MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF COMPUTER SCIENCE AND ELECTRICAL ENGINEERING

6.881 COMPUTATIONAL IMAGING

HWP 1 **Handed out: 2006 Feb. 15th** **Due: 2006 Feb 22nd**

Name: **email:**

Problem 1: In a "Synthetic Aperture Microscope" (SAM) let **c** be the vector of unknown sub-pixel reflectance values, and **b** the vector of measurements of pixel brightness under different textured illumination patterns. Let the matrix T give in each row the sub-pixel illumination of a pixel under a particular textured illumination. Then one can write

$$\mathbf{b} = T\ \mathbf{c}$$

What are the sub-pixel reflectance values when three textures are used with brightness patterns (1 2), (3 4), and (2 1), yielding pixel brightness values 4, 10, 5, respectively?

Hint: this problem is small enough to do by hand if you remember that the inverse of a matrix is the transpose of its cofactors divided by the determinant, so that e.g.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Problem 2: Suppose in a planar (2D) parallel projection system, we sample the Fourier Transform in a regular polar pattern, with constant increments in radius ($\delta\omega$), and constant increments in angle ($\delta\theta$).

What does the Voronoi diagram of the sample points look like?

What is the area associated with each sample (and hence the weight assigned to the corresponding spatial frequency wave in "direct" reconstruction)? (If you like, you may approximate some of the sides of polygons with appropriate circular arcs).

(continued on other side...)

Problem 3: Suppose in a volume (3D) parallel projection system, we take projections in directions that uniformly sample the unit sphere of possible directions. Each such projection gives us information about a slice of the 3D Fourier transform that passes through the origin and is perpendicular to the direction of projection. How does the sampling density vary with radial frequency ω?

If we simply backproject the accumulated projection data without filtering, we will not obtain the correct density distribution. Describe how the resulting estimate of the density distribution is corrupted by describing what happens to low frequency components versus what happens to high frequencies.

What kind of filter is needed to undo this effect? Describe the filter in the frequency domain (you need not find its inverse Fourier transform).

Problem 4:

The Modulation Transfer Function (MTF) of an ideal round lens is the convolution of constant height circular disc with itself. Find the convolution of

$$m(u, v) = \begin{cases} \frac{1}{\pi w_0^2} & \text{for } u^2 + v^2 \leq \omega_0^2 \\ 0 & \text{for } u^2 + v^2 > \omega_0^2 \end{cases}$$

with itself.

Does the transfer function have an absolute limit beyond which no frequency component is passed?

Is the transfer function flat near zero frequency?

Problem 5: In the inverse problem of solving for x given

$$y = \sin x \quad \text{with} \quad -\pi/2 \leq x \leq +\pi/2$$

what is the relationship between variance σ_x^2 in the estimated value of x, and the variance σ_y^2 in noisy measurement of y?

For what values of y is there large amplification of error in the inversion?