Dark Domain (Planning in Belief-Space)

Three-dimensional state space (x_1, x_2, v)

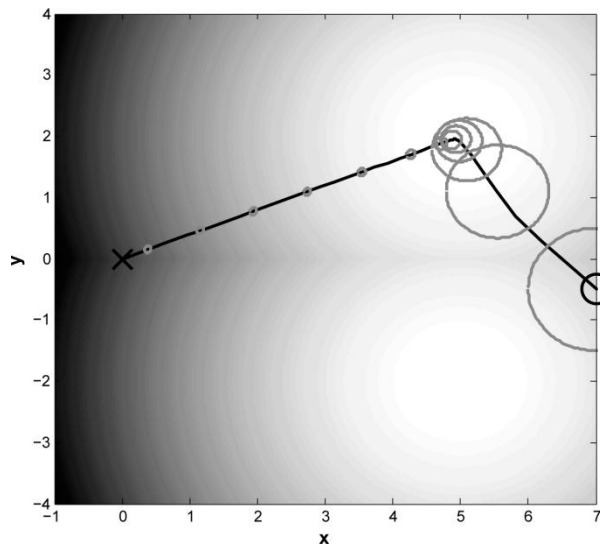
$$w(\mathbf{x}) = \min((\mathbf{x} - \mathbf{b}_1)^2, (\mathbf{x} - \mathbf{b}_2)^2),$$

$$\dot{\mathbf{s}} = (u_1, u_2, \dot{v})^T$$

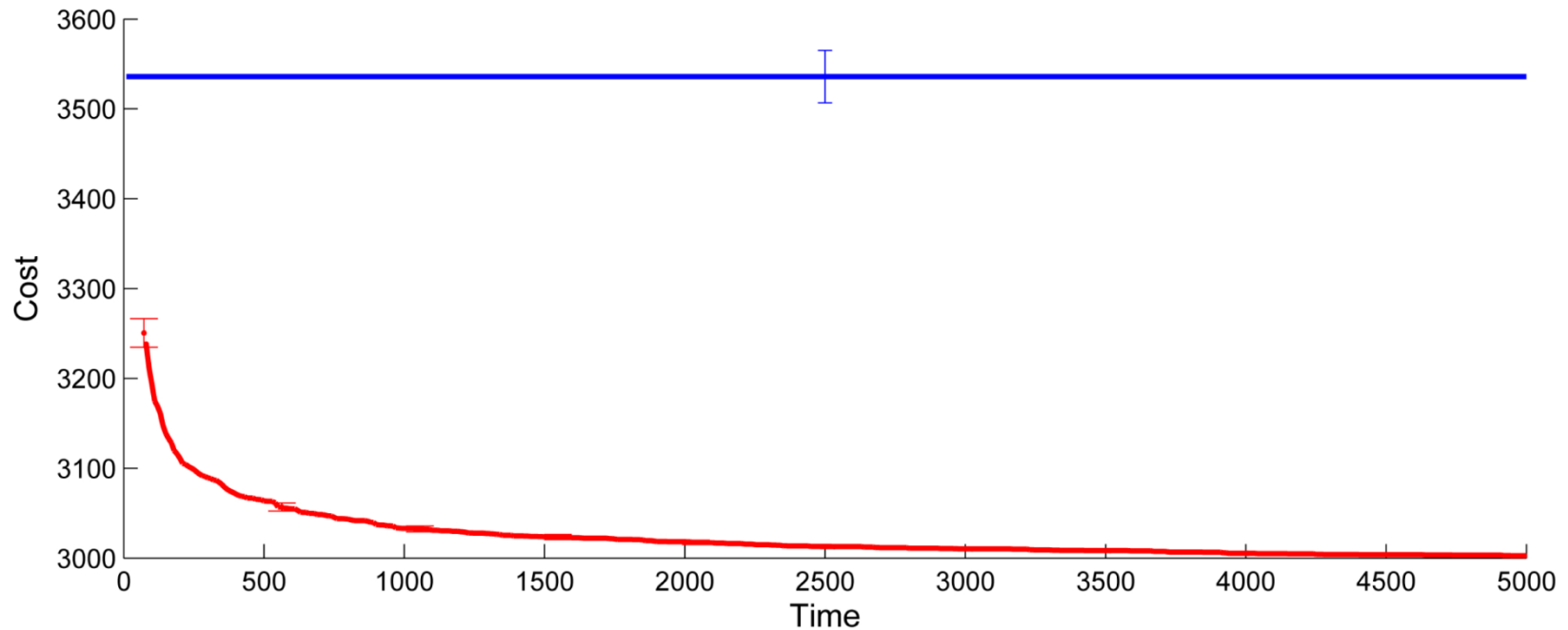
$$\dot{v} = -\frac{v^2}{v + w(\mathbf{x})}$$

Adapted from

R. Platt, R. Tedrake, L. Kaelbling, and T. Lozano-Perez, “Belief space planning assuming maximum likelihood observations,” in *Proceedings of Robotics: Science and Systems*, June 2010.

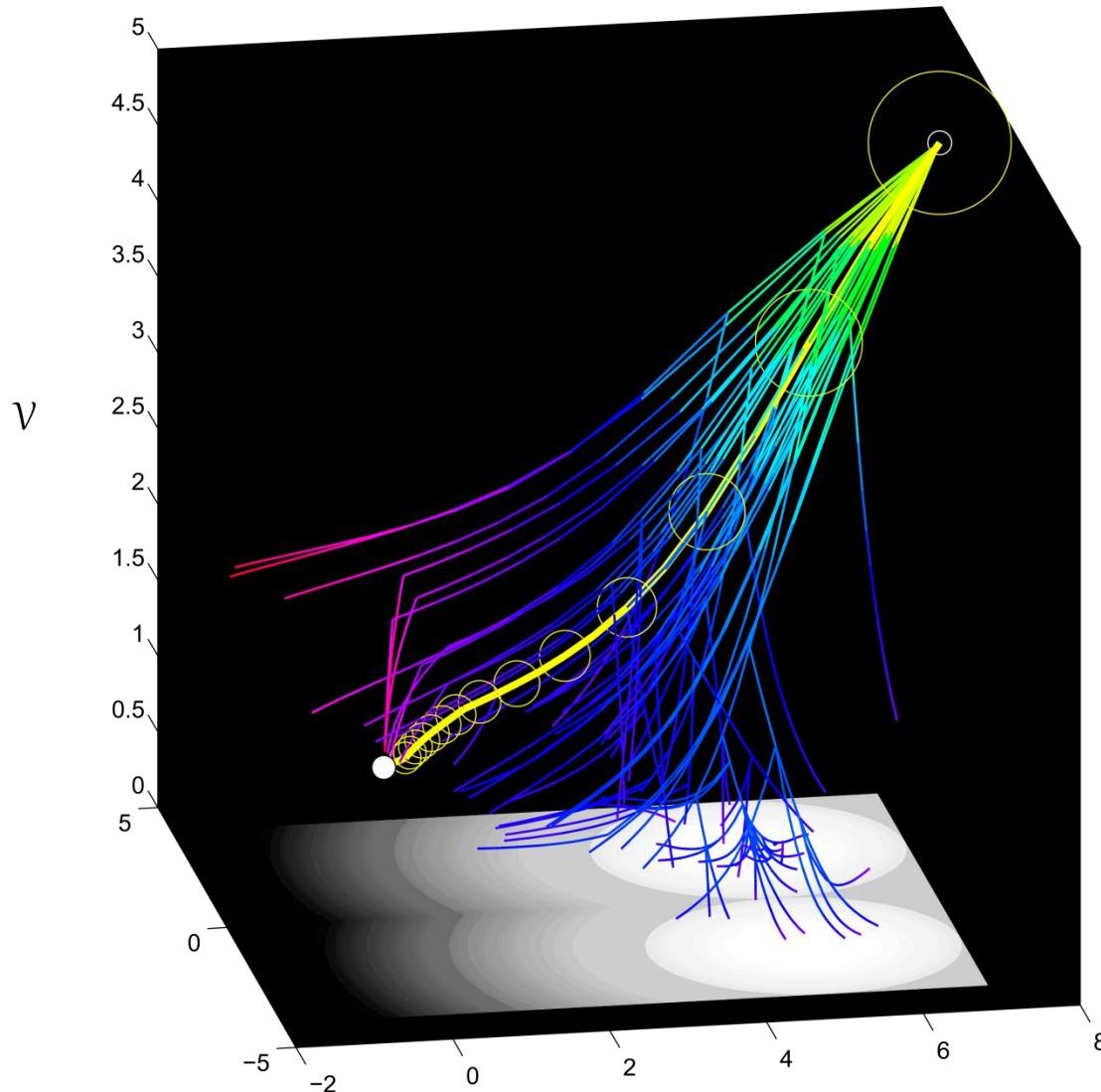


Results: Light-Dark Domain (Planning in Belief-Space)

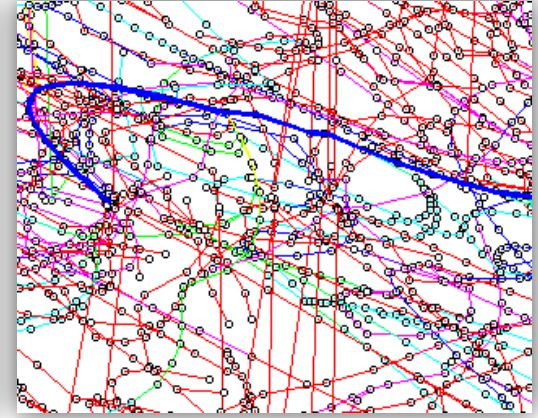
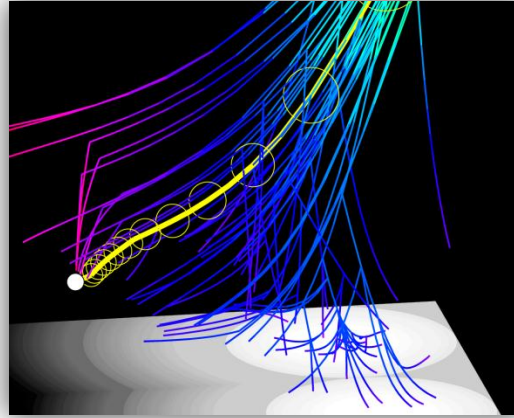
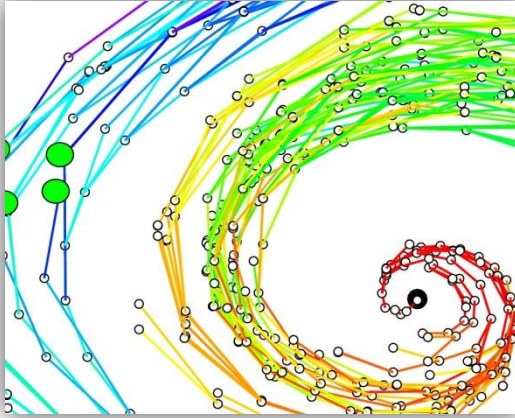


Results: Light-Dark Domain (Planning in Belief-Space)

Iteration: 5000 Nodes: 1578 Radius: 16.7508 Lower Bound: 3006.4426

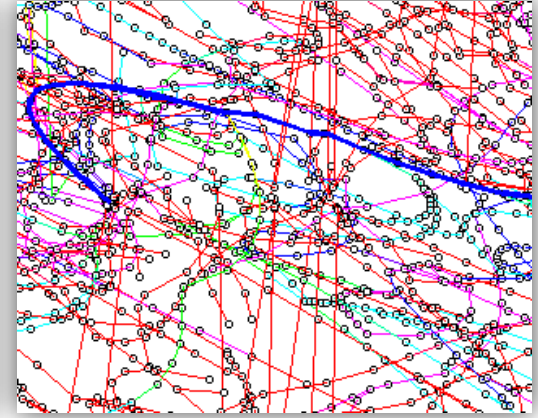
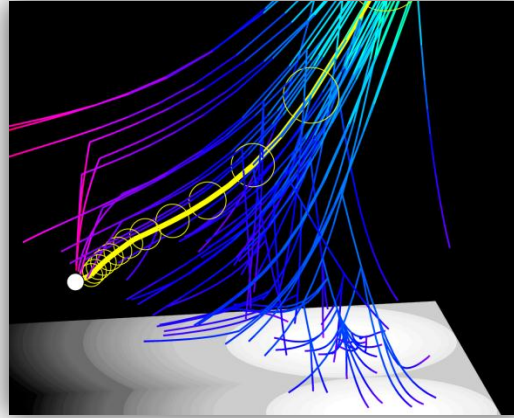
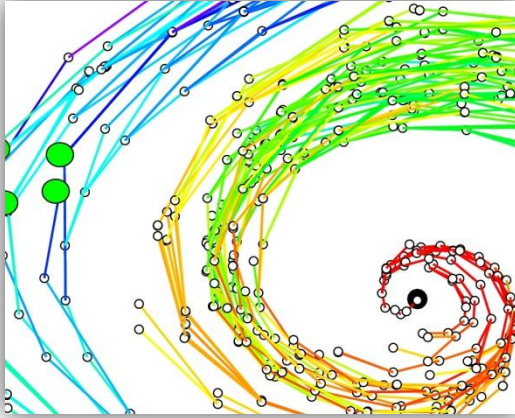


Current Work



- AQR, trajectory optimization by Gustavo Goretkin
- Theoretical Analysis
- Higher Dimensional Domains, Obstacles/Collision Checking

Thanks



For more information:

<http://people.csail.mit.edu/aperez/www/>

<http://lis.csail.mit.edu>

