The dynamic reconstruction technique makes it possible to computationally form images using moving coded apertures. The dynamic reconstruction technique can be illustrated using simple tracing of rays through the mask as shown in the attached figures.

First, Figure 1, shows how radiation from a particular point in the scene interacts with the mask in a particular position. A ray from a point in the scene may, of course, go off in a direction away from the detector system, or may hit an opaque area of the mask, or it may pass through an open area of the mask. It will then end up somewhere in one of the areas highlighted in yellow. Other parts of the detector are shadowed by the mask for this particular source position.

If the scene is static, counts due to radiation from that source position will accumulate in the highlighted areas, but not in other areas. The resulting pattern on the detector is an enlarged and shifted version of the mask pattern itself. The shift of the pattern depends on the position of the source of radiation relative to the center line, while the magnification depends on the distance — it will be larger when the source is closer. Many points in the scene may contribute radiation and so the accumulated counts are due to a superposition of shifted and magnified mask patterns (as well as those due to background radiation).

If the magnification is fixed and known, a simple modification of the traditional coded aperture reconstruction method can be used. If the mask has the ideal auto-correlation property, then the source distribution can be estimated using correlation. The method is like the traditional method except that the correlation mask here has to be magnified appropriately [Accorsi 2001]. If the distance to the scene is relatively small (in relation to the mask to detector distance), then the magnification will be rather large, and “near object” artifacts appear in the estimated source distribution. The low order terms of these artifact patterns can be eliminated using a combination of a mask and its anti-mask [Accorsi 2001].

The traditional coded aperture decoding method, or the above modification, cannot, however, be used if there is relative motion between the detector system and the scene, since the geometry, and hence the lateral shift and magnification of the shadow of the mask are then constantly changing. However, Figure 1 can still be used to understand the basic ray geometry, as long as the relative position and orientation of the components of the system are known at each point in time.
Back-projection Algorithm

The basic idea of the “back projection” algorithm for dynamic reconstruction is illustrated in Figure 2. Suppose that an event — the detection of an X-ray photon say — occurs at a known time at a known position on the detector. The source of that photon must have been somewhere in the environment along a line from this position on the detector through one of the open areas of the mask. Knowing the position and attitude of the detector system in the environment at that time allows one to determine which parts of the scene could have produced that count. The “back projection” algorithm simply adds a “vote” to each such position (shown in green in Figure 2). This means, of course, that not only will the true location get a vote, but so will many other locations. However, as more events are processed, locations in the environment that are providing radiation will accumulate “votes” more rapidly than locations that are not sources of radiation.

It turns out that, for the back-projection algorithm to produce useful results, it also has to casts negative votes in areas that are blocked by the mask (shown in red in Figure 2). The result is that, on average, areas that are not sources of radiation will, after some time, have a near zero total, because negative votes will tend to cancel positive votes. On the other hand, areas that are sources of radiation will have a high total because they will receive many more positive votes.

A simple implementation for a planar reconstruction starts with a two-dimensional accumulator array that is reset to zero. Each event on the detector leads to an update of that array. Using the known position and attitude of the detector system relative to the scene, each event is “back projected” into the scene from the detector position through the open areas of the mask. All accumulator cells that can be “seen” from the given detector position have their votes incremented, while those that are blocked by the mask have their votes decremented. Thus an image is built up incrementally as events are detected. The basic algorithm can be described in pseudo-code as follows:

```
Reset accumulator array to zero;
For each event do {
    For each accumulator cell do {
        if cell to detector ray intersects open cell do {
            increase total in accumulator cell;
        }
        else do {
            decrease total in accumulator cell;
        }
    }
}
```
Now suppose that there is relative motion between the detector system and the scene. In Figure 3, for example, we show a second event that occurs when the detector system has approached the scene more closely. The relative geometry of detector, mask and scene has changed, but once again, we can compute, using simple geometry, which part of the scene could have given rise to an event and which could not. We increment the count in those areas visible through the mask from the position of the event on the detector, and we decrement the count for those areas that are not.

In Figure 4, we show a third event, when the detector system is even closer to the scene. Finally, in Figure 5 we superimpose the previous three figures showing the operation of the back-projection algorithm under the changing geometry. In practice of course, there would be thousands of events, not just three, but the same principle holds.

It can be shown, by the way, that the “back projection” algorithm for dynamic reconstruction produces exactly the same result as the correlation algorithm in the special case that there is no relative motion between the detector system and the scene.

**Computational Cost**

- The computational cost of the back-projection method is the order of \( N \) additions and subtractions per event, where \( N \) is the number of picture elements in the resulting image — and hence elements in the accumulator array. If there are a total of \( M \) events, then the overall computational cost is \( O(NM) \). There is some additional overhead, since the intersection of rays with the mask must be calculated — although this can be done efficiently using pre-computed lookup tables. If events are “batched” by taking \( K \) short exposure detector “images,” instead of back-projecting each event separately, then the computation cost is \( O(NK) \) multiplications and additions instead.

- For comparison, the computational cost of the traditional correlation decoding method is \( O(N^2) \) multiplications and additions, where \( N \) is the number of elements in the mask, since each of the \( N \) pixel in the resulting image requires a correlation touching every one of the \( N \) pixels in the detected signal. The correlation can be performed using multiplication in the Fourier Transform domain, leading to a number of operations the order of \( O(N \log_2 N) \), but for the number of elements found in typical masks, and for the large constant multiplier in the formula for the number of operations, this is rarely advantageous (Note further that \( N \) for an ideal mask is never equal to a power of 2).
Comparison to traditional coded aperture method

- The dynamic reconstruction method can handle relative motion between the detector system and the scene. The traditional correlation method for coded apertures cannot.

- The dynamic reconstruction method builds up an image as events are accumulated. The correlation method computes the image at the end of an exposure period.

- The dynamic reconstruction method can be used with detector systems containing arrays of discrete detectors, but it has special advantages for detector systems that have continuous position readout of event positions — such as Anger cameras — or equivalently, detector systems with very small detector elements. Further, when using discrete detector elements, the size of the detector elements need not be matched to the size of mask element shadows. The correlation method, on the other hand, is based on use of arrays of discrete detector elements matched in size to the mask element shadows. This is a problem when the scene is nearby, since the size of a mask element shadow on the detector array varies with the distance to the scene.

- The dynamic reconstruction method can be used with masks that do not have the ideal two-valued auto-correlation property. This can be used to advantage for example when the detector system is not square or near square in aspect ratio. The correlation method works only with ideal masks, and known designs for ideal masks are square or nearly square.

- The dynamic reconstruction method can be used with masks that have other than the traditional 50% fill factor. This is an advantage if the scene does not consist of isolated point sources. It has been shown that better signal-to-noise ratio can be expected in this case by using masks with lower fill factors [Accorsi 2001]. Masks with ideal correlation properties needed for the correlation method are known only for near 50% fill factor.

- The dynamic reconstruction method does not yet have a solid mathematical foundation, and may need to be further modified to attain such a basis. The correlation method does have a clear and interesting mathematical foundation (although this has not yet been fully explored either).
Example

In previous work, a system using a (one-dimensional) side-looking detector behind a (one-dimensional) mask mounted in a vehicle has been described [Ziock et al. 2003], [Ziock et al. 2004]. In this system a two-dimensional image is developed as the vehicle moves. One dimension is “depth” — i.e. cross-track distance — while the other it along-track distance.

Sample images from a simulation of the dynamic reconstruction algorithm for coded aperture imaging using this geometry are shown in Figure 6. The vehicle appears in a fixed position at the bottom center of the rectangles. The reconstructed scene sweeps past the vehicle to the left as the vehicle moves. The height of the image corresponds to just over 100 meters.

The image is "developed" as the vehicle moves from left to right (subfigures are arranged left-to-right and top-to-bottom). The reconstructed image is slid off to the left at the same rate, so that the detector array appears to remain stationary while the recovered image moves to the left.

Detected in sequence are: a 2 mCurie source at 100 m, a 0.5 mCurie source at 25 m, followed by a 1 mCurie source at 50 m. Total detected counts are fewer than 18,000 with almost 10,000 of those being from background radiation. During most time steps individual detectors pick up only between 0 and 3 or 4 counts.

Note that, not surprisingly, resolution decreases with distance, as seen in the different sizes of the responses to each of the three sources at different distances. Also note the characteristic “bow tie” shape of the response to each of the sources resulting from the fact that the response to a source is maximal at its distance, but is still present — although “out of focus” — in neighboring depth planes.

References


Sample reconstruction animation may be found at: http://csail.mit.edu/~bkph/movies/backprojection.html
Appendix — Additional Details

The above outlines the basic operation of the back-projection algorithm for dynamic reconstruction. There are some additional details to note:

- For best result, the positive and negative votes should actually not be exactly equal in magnitude. The relative magnitudes of positive and negative votes should be proportional to the relative numbers of closed and open cells in the mask. In the case of masks with ideal correlation properties, the number of open cells is almost, but not quite, the same as the number of closed cells. The reason is that such masks always have an odd number of cells overall. The actual positive and negative weights to use are proportional to the number of closed and open cells in one repetition of the mask.

- If the scene is static, then, with this minor modification, the reconstructed image is identical to the one that would have been obtained using the traditional correlation method. Of course, the dynamic reconstruction method really comes into its own when the scene is not static, in which case the traditional correlation decoding method cannot be used.

- The computation can be organized in different ways. For example, for each event, one might step through all mask elements and project a ray from the detector position through that mask element onto the accumulator array. It is, however, more efficient to instead organize the computation around stepping through the cells in the accumulator array and figuring out for each one whether it is visible from the detector position.

- If events occur at a high rate, then relative motion between detector system and scene will be small between events and, for efficiency, events may be “batched.” That is, rather than increment and decrement the counts in the accumulator array for every single event, events can be counted in finite sized detector elements over a short period of time, and those counts then back-projected instead of individual events. In essence, an array of detectors can be used to obtain an image over a sufficiently short exposure time and then this image back-projected. This works because the geometry can be assumed to be more or less constant if the exposure time is short enough.

- The simple two-dimensional reconstruction using a two-dimensional accumulator array is appropriate when the differences between distances to different parts of the scene is relatively small compared to the distance to the scene. If there is a large range of distances to different parts of the scene, then the back-projection method can be applied in several depth planes. Each plane will show “in focus” what is at a given distance, while contributions from other distances will be out of focus or blurred. This is entirely analogous to an optical microscope.
where one depth plane is in focus for a given setting of the optics. Other depth planes still contribute to the image, but features in them are out of focus.

- In the limit as one considers many such depth planes, one arrives at a three-dimensional accumulator array. This permits separation of scene elements that occur at different distances. It has to be remembered though that the result is like a “through focus” stack of microscopic images, not quite a true three-dimensional reconstruction (as in CT or MRI), since “out of plane” scene elements are merely out of focus, but still contribute to the image.

- The back projection contribution or “vote” should ideally be weighted differently depending on distance from the detector system in order to get close to a faithful reconstruction.

- In the case of a detector system consisting of a discrete arrays of individual detectors, the contribution or “vote” should be weighted by how much of the detector is visible through the mask from the point in the scene to which a vote is contributed. This factor can be ascertained using simple geometry. It can also be precomputed and stored in a lookup table.

- For the traditional coded aperture mask method, masks with ideal correlation properties have to be used. Such mask patterns have the feature that they produce the same correlation for any non-zero shift. That is, they produce only two different auto correlation results — one value for zero shift and another for any other shift. Interesting mathematical techniques have been discovered for creating such patterns, but they only apply for certain sizes (e.g. twin primes) and one fill factor (ratio of open mask elements to total number of mask elements.)

- Essentially all such ideal masks have about one half of the cells open and one half closed. Currently there seem to be no methods known for generating masks with ideal auto-correlation properties other than those with approximately 50% fill factor. For isolated point sources, “half open” masks are ideal. But for distributed sources, masks with lower fill factors, if they existed, would actually lead to better signal to noise ratio [Accorsi 2001].

- The back-projection method does not directly depend on masks having ideal correlation properties, although, generally it appears that the quality of reconstruction is best with ideal or near ideal masks. Nevertheless, back-projection can be used with “non ideal” masks, such as ones with a lower fill factor.

- The back-projection method does not directly depend on the trajectory of the relative motion between the detector system and the scene. Generally, it appears that certain straight line trajectories may give good results, but this question has not been explored in detail. So far, only linear motion with a “side looking” geometry has been explored in detail.