University of California at Berkeley

Department of Computer Science and Electrical Engineering

CS-294-6 Computational Imaging

HWP 2 **Handed out: 2002 Mar 13th Due: 2002 Mar 20th**

Problem 1: Suppose a synthetic aperture microscope (SAM) has N beams which impinge with the same angle of incidence θ on the reference plane and are equally spaced around a ring. Let the wavelength of the monochromatic coherent light be λ. Show that the highest spatial frequency term in the interference pattern will be

$$\omega_{\text{high}} = \frac{4\pi}{\lambda} \cos\theta$$

if N is even.

What is the spacing between peaks in this component of the interference pattern? Express this spacing in terms of the wavelength λ of the monochromatic light. What is the highest frequency term when N is odd?

Show that (aside from the zero frequency or "D.C." term) the lowest spatial frequency in the interference pattern will be

$$\omega_{\text{low}} = \frac{4\pi}{\lambda} \cos\theta \sin\frac{\pi}{N}$$

What is the ratio of the wavelength λ_{low} corresponding to this frequency and the wavelength λ of the monochromatic light? Given this result, comment on the properties of an Computed image derived from data collected by such a device.

Problem 2: The frequency $\omega(\theta)$ of the interference pattern between two beams varies with the angle θ between the beams as described in the previous problem. Here we consider the variation in sampling of the transform domain in the limit when there are very many beams.

Express the density of sampling of the frequency domain at radial frequency ω as a function of the angle θ. Note that density equals number of samples per unit area, and that the area allocated to a sample is proportional to $r\,dr/d\theta$.

Express the density of sampling as a function of radial frequency ω instead of the angle θ.

Clearly the sampling density is high near the lowest and near the highest frequencies. Why do we nevertheless consider sampling to be poor in those areas when compared to the mid frequency range?

Problem 3: To get rid of the requirement for a reference image, consider the following simple method for calibrating a SAM using a microbead: Arbitrarily treat the zero-th beam as a reference beam that will be on throughout the calibration. With just one beam, how does the surface illumination vary with position?

Now add one other beam to create a simple interfenence pattern. Adjust the phase of the second beam until the brightness in the pixel containing the image of the microbead is maximum. What is the phase relationship of the two beams at the microbead? Why might it be better to adjust the phase so as to minimize the brightness instead?

Repeat for each of the remaining beams. Suppose the phases so recovered (for maximum brightness) are ϕ_l for $l = 1, \ldots N-1$, with $\phi_0 = 0$.

What is the brightness of the interference pattern at (x, y), when we command the phase of each beam to be $\phi_l + \delta\phi_l$ Assume the microbead is at (x_b, y_b) and that the wave-numbers of the beams are $\mathbf{k}_l$.

Now while we know from the image which pixel the microbead is in, we do not know where in the pixel it lies. What is the effect of this uncertainty in the compted image recovered from the response to a large number of finely textured illumination patterns?

Problem 4: Suppose you wish to produce two spots of light at

$$(x, y) = (\pm L/2, 0)$$

in a synthetic aperture device by simply adding beam complex amplitudes computed for producing the two isolated spots separately. Suppose there are N equally spaced beams and that the wavelength on the reference plane is $\lambda' = \lambda / \cos\theta$.

First determine what the complex amplitude of the beams should be as a function of the angle ϕ that the beam makes with the x-axis in order to produce a bright spot at $(L/2, 0)$. Repeat for the spot at $(-L/2, 0)$.

Now combine the two results while shifting the phase of the spot at $(+L/2, 0)$ by $+\alpha/2$ and shifting the phase of the spot at $(-L/2, 0)$ by $-\alpha/2$.

For what values of α do the beams appear to interfere destructively at the origin?

(Please see next page for Problem 5)

Problem 5:

(a) Flip a fair coin 19 times and record the results. Taking "0" to represent tails and "1" to represent heads, construct a binary vector of length 19. Now produce a periodic binary pattern by laying copies of the vector so constructed next to one another. Find the correlation of this periodic pattern with the original vector for all shifts. (You may find it useful to write the 19 bits on a strip of paper that can be moved along another the periodic binary pattern).

Now instead use the quadratic residues for $n = 19$ to define the "1"s in a binary array of length 19. Again, produce a periodic pattern from this vector and find the correlation of this periodic pattern with the original vector for all shifts. (Refer to the Math World web site for definition and properties of quadratic residues).

Comment on the differences between the two results and the advantages or disadvantages of one or the other in one-dimensional "coded aperture" imaging.

(a) Consider now a one dimensional "random binary mask" of length N where the probability that a particular cell has value "1" is p — and correspondingly the probability that it has value "0" is $(1 - p)$. The probabilities of having a "1" at different positions are independent.

Use this binary vector of length N to create a periodic pattern as above. We'll now correlate the perdiodic pattern so created with the original vector of length N. What is the expected value of the correlation for zero shift?

What is the expected value of the correlation with shift by a number of cells that is not an integer multiple of N?

What is the variance of the value of the correlation in this case? Let us consider the ratio of the peak response to the standard deviation of these "side lobes" as the signal-to-noise ratio (SNR). Show that the SNR so defined is proportional to $\sqrt{N}$. How does the SNR so defined vary with p?