MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF COMPUTER SCIENCE AND ELECTRICAL ENGINEERING

6.801/6.866 MACHINE VISION

Problem 1 solution: We wish to add the ability to recover a rotational component to the translation component of a "fixed flow." We add rotation ω about the point (x_0, y_0) to translation (u_0, v_0) and obtain

$$u = u_0 - \omega(y - y_0)$$
$$v = v_0 + \omega(x - x_0)$$

for the motion field. The brightness change constraint equation becomes

$$E_t + (u_0 - \omega y')E_x + (v_0 + \omega x')E_y = 0$$

where $x' = x - x_0$ and $y' = y - y_0$. Typically (x_0, y_0) would be chosen to be in the middle of the sensor array. For example, we could chose $x_0 = m/2$ and $y_0 = n/2$, for an $n \times m$ array. We minimize

$$\iint \left(E_t + (u_0 - \omega y')E_x + (v_0 + \omega x')E_y\right)^2 \, dx\, dy$$

differentiation w.r.t u_0, v_0 and ω and setting the results equal to zero we obtain

$$u_0 \iint E_x^2 + v_0 \iint E_x E_y + \omega \iint (x'E_y - y'E_x)E_x = 0,$$
$$u_0 \iint E_x E_y + v_0 \iint E_y^2 + \omega \iint (x'E_y - y'E_x)E_y = 0,$$
$$u_0 \iint (x'E_y - y'E_x)E_x + v_0 \iint (x'E_y - y'E_x)E_y + \omega \iint (x'E_y - y'E_x)^2 = 0.$$

We can solve these three linear equations for the three unknowns u_0, v_0 and ω provided the determinant of the 3×3 coefficient matrix is non-zero.

Not surprisingly, the method fails when the brightness pattern is rotationally symmetric about the point (x_0, y_0) since then $x'E_y - y'E_x = 0$ everywhere.