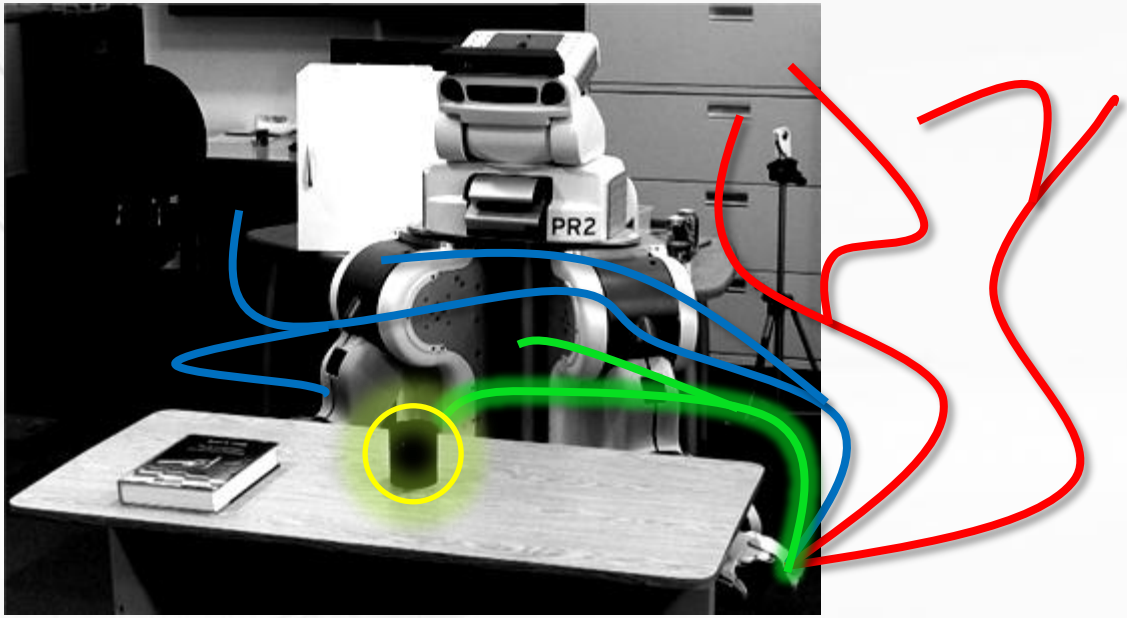
Asymptotically-optimal Manipulation Planning using Incremental Sampling-based Algorithms



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[1] MIT/CSAIL [2] MIT/LIDS

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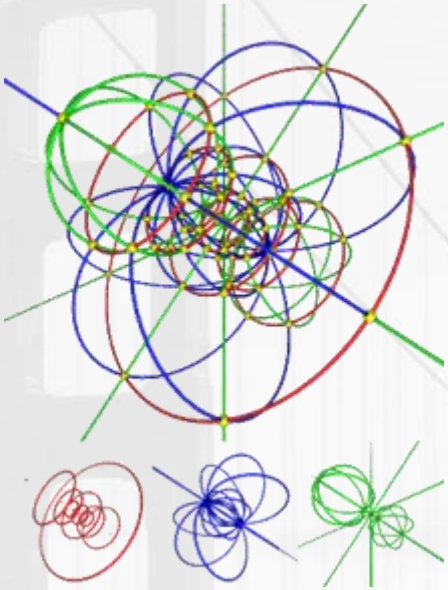
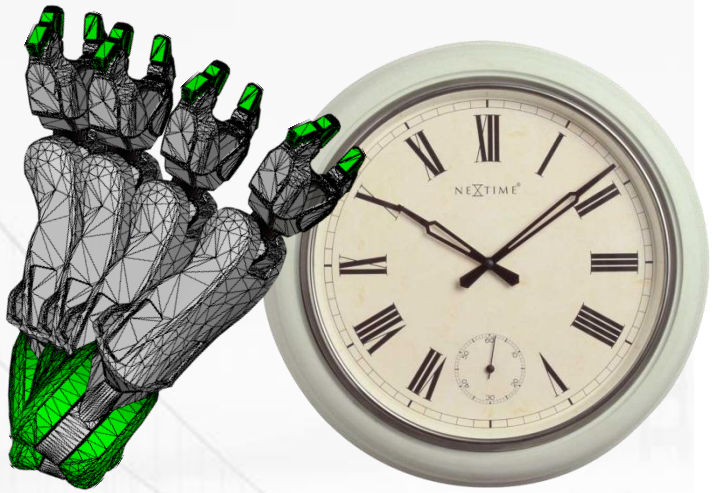
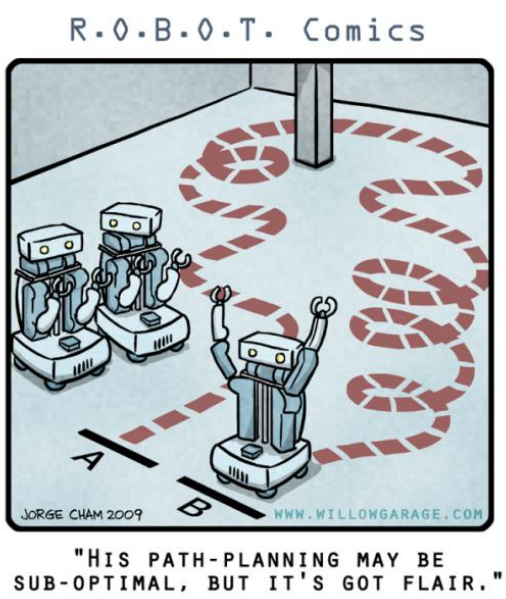
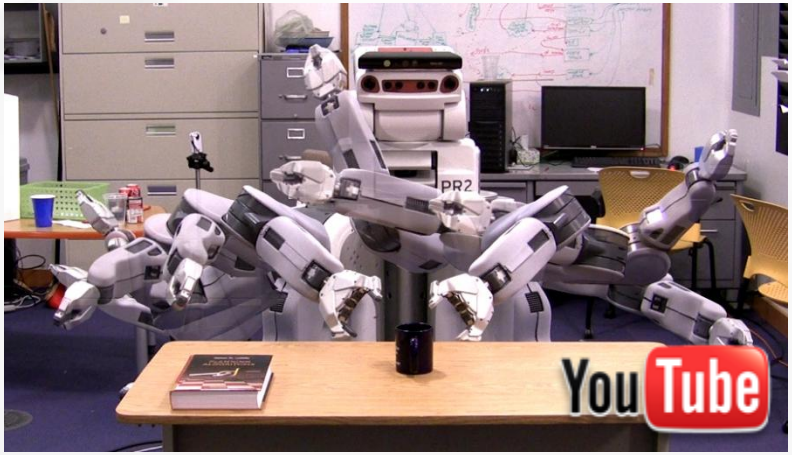
Manipulation Planning

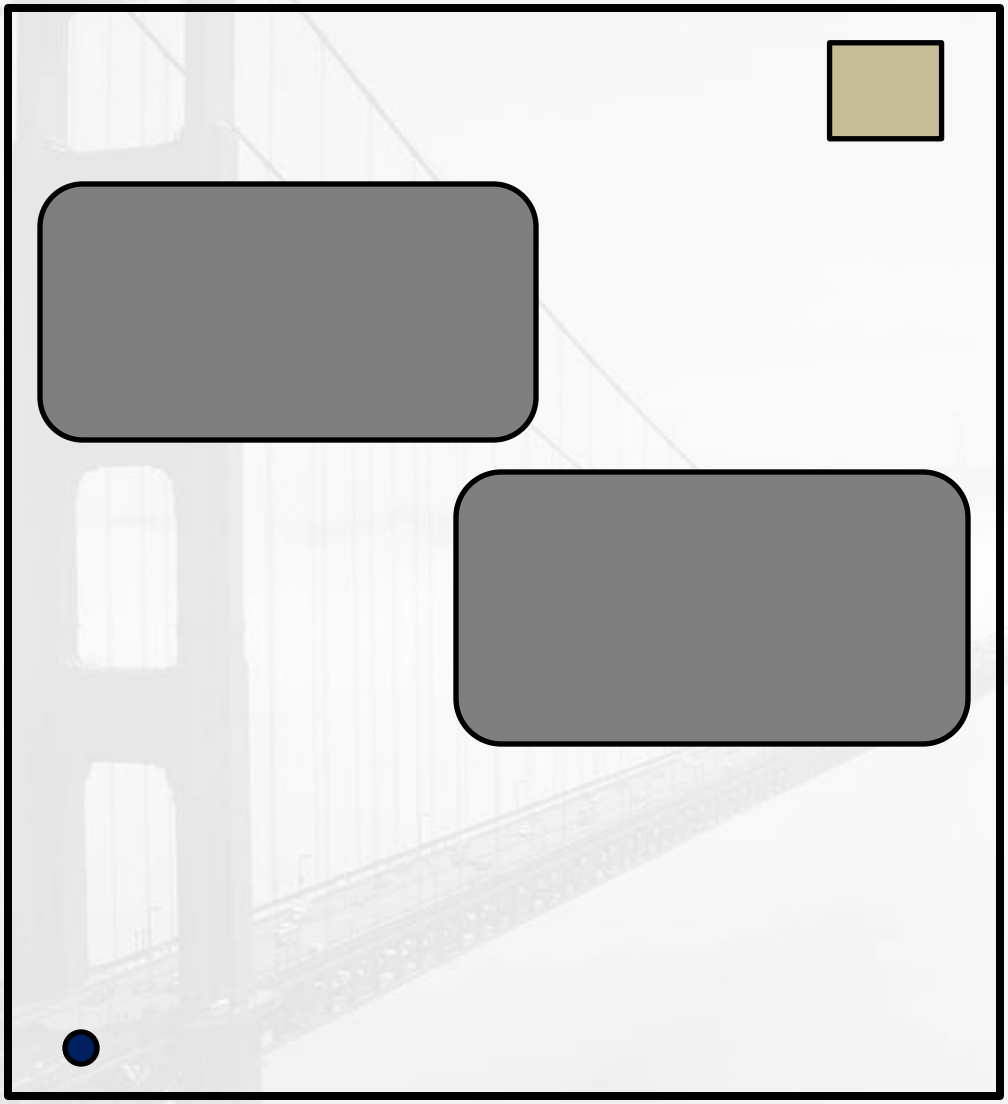








RRT* with Ball Trees



The RRT* Algorithm

$V \leftarrow \{x_{\text{init}}\}$; $E \leftarrow \emptyset$; $T \leftarrow (V, E)$;
for $i = 1$ *to* N **do**
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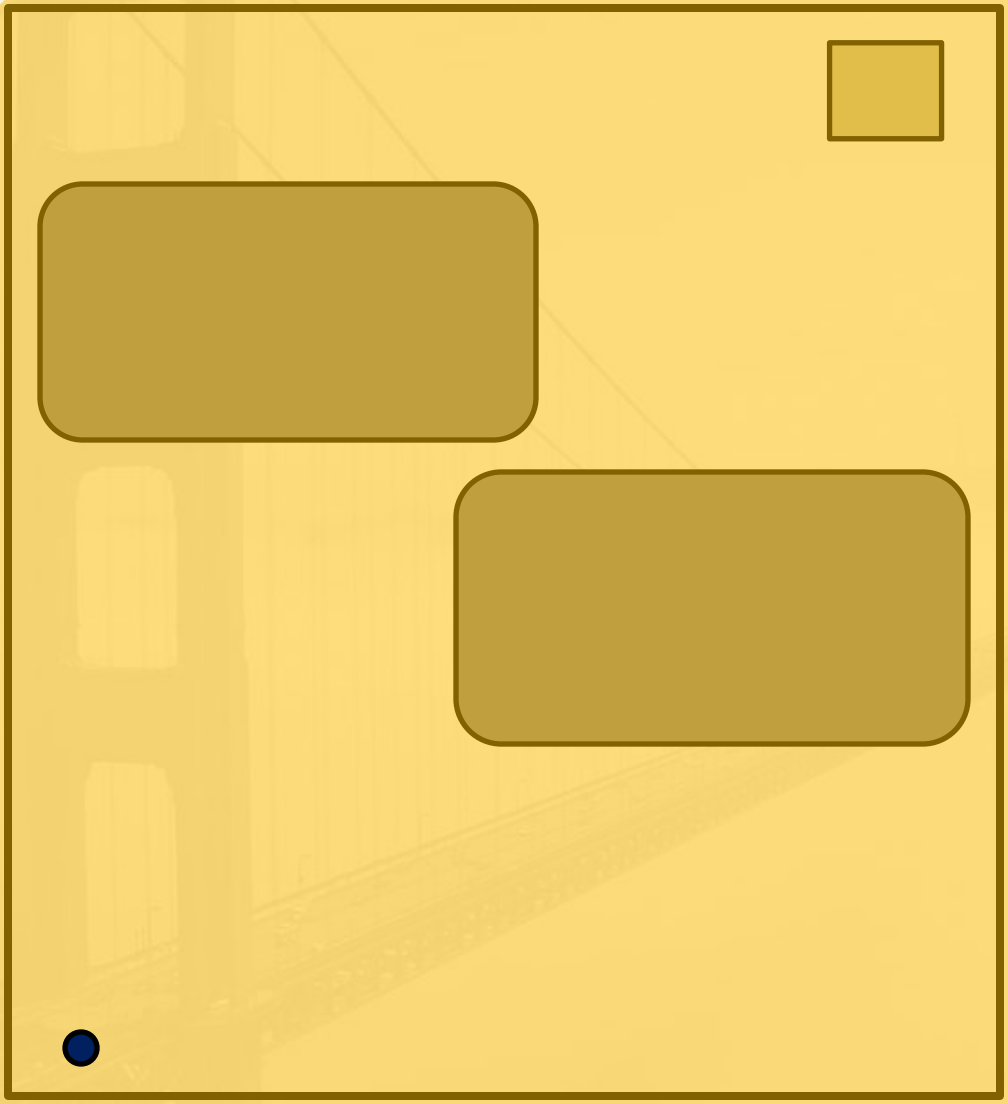

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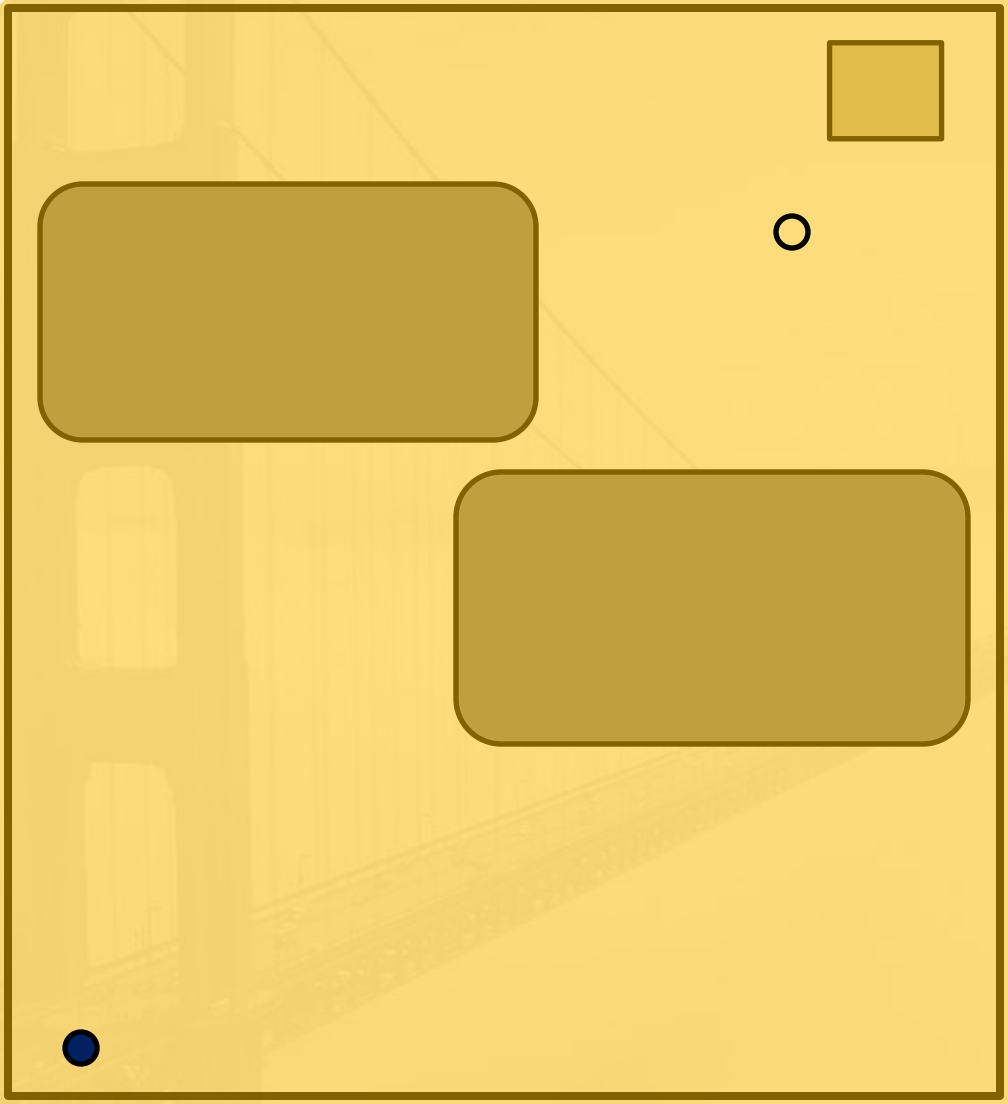

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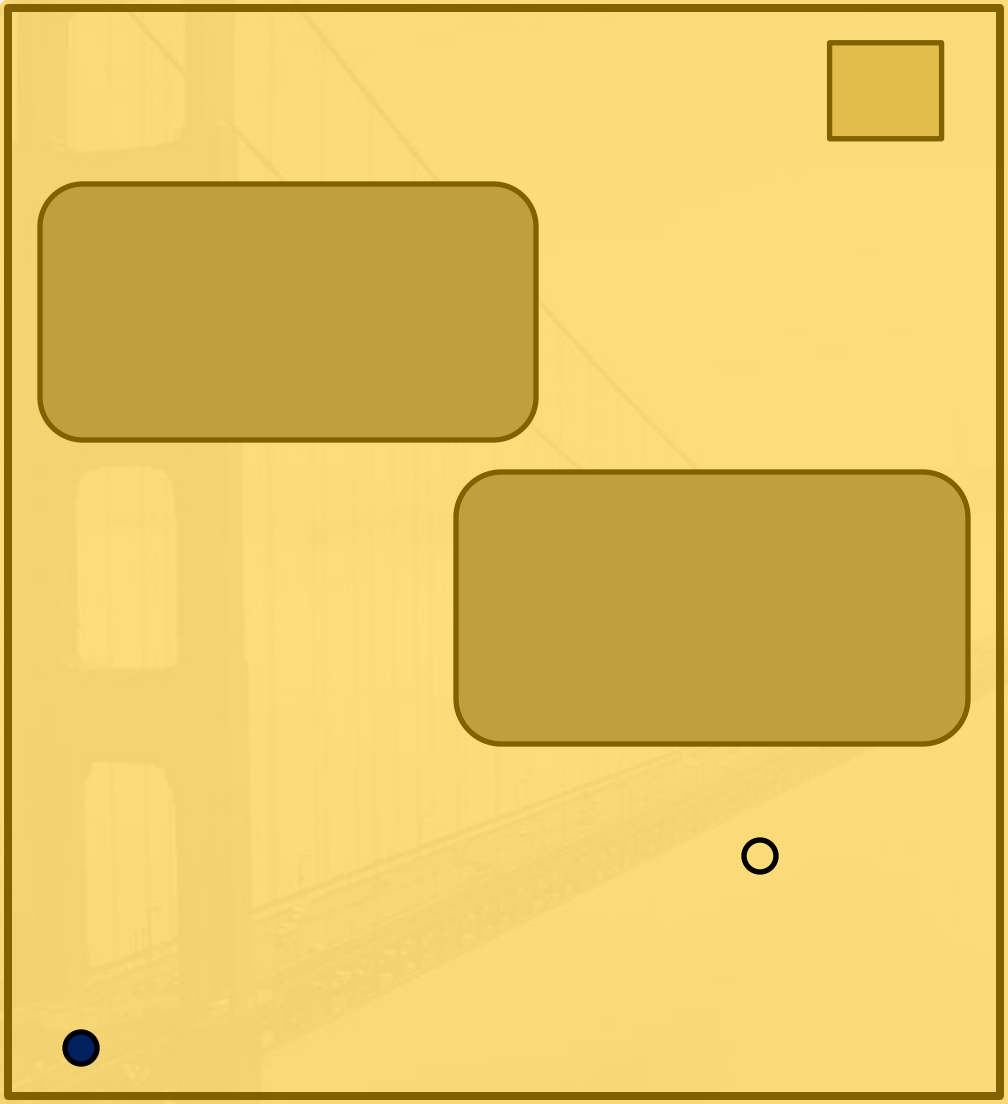

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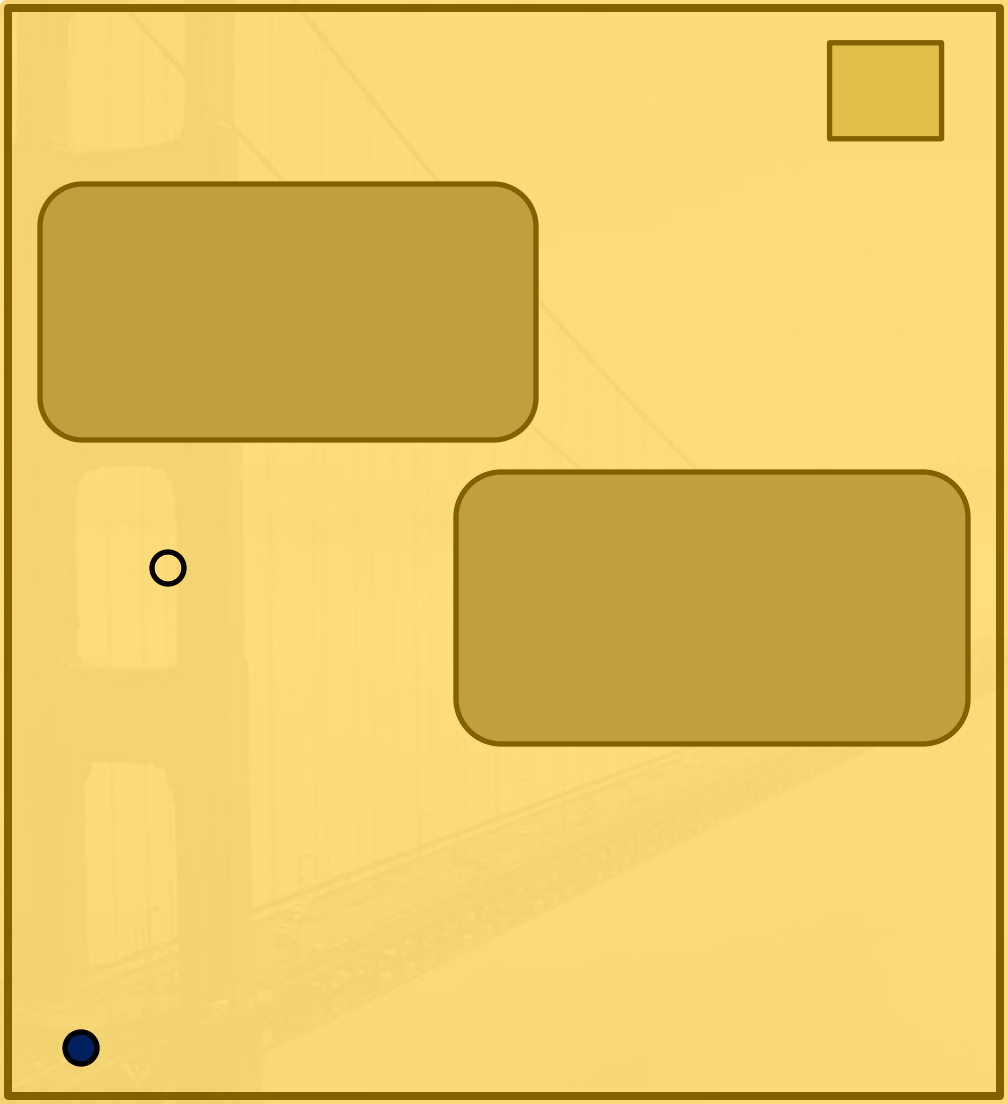

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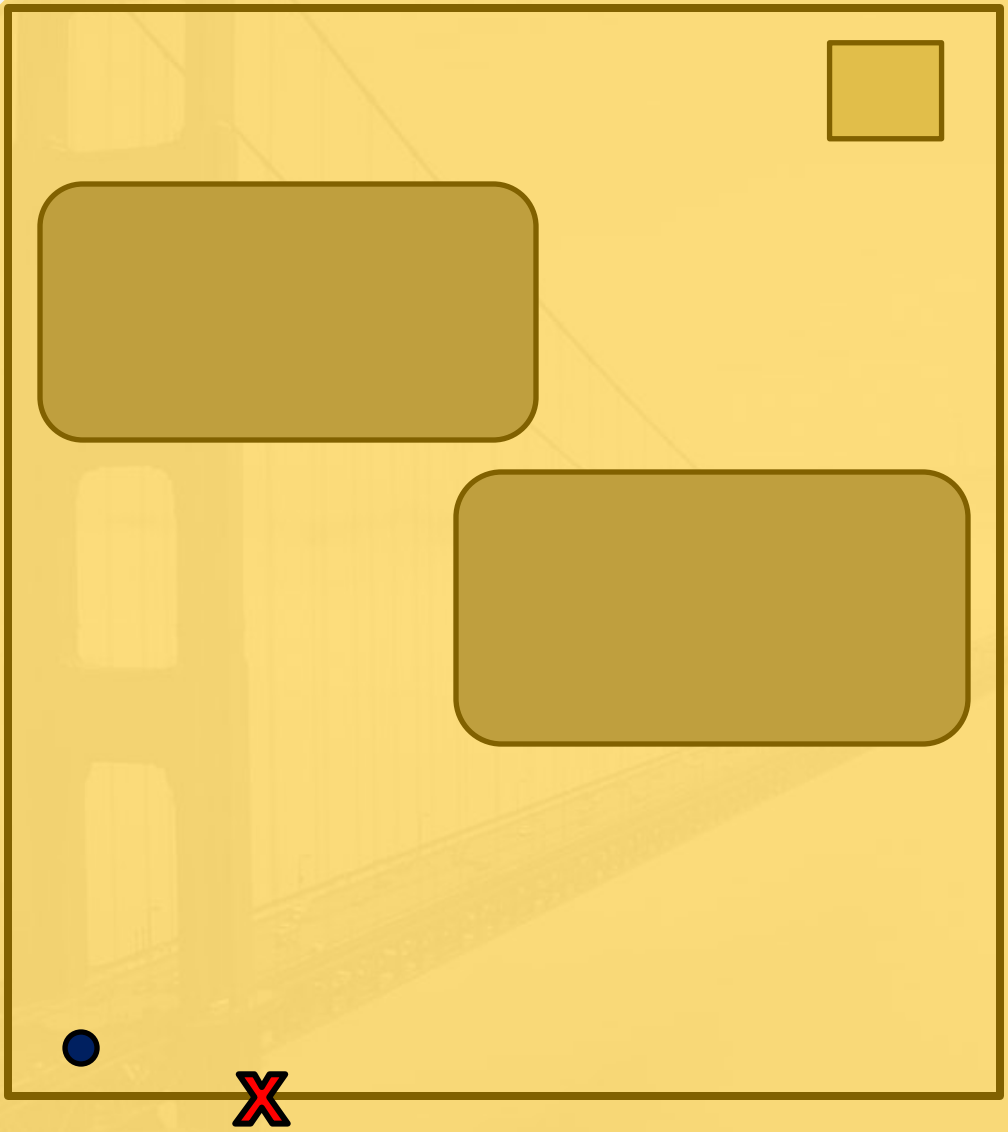

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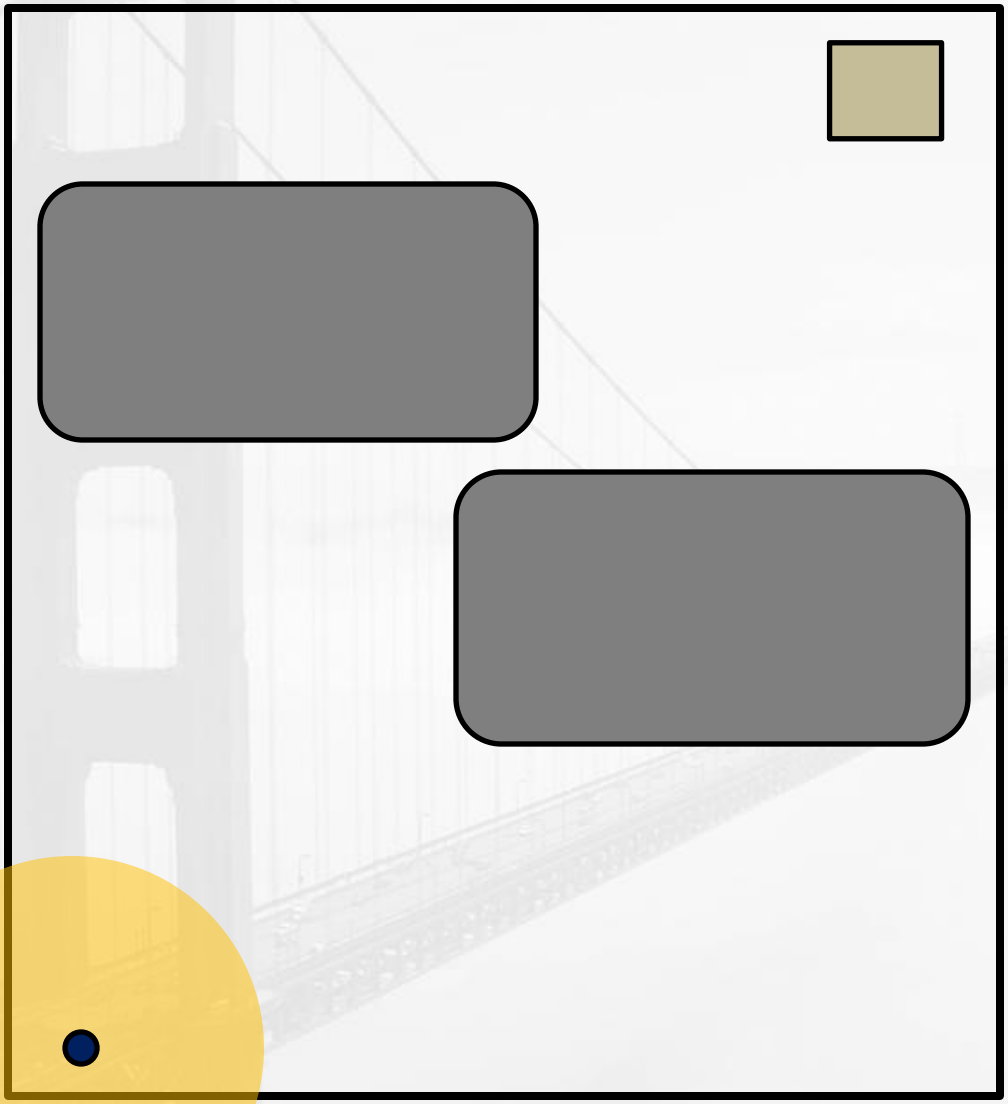

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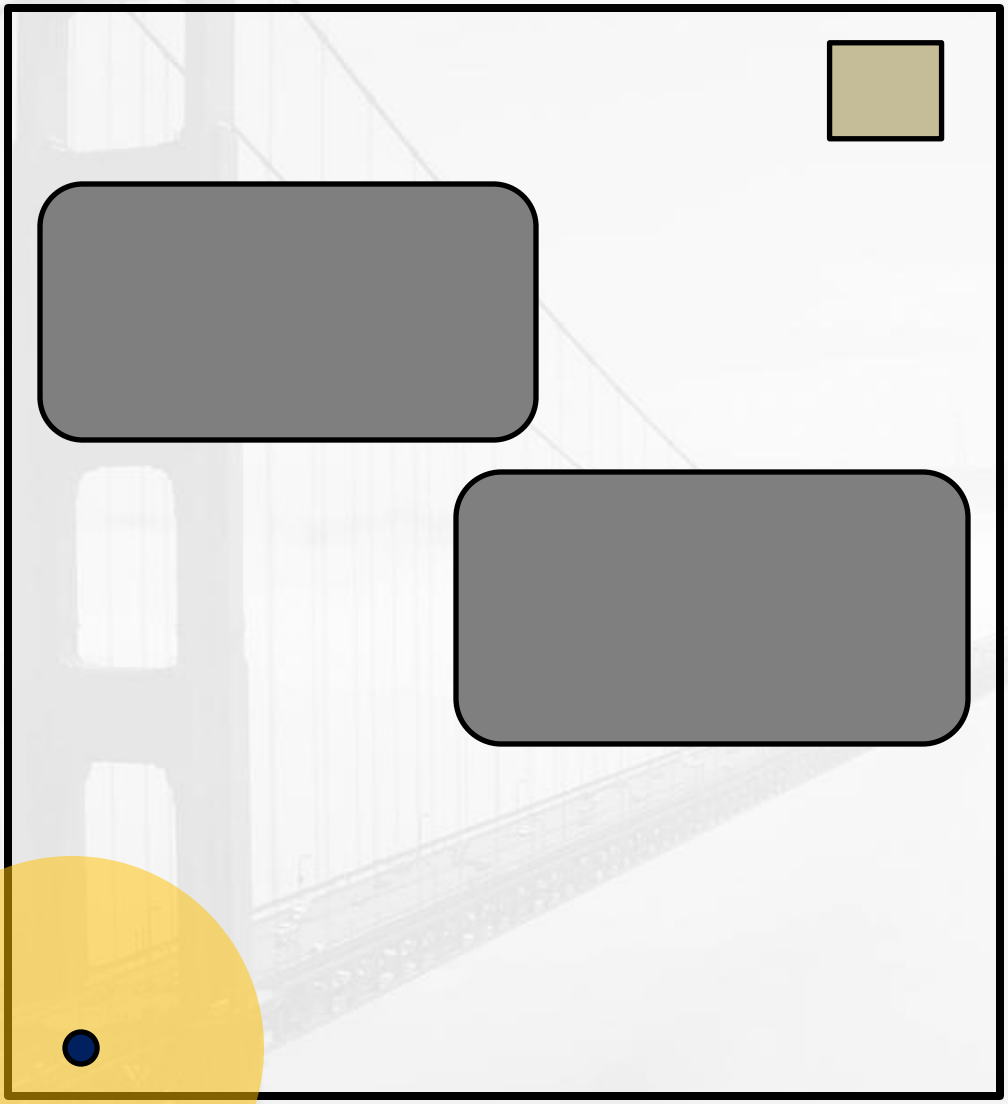

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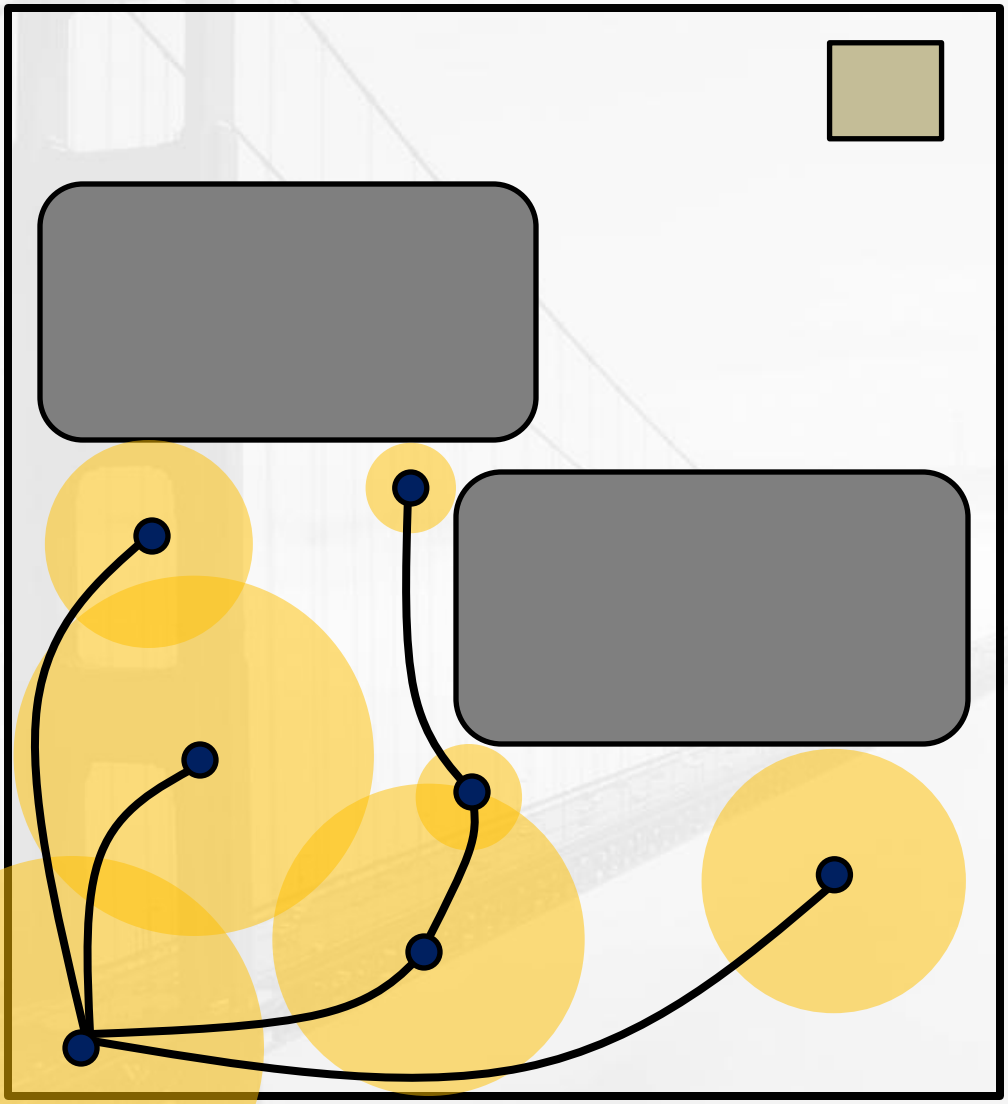

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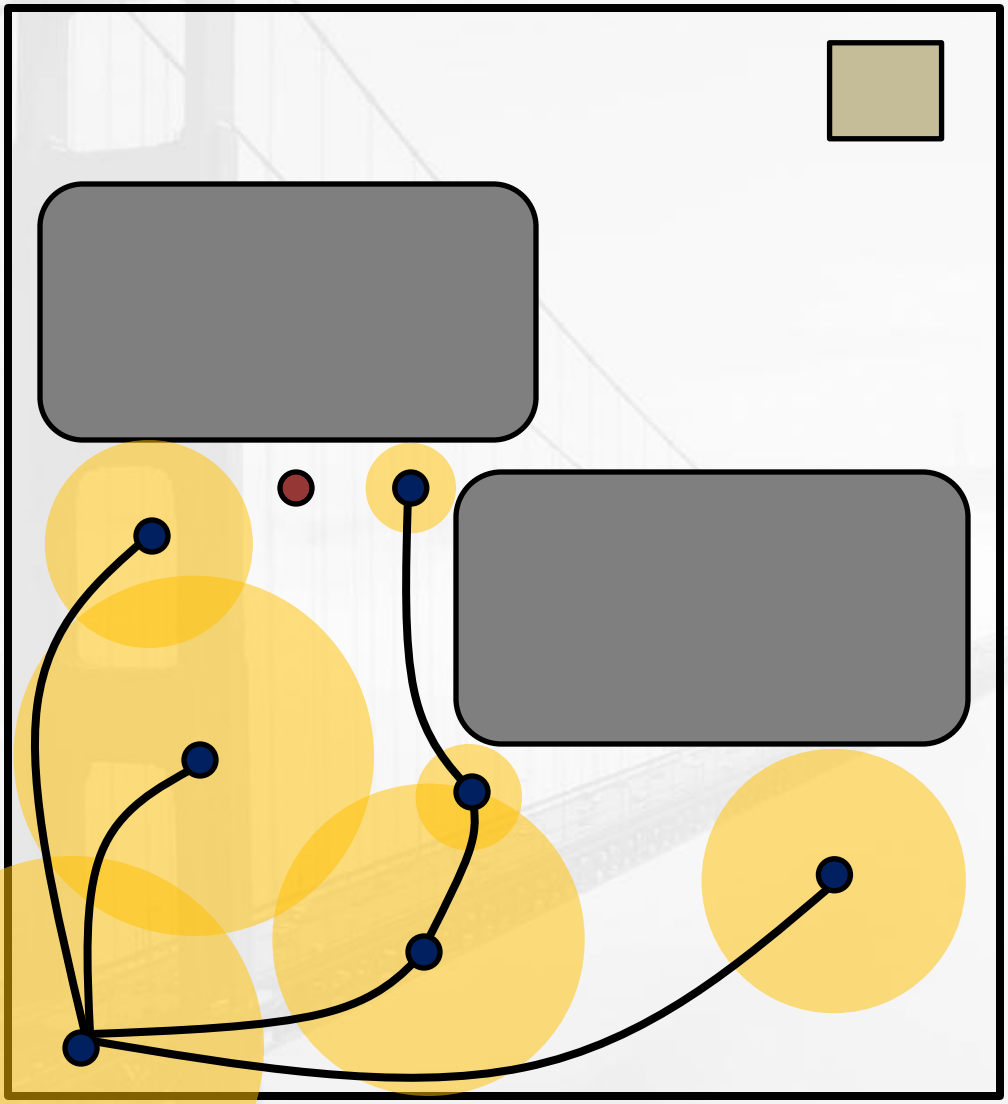

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        TrimRadius$(x_{\text{nearest}}, \|x_{\text{new}} - x_{\text{nearest}}\|)$;
    else
      break;
  $x_{\text{nearest}} \leftarrow \texttt{NearestBall}(V, x_{\text{new}})$;
  $\sigma \leftarrow \texttt{Steer}(x_{\text{new}}, x_{\text{nearest}})$;
  if CollisionFree(σ) **then**
    V.add(x_{new}, r_0);
    E.add$(\,(x_{\text{nearest}}, x_{\text{new}})\,)$;
  else
    TrimRadius$(x_{\text{nearest}}, \|x_{\text{new}} - x_{\text{nearest}}\|)$;
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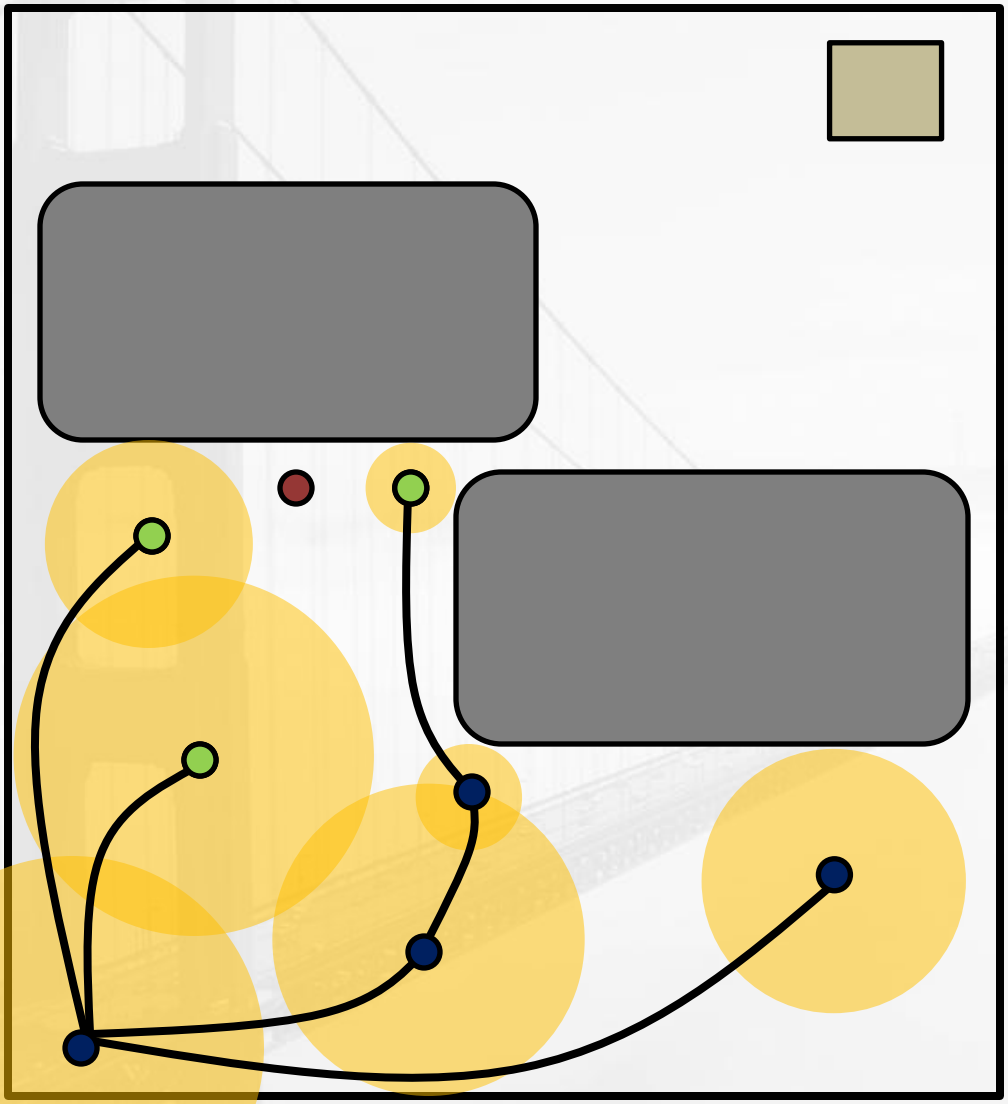
RRT* with Ball Trees



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$V \leftarrow \{x_{\text{init}}, r = 0\}$; $E \leftarrow \emptyset$; $T \leftarrow (V, E)$;
for $i = 1$ *to* N **do**
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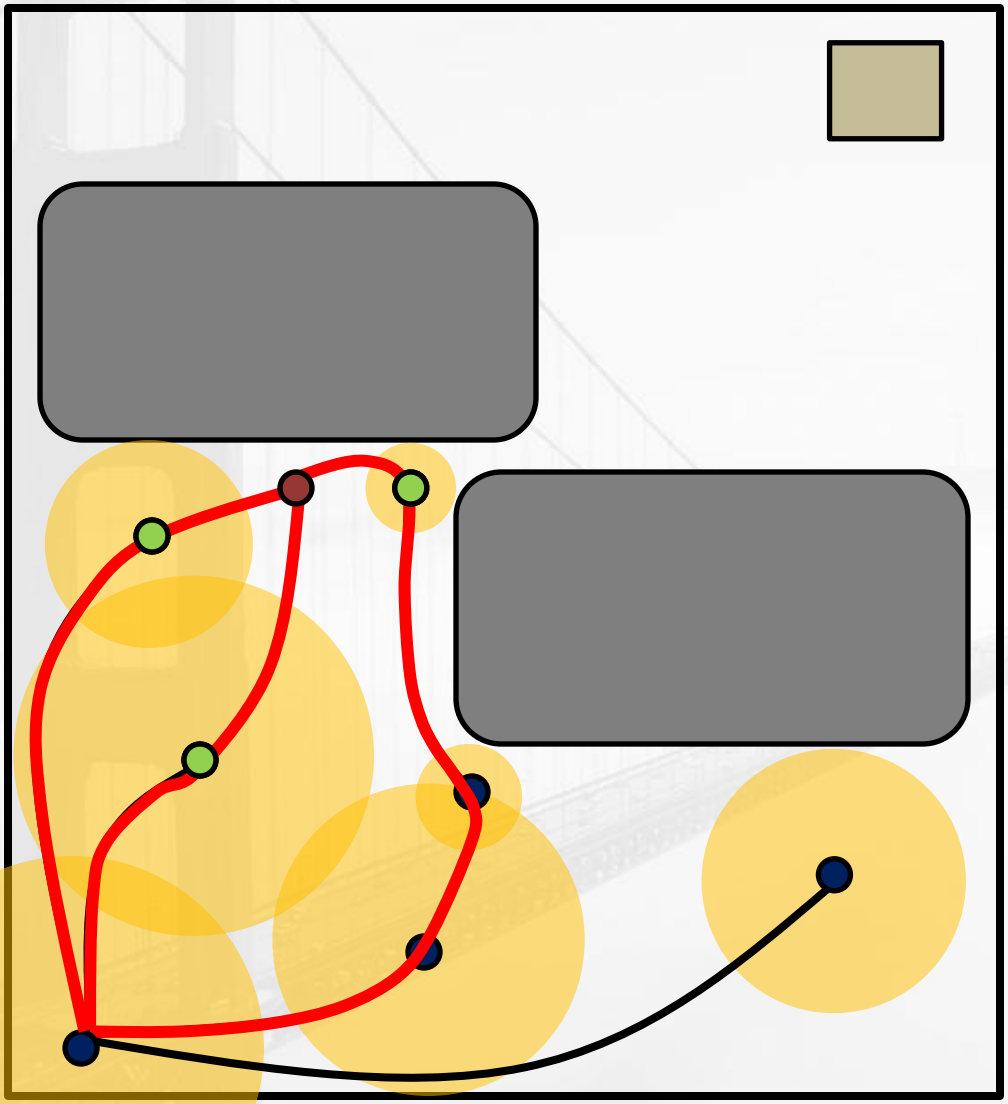

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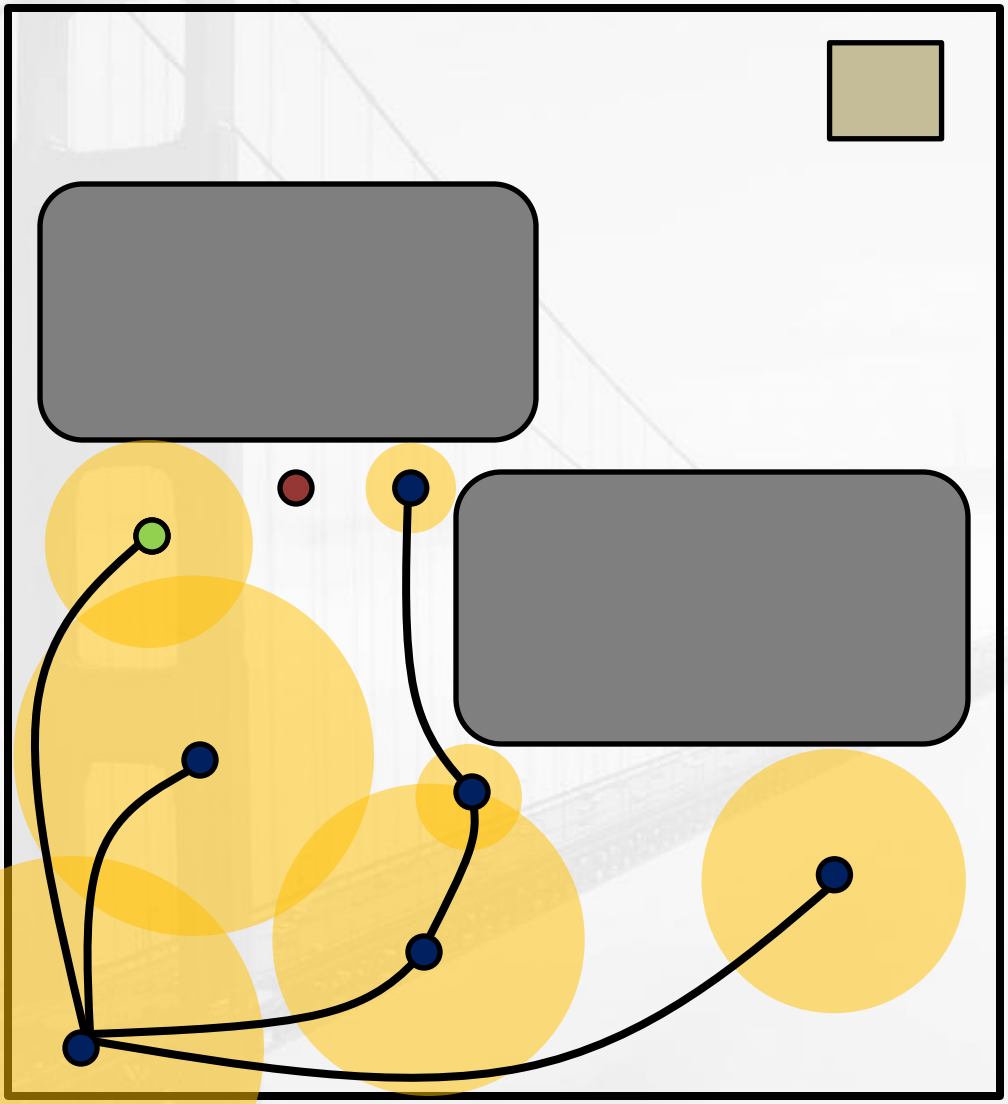

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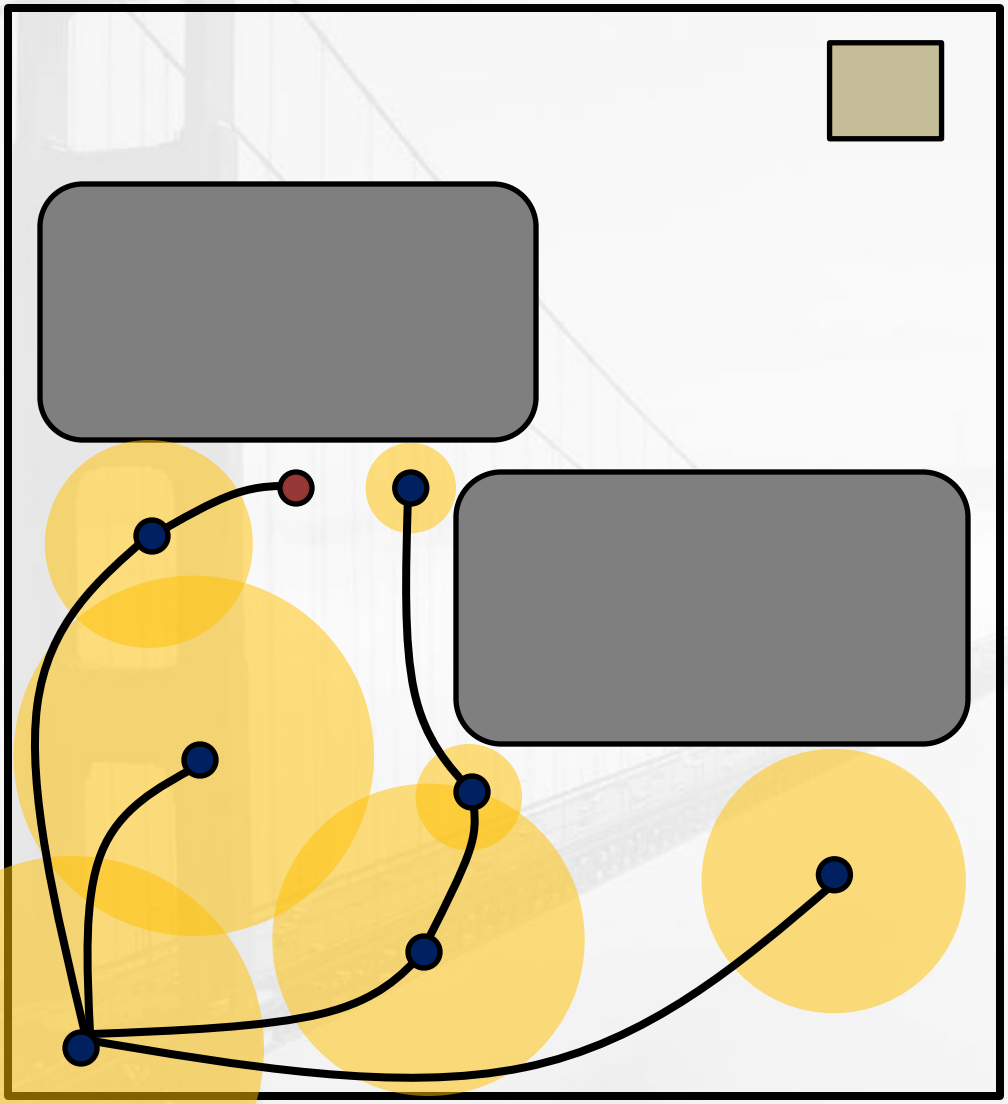

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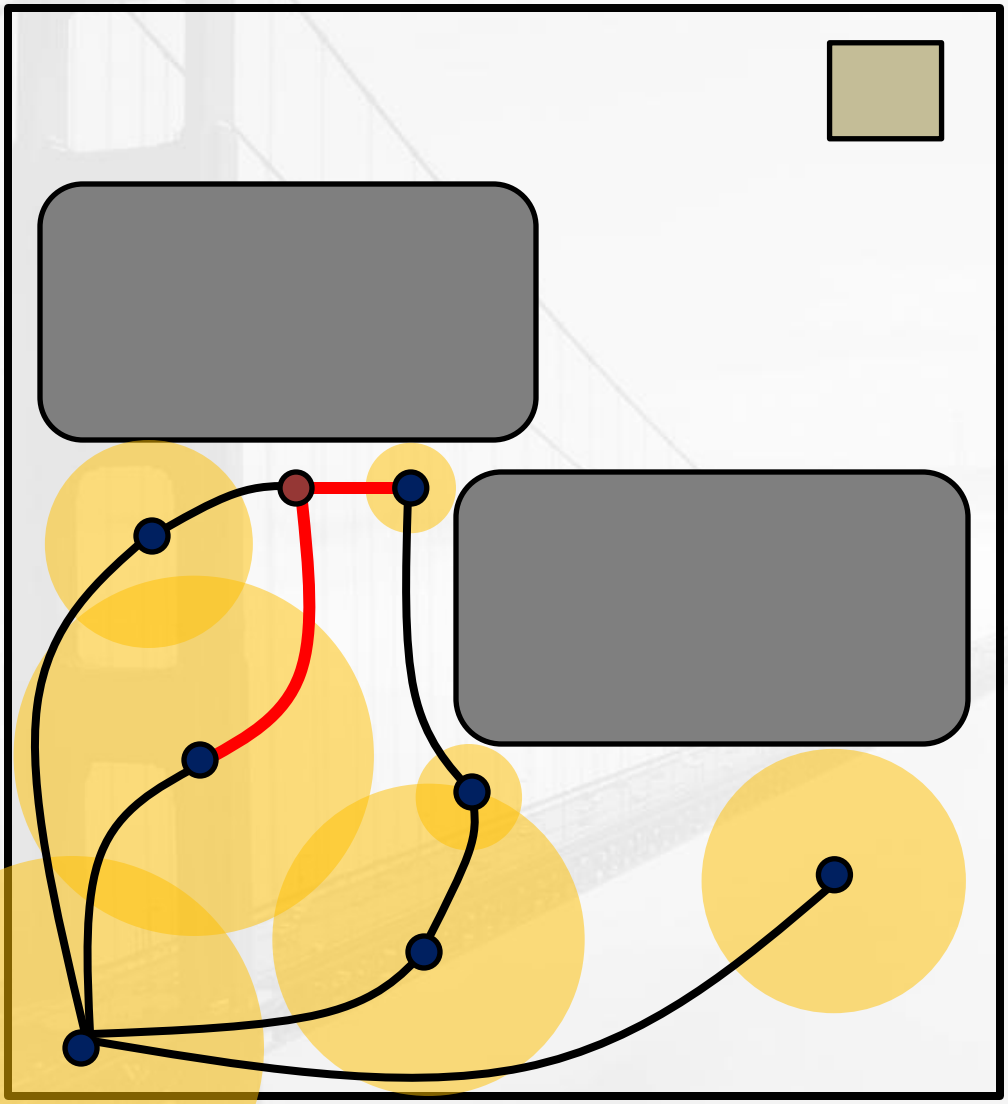

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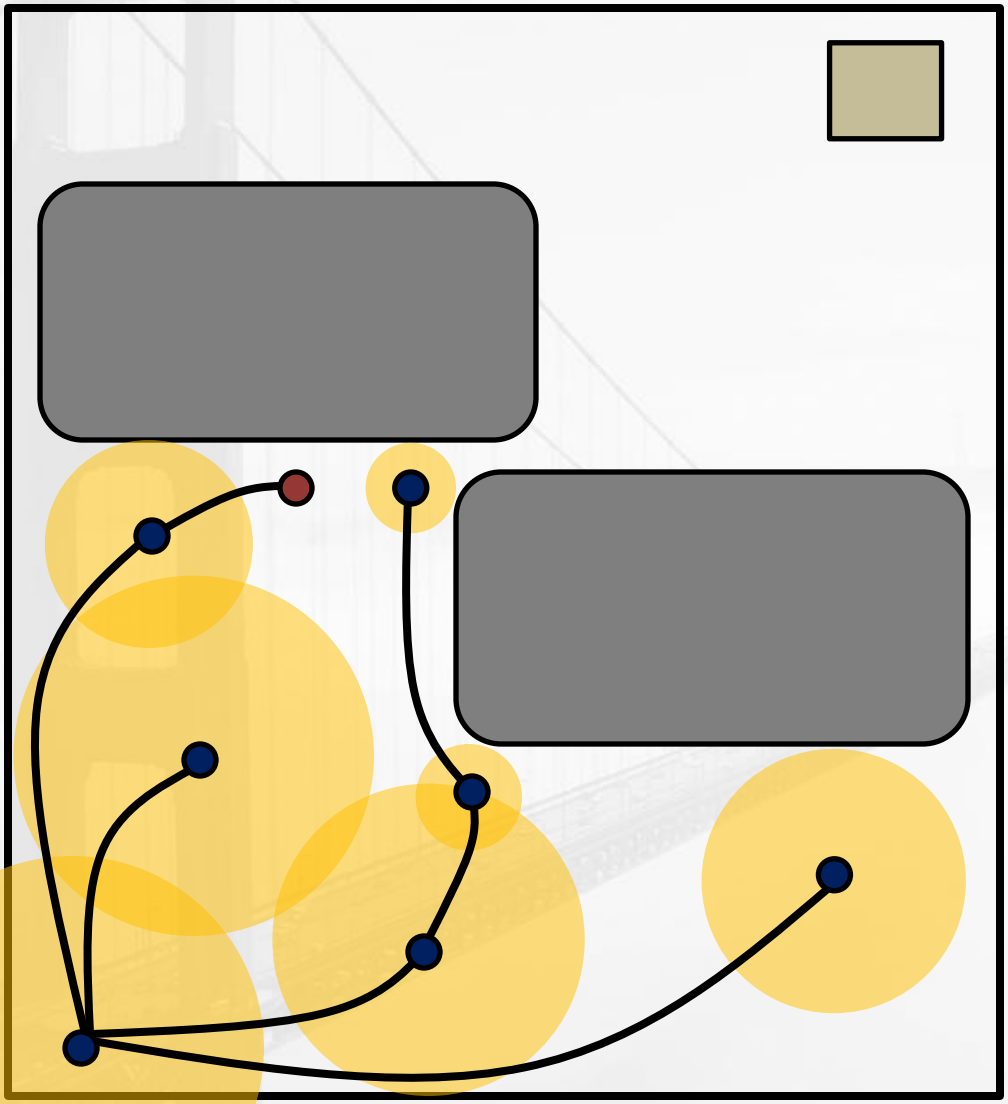

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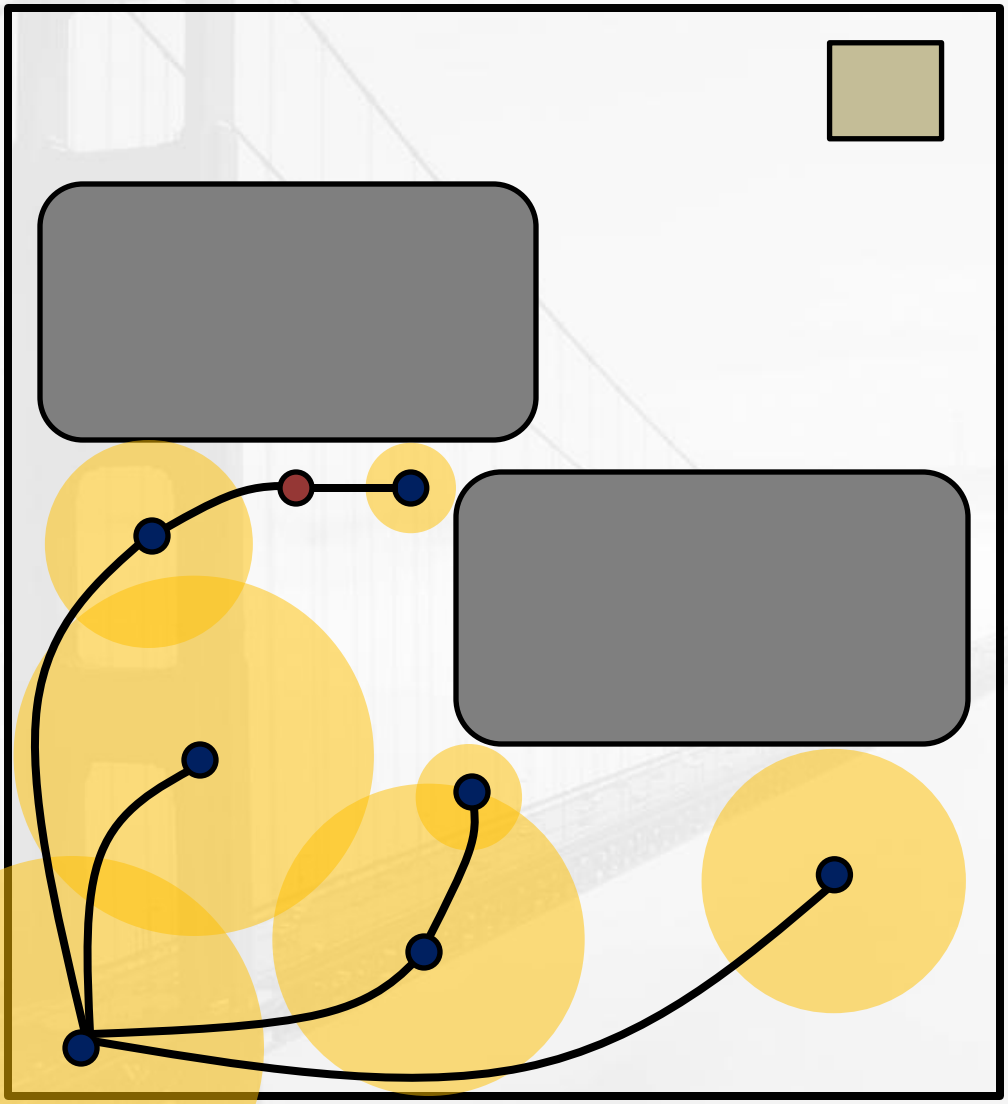

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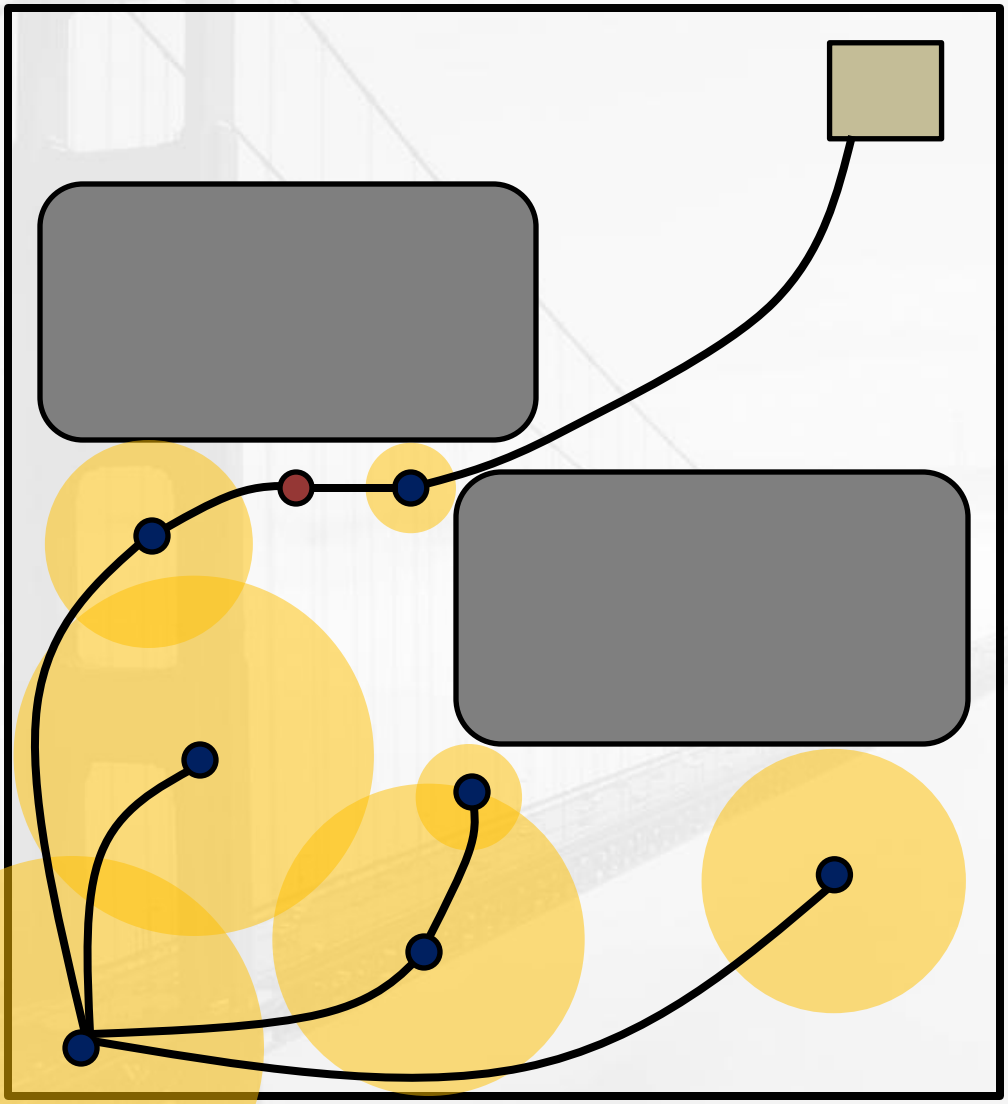

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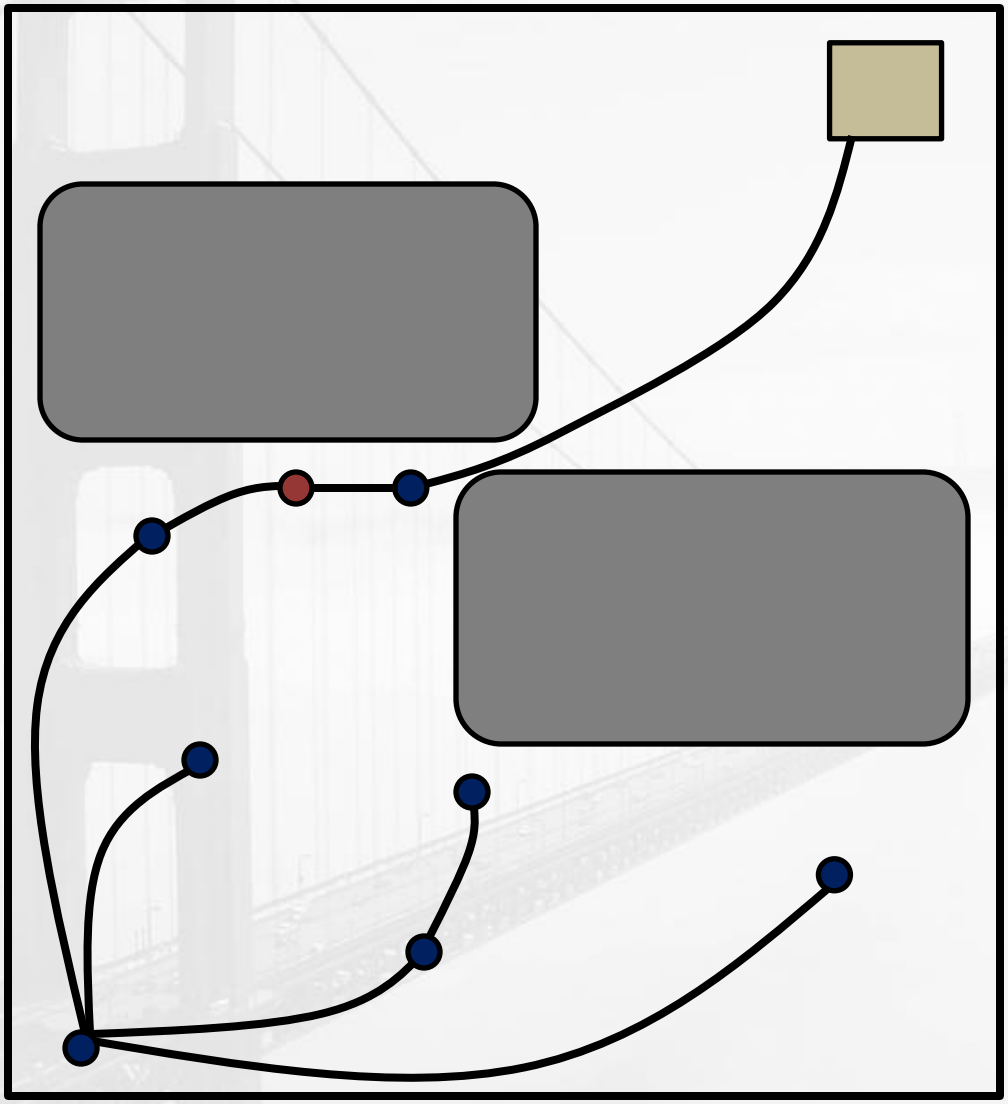

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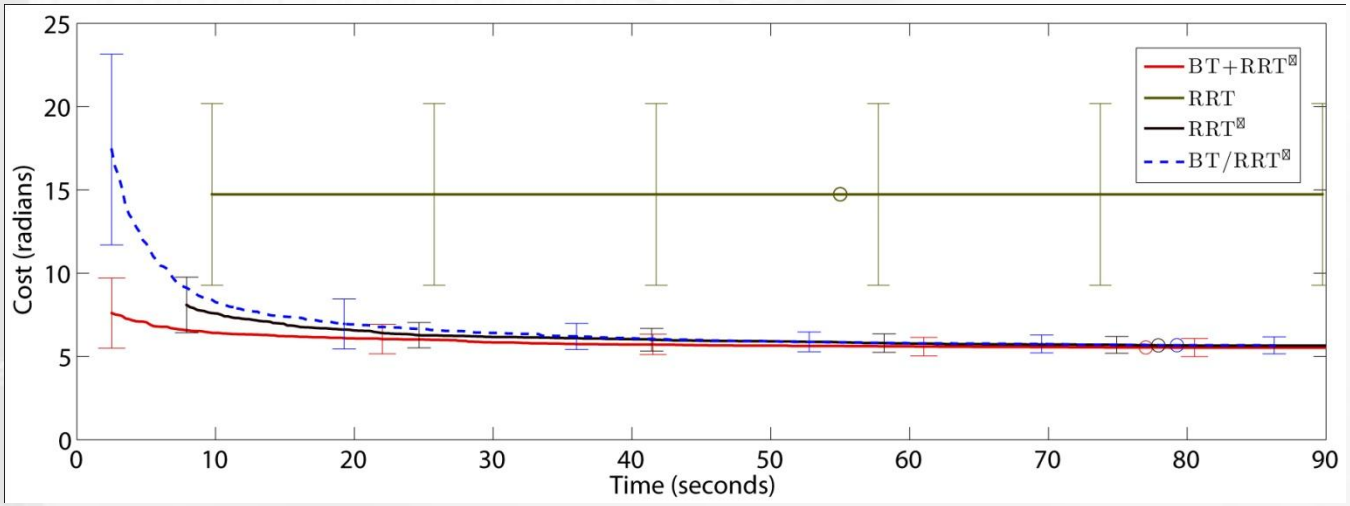
Results (7DOF)

RRT
First solution time: 9.75 s
First solution cost: 14.73
Final solution time: 54.96 s
Final solution cost: 14.73

BT+RRT*
First solution time: 2.52 s
First solution cost: 7.61
Final solution time: 77.14 s
Final solution cost: 5.52



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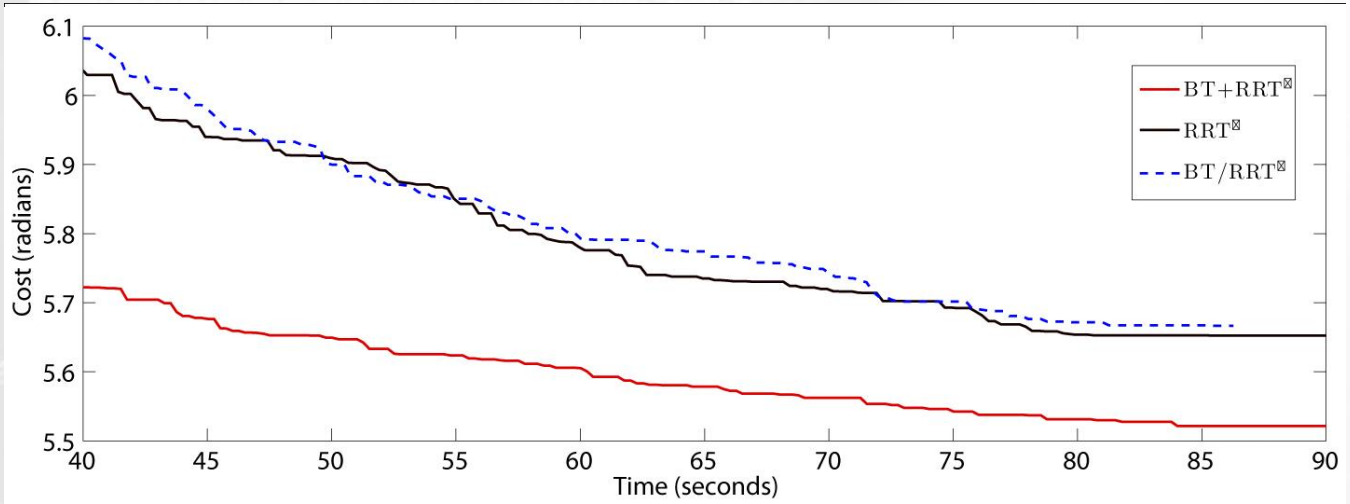

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TABLE I

SEVEN DEGREE OF FREEDOM MONTE CARLO RESULTS

		BT+RRT*	RRT	RRT*	BT/RRT*
Success Rate (100 runs)		100.00%	87.00%	99.00%	100.00%
First Solution	Time (s)	2.52 (3.07)	9.75 (12.52)	7.92 (10.97)	2.51 (2.48)
	Cost	7.61 (2.11)	14.73 (5.49)	8.11 (1.67)	17.99 (5.63)
Final Solution	Time (s)	77.14 (4.49)	54.96 (4.75)	77.85 (3.95)	79.21 (4.47)
	Cost	5.52 (0.53)	14.73 (5.49)	5.65 (0.50)	5.67 (0.51)
Time per Iteration (ms)		19.33 (1.13)	13.78 (1.19)	19.51 (0.99)	19.85 (1.12)

Results (12DOF)

RRT
First solution time: 29.92 s
First solution cost: 19.76
Final solution time: 112.41 s
Final solution cost: 19.76

BT+RRT*
First solution time: 9.74 s
First solution cost: 8.59
Final solution time: 135.28 s
Final solution cost: 7.53

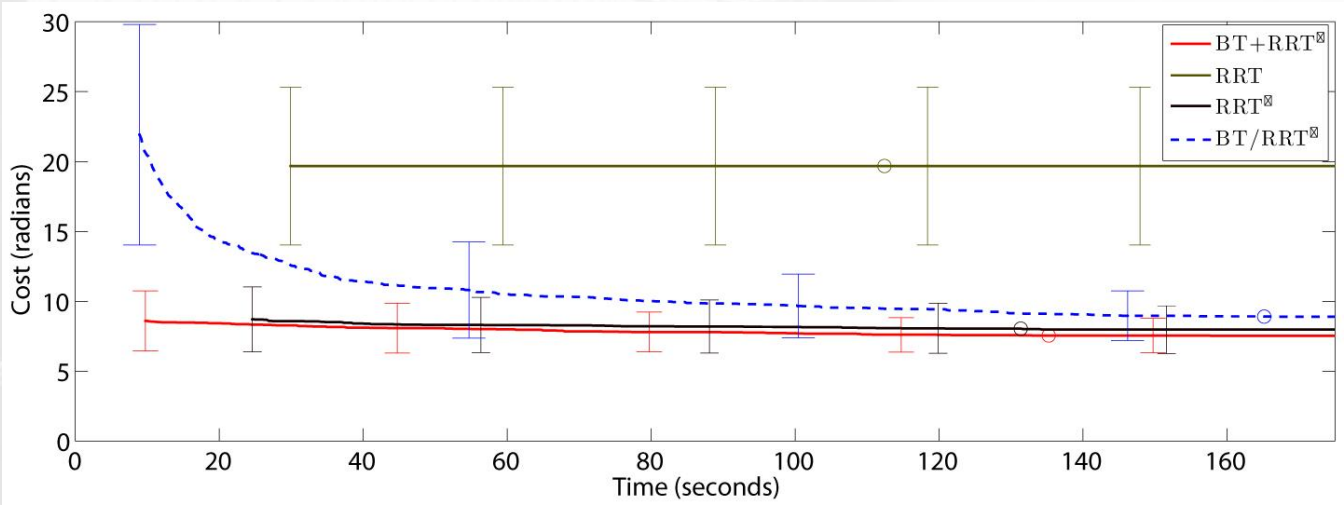

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TABLE II

TWELVE DEGREE OF FREEDOM MONTE CARLO RESULTS

		BT+RRT*	RRT	RRT*	BT/RRT*
Success Rate (100 runs)		100.00%	58.00%	85.00%	100.00%
First Solution	Time (s)	9.74 (12.84)	29.92 (34.05)	24.61 (32.09)	8.94 (11.06)
	Cost (rad)	8.59 (2.16)	19.76 (5.69)	8.71 (2.34)	22.13 (7.72)
Final Solution	Time (s)	135.28 (15.08)	112.41 (19.46)	131.38 (14.49)	165.28 (28.16)
	Cost (rad)	7.53 (1.21)	19.76 (5.69)	7.97 (1.71)	8.83 (1.73)
Time per Iteration (ms)		22.58 (2.52)	18.77 (3.25)	21.93 (2.42)	27.59 (4.70)

IROS 2011 Video

For more information:

Perez, A., Karaman, S., Shkolnik, A., Frazzoli E., Teller, S. and Walter, M., "Asymptotically-optimal Manipulation Planning using Incremental Sampling-based Algorithms," in IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), 2011.

http://ares.lids.mit.edu/manipulation_planning

Interactive Session (Golden Gate Room)

The PR2 Workshop

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