University of California at Berkeley

Department of Computer Science and Electrical Engineering

CS-280 Computer Vision

Handed out: 2002 Aug. 28th **Due on: 2002 Sep. 6th**

Problem 1: In class we solved the least squares problem of estimating the speed at which an image moves in a one dimensional camera based on the constant brightness assumption. The constant brightness assumption (in one dimension) can be expressed in the form

$$uE_x(x,t) + E_t(x,t) = 0,$$

where E_x and E_t are the x and t derivatives of image brightness $E(x,t)$, while u is the speed of the image motion.

Now suppose that the illumination is *not* constant, but changing in such a way that $uE_x + E_t = k$ (rather than being equal to zero). Clearly this is equivalent to using the incorrect value $E_t' = E_t - k$ for the time derivative of the brightness in the original equation. Suppose we do not know the value of k ahead of time.

Lets first blindly apply the method worked out in class. Suppose that the "image" runs from x_1 (with brighntess E_1) to x_2 (with brighntess E_2).

(a) What is the least squares solution for u we would get based on the "corrupted" time derivative (E_t') of brightness?

(b) Is there any error in the estimated speed u if the brightness at the right end of the image matches that at the left end?

Now let us assume that we *do* know that there is a change of overall illumination with time, but we do not know the magnitude of k. Let us try and estimate both u and k using a least squares method.

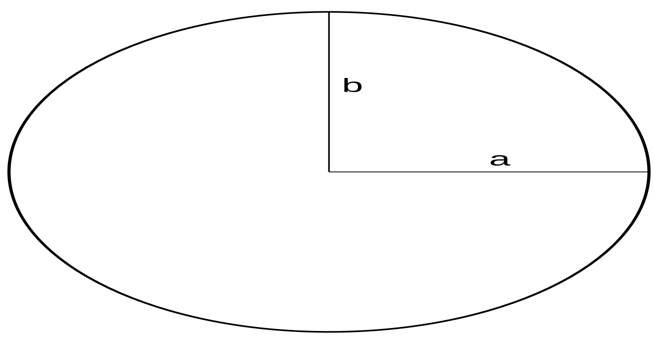
(c) What are the best fit values for u and k? Assume that the linear "image" runs from x_1 (with brighntess E_1) to x_2 (with brighntess E_2).

(d) How do the expressions for u and k simplify if the brightness at the right end of the image matches that at the left end?

(continued...)

Problem 2: Consider a monocular system for estimating the oriention of a planar surface based on an image of a circle drawn on that surface. If we view the plane "straight on", the circle will be imaged as a circle (by "straight on" we mean that the optical axis is perpendicular to the plane, and the optical axis passes through the center of the circle). We get an ellipse in the image if the plane is tilted so that its normal vector is no longer parallel to the image plane. From the eccentricity of the ellipse we can determine how far the plane is tilted relative to the "straight on" orientation.

To simplify matters, we here consider *orthographic* instead of *perspective* projection. Imagine parallel rays perpendicular to the image plane arriving from a very distant light source. A circular disk is suspended somewhere above the image plane blocking some of the incident light. The result is an elliptical shadow in the image. The angle between the image plane and the plane containing the circular disk is θ (see Figure).



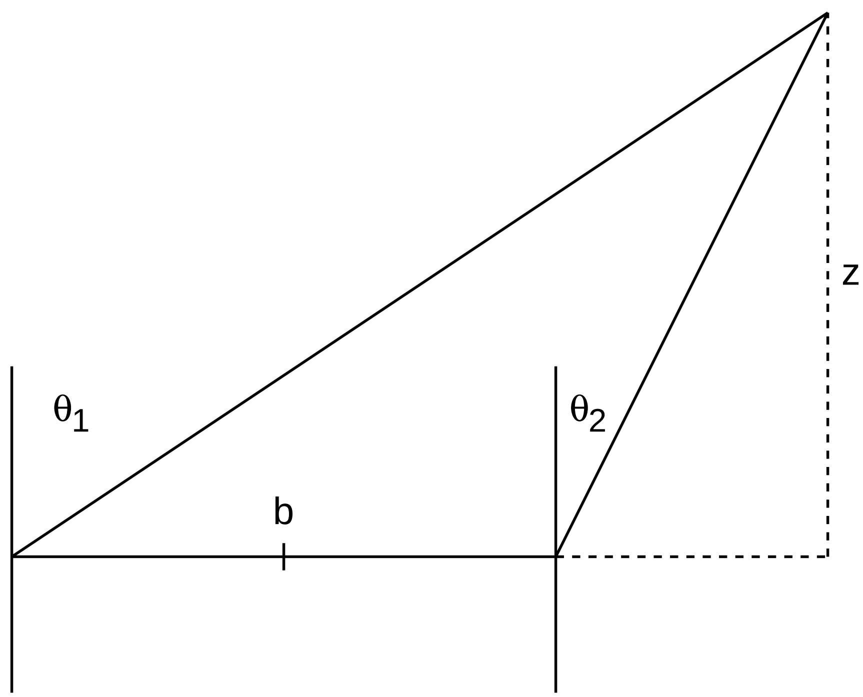
(a) Find the length of the minor axis b in terms of the length of the major axis a and the angle θ.

The eccentricity e of an ellipse is defined by $b^2 = a^2(1 - e^2)$. Express the eccentricity of the ellipse in the image in terms of the angle θ.

(c) Image measurements are hard to make accurately. If we are to estimate θ based on the ratio of b to a, then we should also consider the effect of small errors in the measurement of b (assume a is known accurately). What is the relationship between small errors in measurement δb and corresponding errors in the estimated orientation of the plane $\delta\theta$? Give an expression the for error sensitivity $d\theta/db$ as a function of the eccentricity e of the ellipse.

(c) If you had to select a good position for a camera in a monocular visual wheel alignment system based on imaging the circular rim of the wheel, would you put the camera somewhere along the line passing through the axis of the wheel?

Problem 3: A simple planar binocular stereo system has a baseline of length b and two 'cameras' measuring the angles θ_1 and θ_2 between rays to objects in the world and the baseline. We erect a coordinate system with the x-axis lined up with the baseline and the origin at the midpoint of the baseline. The z-axis is perpendicular to the x-axis.



(a) Show that the intersection of the two rays lies at

$$x = \frac{b}{2}\frac{\sin(\theta_2 + \theta_1)}{\sin(\theta_2 - \theta_1)} \qquad \text{and} \qquad z = b\frac{\sin\theta_1 \sin\theta_2}{\sin(\theta_2 - \theta_1)}$$

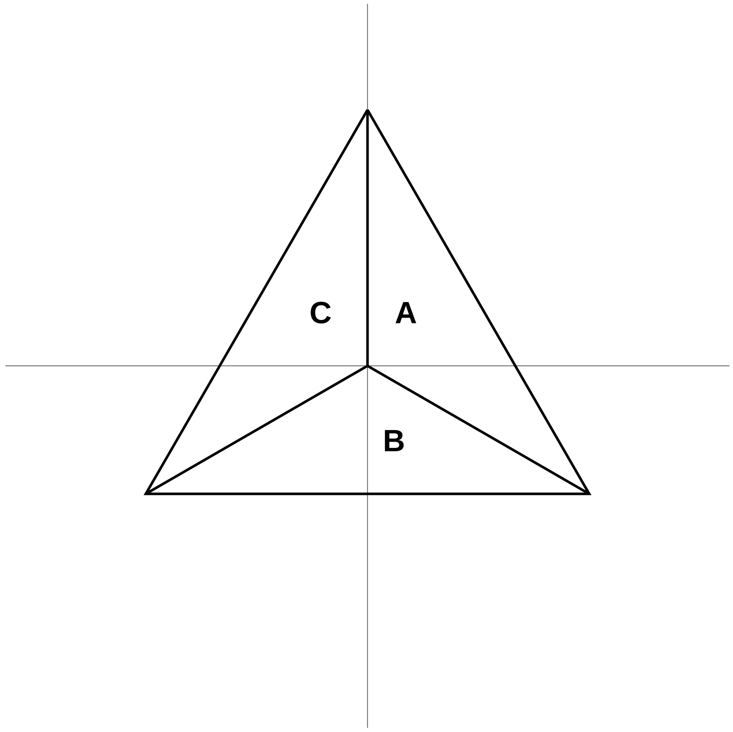
(b) Show that

$$z \approx b\frac{1}{\sin(\theta_2 - \theta_1)}$$

when $\theta_1 \approx \pi/2$ and $\theta_2 \approx \pi/2$. Hence depth z is inversely proportional to disparity $(\theta_2 - \theta_1)$ (and proportional to the baseline b).

(c) The measurements of ray directions are never perfectly accurate. Estimate the errors in z resulting from errors δ_1 in measuring θ_1, and δ_2 in measuring θ_2. Show that the absolute error in estimating depth grows as z^2/b. Consequently the relative error (i.e. $\delta z/z$) grows as z/b (Hint: differentiate w.r.t. the angles).

Problem 4: Consider a pyramid of unknown height h with an equilateral base in the x-y plane as shown.



One edge of the base is parallel to the x-axis. The three isosceles triangles forming the sides of the pyramid are coated with matte white paint that obeys Lambert's 'law' and is illuminated by a distant light source of unknown brightness lying in direction $(1/\sqrt{2}, 0, 1/\sqrt{2})^T$. Now suppose that the observed brightnesses of the three facets are in the ratio

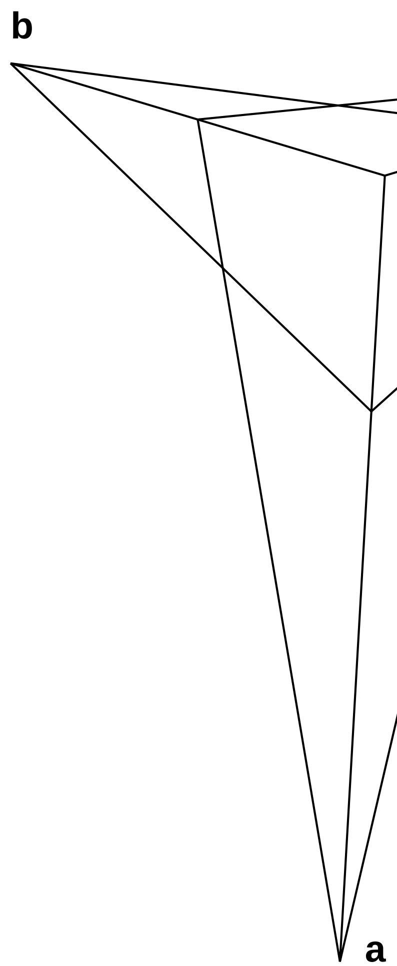
$$E_A : E_B : E_C = 3 : 1 : 2$$

What is the height h of the pyramid as a multiple of the length of an edge of the base?

Problem 5: Consider a perspective image of a rectangular brick. Three vanishing points can be obtained by intersecting extended image lines corresponding to parallel edges on the object (see Figure).

(a) Suppose that the vanishing points are at $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ in the image plane. Suppose that the center of projection is at $\mathbf{r}$ above the image plane. Write down three (quadratic) equations involving the vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{r}$, based on the expected angles between lines from the center of projection to the vanishing points in the image plane.

(b) Eliminate the quadratic terms to obtain three (redundant) linear equations in $\mathbf{r}$.



(c) Consider the triangle formed in the image plane by connecting the vanishing points. Show that the principal point $\mathbf{r}'$ (foot of the perpendicular from the center of projection onto the image plane) lies at the intersection of the perpendiculars dropped from the three vertices onto the opposite sides. (Hint: $\mathbf{r} = \mathbf{r}' + f\hat{\mathbf{z}}$ where f is the principal distance, that is the height of the center of projection above the image plane, and $\hat{\mathbf{z}}$ is a unit vector perpendicular to the image plane).

(d) Give a (non-trivial) bound on the principal distance in terms of the maximum or minimum of the lengths of the sides of the triangle.

(e) Consider the special case when the vanishing points form an equilateral triangle. Show that the principal distance f equals kl, where l is the length of the sides. What is the value of k?