Fresnel Transformations of Images

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Summary—An indirect procedure for image measurement is presented, with applications ranging from X-rays to the infra-red. The procedure is based on measurement of a Fresnel transformation of the image, and resembles Gabor's holograms[1]. However, since diffraction is not used to produce the holograms, no coherent illumination is needed, and resolution is not limited by chromatic aberrations. Refinements of the holograms and the image reconstruction techniques allow background control in the final image.

One application to X-ray star photography may be used to illustrate the procedure. In these spectral regions we can construct a large aperture, high resolution, large field camera with neither refracting nor reflecting components. Conventional cameras are impossible since such components are unavailable for these short wavelengths.

Résumé—Une manière indirecte de mesure d'image est presentée avec exemples depuis les rayons X jusqu'aux infrarouges. Le procédé est basé sur la mesure de la transformation de Fresnel d'une image, et resemble aux holograms de Gabor[1]. Cependant, puisque le phenomene de diffraction n'est pas utilisé pour produire des holograms, il n'est pas nécéssaire d'avoir une illumination cohérente et la résolution n'est pas limité par les abérations chromatiques. Les perfectionment des holograms ainsi que les techniques de reconstruction d'image permettent un control du fond de l'image final.

Une application à la photographie des étoiles par les rayons X sert à illustrer le procédé. Pour les rayons X l'on peut construire un appareil photographique de large ouverture, grande résolution et de large champ sans utiliser aucun élément réflechissant où refractant. Il n'est pas possible d'utiliser un appareil photo ordinaire car il n'éxiste pas d'element pour ces longueurs d'onde.

Zusammenfassung—Ein indirektes Verfahren fuer Bildmessungen findet Anwendung in den Bereichen von X-Ray zu infra red. Dieses Verfahren hat seinen Ursprung in den Messungen von Bildern, mit Hilfe der Fresnel Transformation, und ist einem Gabor Hologram aehnlich[1].

Aber seitdem die Diffraktion nicht mehr benutzt wird um ein Hologram zu produzieren, ist keine coherente illumination mehr notwendig. Ausserdem ist das Ergebnis nicht begrenzt, (durch chromatische Abweichung). Verfeinerungen des Holograms, und Verfeinerungen des Rekonstruktionverfahrens der Bilder ermoeglichen eine Hintergrund-Steuerung (background-control) an dem endgueltigen Bild.

Eine Anwendung dieses Verfahrens in Stern Fotografie kann man zur Illustration benutzen. In diesen Spektralbereichen kann man eine Kamera konstruieren, mit grossem Bildwinkel, grosser Oeffnung und guter Aufloesung, ohne brechende und spiegelnde Teile. Mit herkoemmlichen Kameras ist es unmoeglich, da die oben angefuehrten Teile fuer solche kurzen Wellenlaengen micht zu erhalten sind.

In order to introduce the concepts and purposes of Fresnel transformation I would like to start with a physical example. This example is an X-ray star camera. A normal camera at such wavelengths would be faced with the problem that niether refracting nor reflecting materials are available to provide focusing. Pinhole cameras, although they would give adequate resolution, do not have sufficient aperture to acquire a statistically large number of photons. ???

Our approach is to replace the lens of a normal visible light camera with an etched self supporting zone plate. It is to be emphasized that this is a large

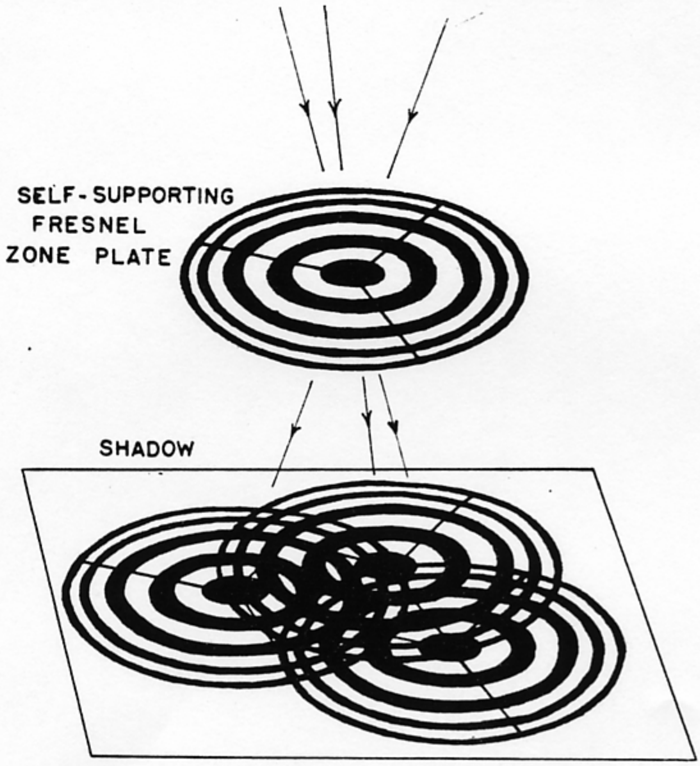

FIG. 1. X-ray star camera.

coarse zone plate and that it is not used to focus the X-rays. Instead, it casts shadows; one shadow for each star. Indeed, with such short wavelengths these shadows are not significantly blurred by diffraction and the shadows are achromatic. The arrangement is illustrated in Fig. 1. It might be mentioned that off-axis stars give circular and not elliptical zone shadows. This independence of direction for the shadow contours will give a large usable field of view, facilitating extensive sky coverage. Incidently, we still have to place the camera above the atmosphere, and we still have to select our wavelength sensitivity by appropriate filtering.

Clearly the photographic result is not the desired image, but is an ensemble of Fresnel zone plates. Because of similarities to Gabor's experiments, these patterns shall be called holograms. It is possible to reconstruct the desired star images from this hologram.

A simple reconstruction technique for hologram originals is to make a reduced size transparency of the hologram; the minified copy should be such that individual zone plates of the ensemble have conveniently short focal lengths for visible light. In this case one can simply use this hologram transparency as a lens to form an image of a monochromatic point source in the laboratory. Each zone plate of the ensemble acts independently of the others, and will focus like a lens. Thus, each zone plate forms a point image of the laboratory source, and we find that the original array of bright points, stars, is reconstructed.

As a demonstration, we simulated some stars by illuminating pinholes in an opaque sheet and used visible light rather that X-rays to form the hologram. The results are shown in Fig. 2. At the upper left are shown the n

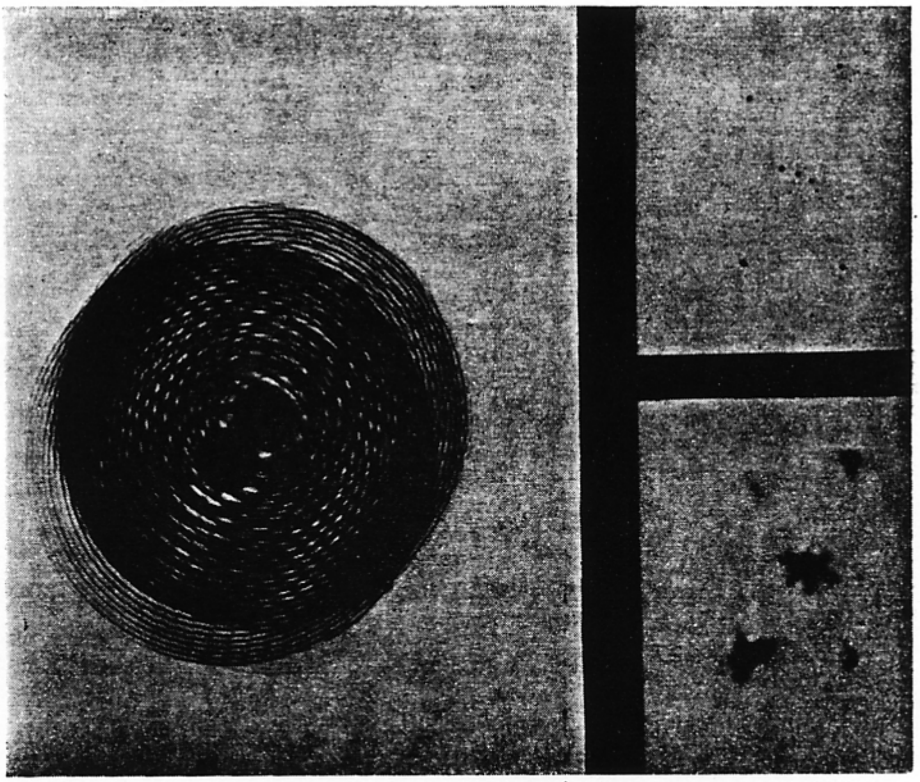

FIG. 2. Illustrative sample of optical Fresnel transformations.

pinholes (negative for clarity). On the right is shown the n distinct shadows of the zone plate; this is our hologram. At the lower left is the reconstruction by diffraction from a reduced copy of this hologram. We would like to call attention to the extensive overlap of the zone plates in the hologram. The fuzziness of the reconstruction is partly due to the long wavelength of visible light preventing good shadowcasting of the zone plate in the original exposure.

Now, in practice, using the hologram as a lens for the reconstruction passes a considerable amount of undiffracted light which raises the background level. Two approaches are used together to suppress this background. One approach is to bleach the hologram transparency. By bleaching we get a phase modulating transparency much like R. W. Wood's phase zone plates. In this manner, the reconstructing efficiency is improved relative to the undiffracted light. Alternatively, a focal isolation technique is employed to remove the undiffracted light. The reconstruction technique becomes as illustrated in Fig. 3. Considering the system without the hologram, the lens focuses the point source on to the obstructing spot and little light gets to the eyepiece. When the hologram transparency is inserted, however, its zone plates act as negative lenses focusing the source onto a plane where the reconstructed image can be viewed with an eyepiece.

The zone plates also act as positive lenses and create large dim halos around each star image. These halos are, in fact, another hologram superimposed on the reconstructed image. The nature and removal of this extra

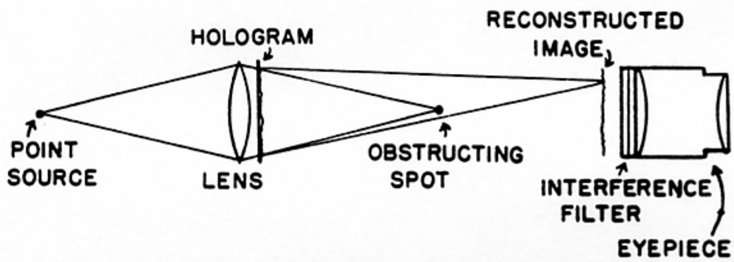

FIG. 3. Hologram reconstruction.

hologram has been discussed by Bragg[(2)] and Rogers[(3)]. We have found that this additional hologram is less troublesome in practice than the undiffracted background, which we have succeeded in eliminating.

Unfortunately along with background elimination we remove the low spatial frequency components of the original image, so that only stars are well reconstructed. The transfer function properties and their indications will be discussed later.

In order to develop a quantitative interpretation of the overall process we find it convenient to introduce to concept of Fresnel transformation. This concept of Fresnel transformation will have far wider analytical application than merely the interpretation of the camera just described. Fresnel transformation is similar to Fourier transformation except that instead of considering an image as the sum of spatial frequency components, we consider the image as the sum of Fresnel zone plate components. Hence the name Fresnel transformation. Just as spatial frequency components of differing frequency are orthogonal to one another, so Fresnel zone plate components which are displaced from one another are orthogonal. By orthogonality, it is meant that no cross correlation exists between the components. For one dimensional zone plates the orthogonality relation takes the form

$$\lim_{L\to\infty} \frac{1}{L}\int_{-L}^{+L} \cos(x+a)^2 \cos(x-a)^2\, dx = \begin{matrix} 1, a = 0 \\ 0, a \neq 0. \end{matrix} \tag{1}$$

Here, the cos $(\ )^2$ functions represent two one-dimensional zone plate components displaced from one another by $2a$. The limits $\pm L$ are included, since the results are exact only for an infinite number of rings in the zone plates. These limits require the normalizing factor $1/L$. For non-infinite limits, i.e. a finite number of zones, the above relation looks somewhat like a diffraction pattern centered around $a = 0$.

If we omit the limits for the sake of clarity, the following self-reciprocal transformation follows directly from the above orthogonality relation.

if
$$F(x) = \int f(y) \cos (x + y)^2 \, dy$$

then
$$f(y) = \int F(x) \cos (x + y)^2 \, dx. \tag{2}$$

Notice the similarity and differences compared to Fourier transformation. The similarity is evident in the form and self-reciprocity. The difference is that the two parameters x and y appear as a sum rather than a product. The Fresnel transformation is in the form of a convolution transform, where the arbitrary function $f(y)$ is convolved with a Fresnel zone plate $\cos (y)^2$.

Although this formulation lacks rigor, it is undoubtedly valid in its essentials. A more general and more rigorous extension is a complex formulation developed by O'Neill[4].

Returning to the case in point, it is important to know just how far finite zone plate components must be displaced before they become independent, or without cross correlation. If one expands the integrand of the orthogonality relation one finds a beat frequency proportional to the displacement $2a$. Such beat frequencies are readily observed as Moiré fringes between two Fresnel zone plates. When we have one full cycle of beat frequency across the zone plates we have zero cross correlation, and this occurs when the zone plates are displaced by the width of their outermost ring.

In the X-ray camera application we expect diffraction patterns around each reconstructed star image with the first null defined by the above zero of cross correlation. Let us now consider the transfer function of the camera.

The shadows may be thought of as consisting of two parts. One part is the Fresnel zones up to the given diameter. The second part is a bias level, so that the ideally negative Fresnel zones are biased up to zero. The transmission varies from zero to one, rather than from minus one to plus one, as the Fresnel kernel ought to vary.

We take the Fourier transforms of these two parts and sum their squares to get the power spectrum or transfer function. For the Fresnel part we get another Fresnel kernel (though with a phase shift), which becomes squared and so looks like our original biased zone plate. The peaks of this are uniformly high out to some maximum frequency determined by the size of the original zone plate. The bias on the other hand is largely d.c. with some low frequency sidebands due to the cut off at the edge of the original zone plate.

The focal isolation reconstruction technique described earlier may also be interpreted as spatial frequency filtering to remove the d.c. spike in the transfer function.

As far as the noise of the overall system is concerned, we can estimate a comparison with an equivalent pinhole camera. Let R be the ratio of the zone plate diameter to the width of its outermost ring (this width would be the diameter of the pinhole of equivalent spatial resolution). In the reconstruction we average over R^2 times the film area and so grain fluctuations should be down by R. However, according to R. C. Jones[5] the granularity will be higher by about $(R^2)^{0 \cdot 23}$ due to the increased exposure, leaving a net gain of only about $R^{0 \cdot 5}$ over the equivalent pinhole camera.

On the other hand, it seems that the information capacity of the film can be better used, because the hologram distributes the information storage rather uniformly over the whole photograph.

Actually we will have to refine our experimental techniques, to remove the many other extraneous noise sources before the anticipated granularity limit can be achieved.

In 1951, Golay[6] used an equivalent transformation technique based on convolution with a random variable function rather than a Fresnel zone plate. The power spectrum of such a random variable function is also basically flat out to some high cutoff frequency and his transformation is also self-reciprocal. Diffraction processes are not however suitable for the reconstruction.

Actually the self-reciprocity of any of these transforms, Fresnel, Fourier, or Golay, indicates that any method used to perform the transformation may be used to carry out the reconstruction. This is frequently awkward in practice. With the Fresnel transform numerous of the shadowcasting techniques are adequate. although all have the undesired d.c. spike. A scanning technique which should avoid this is to use alternate light and dark centre Fresnel zone plates as a chopping scanning aperture. Any differencing technique which permits going negative, such as mentioned, circumvents the d.c. bias problem of the spike in the transfer function.

Other applications of Fresnel transformations are to infra-red image scanning[7], to the Girard grill[8], to the spectrography of diffuse sources[9], to the study of out-of-focus light distributions, to 'chirp' radar[10] and to 'chirp' seismology.

Finally, we would like to express our appreciation to the Geophysics Research Directorate for their support of this research.

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