Contents

Chapter 1

X-ray wide-field camera imaging concept

Abstract – The concept of coded-mask imaging, as applied to X-ray wide-field imaging, is reviewed. The three principal components of a coded-mask imager (mask, detector and reconstruction) are discussed and requirements are introduced to arrive at an optimum imaging capability.

1.1 Introduction: multiplexing techniques as alternative to focusing techniques

In the energy range under consideration (2−30 keV) it is only partly possible to make use of traditional methods to image the sky. Focusing of X-rays is technically feasible for photon energies up to about 10 keV through grazing incidence reflection (Aschenbach 1985). This method can provide a very good angular resolution, i.e. down to $0.5''$ which is the value proposed for AXAF (van Speybroeck 1987). The sensitivity is optimized through the use of nested grazing incidence mirrors. The field of view (FOV) is limited by the grazing incidence to about 1°, but can be enlarged by using a special configuration of the mirrors ('Lobster-eye' telescopes, Aschenbach 1985).

An alternative class of imaging techniques employs straight line ray optics that offer the opportunity to image at higher photon energies and over larger FOV's. These techniques have one common signature: the direction of the incoming rays is, before detection, encoded; the image of the sky has to be reconstructed by decoding the observation afterwards. It is apparent that this method of producing sky images is a two-step procedure, in contrast to the direct or one-step imaging procedure of focusing techniques. These alternative techniques are referred to as MULTIPLEXING techniques.

Multiplexing techniques can be divided in two classes: those based on TEMPORAL multiplexing and those on SPATIAL multiplexing (Caroli *et al.* 1987). A straightforward example of temporal multiplexing is the scanning collimator: when the direction of a collimator is moved across a part of the sky which contains an X-ray point source, the number of counts per second that is detected as a function of time has a triangular shape. The position of the maximum of the triangle is set by the position of the source along the scanning direction and the height of the triangle is set by the flux of the source. A second scan along another direction completes the two-dimensional position determination of the source. More scans may be necessary if the source is extended or when there are more sources in the FOV of the collimator. The Large Area Counter (LAC) of the Japanese X-ray satellite *Ginga* (Makino *et al.* 1987 and Turner *et al.* 1989) is a recent example of an instrument employing a collimator. Another obvious example of time multiplexing is the covering of an X-ray source by the moving moon.

A more complex but popular device that is based on time multiplexing was introduced by Mertz (1968) and further described by Schnopper *et al.* (1968): the rotation modulation collimator (RMC). RMCs are often used as all-sky monitor. Several RMCs have flown, for instance in *Ariel-V* (Sanford

1975), *SAS-3* (Mayer 1972) and *Hakucho* (Kondo *et al.* 1981) and in several balloon experiments (see e.g. Theinhardt *et al.* 1984). The most recent example is the *Granat* observatory which carries 4 RMCs (Brandt *et al.* 1990). In its basic form an RMC has the disadvantage of being insensitive to short term (with respect to the rotation period of the aperture) fluctuations of X-ray intensity, because the temporal information must be used for reconstructing the position of sources. However, techniques to circumvent this problem have been developed (Lund 1985).

Temporal multiplexing techniques in principle do not need a position-sensitive detector, contrary to spatial multiplexing techniques. Spatial multiplexing techniques can be divided in two subclasses: in the first subclass two or more collimator grids, widely separated, are placed in front of a detector, and in the second subclass one or more arrays of opaque and transparent elements are placed there. Instruments of the former class are called 'Fourier transform imagers' (Makishima *et al.* 1978 and Palmer & Prince 1987). These instruments record a number of components of the Fourier transform of the observed sky, and the observed sky can be reconstructed by inverse Fourier transform in a way that is common in radio astronomy ('CLEAN' algorithm).

Instruments of the second subclass are called 'coded-mask systems'. *COMIS* belongs to this subclass. In the following chapter, a short review is given on the imaging concept of coded-mask systems, dealing separately with each important component of such a system. This includes requirements to arrive at an optimum imaging capability of the whole system.

1.2 Basic concept of coded-mask imaging

A coded-mask camera, intended to image the sky in the photon energy band E_1 to E_2, basically consists of:

- a coded mask, i.e. a plate with areas that are transparent or opaque to photons in the energy range E_1 to E_2. The transparent and opaque areas, or shortly 'mask elements', all have an equal size and are distributed in a pre-determined pattern, which is placed on a regular grid. The form of a mask element may be arbitrary.

- a position-sensitive detector, whose spatial resolution is sufficiently matched to the mask-pattern grid size and which is sensitive to photons with energies between E_1 and E_2.

The principle of the camera is straightforward (Fig. 1.1): photons from a certain direction in the sky project the mask on the detector; this projection has the same coding as the mask pattern, but is shifted relative to the central position over a distance uniquely correspondent to the direction of the photons. The detector accumulates the sum of a number of shifted mask patterns. Each shift encodes the position and its strength encodes the intensity of the sky at that position. It is clear that each part of the detector may detect photons incident from any position within the observed sky. After a certain illumination period, the accumulated detector image may be decoded to a sky image by determining the strength of EVERY possible shifted mask pattern.

Proper performance of a coded-mask camera requires that every sky position is encoded on the detector in a unique way. This can be stated in terms of the autocorrelation function of the mask pattern: this should consist of a single peak and flat side-lobes. This puts demands on the type of mask pattern and on the way its (shifted) projections are detected.

An important difference to direct-imaging systems is the fact that Poisson noise from any source in the observed sky is, in principle, induced at ANY other position in the reconstructed sky.

The imaging quality of the camera is determined by the type of mask pattern, the optical design of the camera, the spatial response of the detector and the decoding (or reconstruction) method. These items will be discussed in the remainder of this chapter.

1.3 Mask pattern

In view of the imaging quality, one would want the mask pattern to satisfy the following conditions:

Figure 1.1: Basic concept of coded-mask imaging. Two point sources illuminate a position-sensitive detector through a mask. The detector thus records two projections of the mask pattern. The shift of each projection encodes the position of the corresponding point source in the sky; the 'strength' of each projection encodes the intensity of the point source

1) the autocorrelation function of the mask pattern should be a single peak with perfectly flat side-lobes;
2) the signal-to-noise ratio of a coded sky source should be optimum.

In the early days of multiplex imaging, two types of mask patterns were proposed: a pattern of Fresnel zones (Mertz & Young 1961) and the random pinhole pattern (Dicke 1968 and Ables 1968). While a camera with a Fresnel zone plate was never applied in X-ray astronomy (except for solar studies in the early 1970's), concepts based on the random pinhole pattern have. The random pinhole pattern was proposed as an extension of a pinhole camera. The pinhole camera has ideal imaging properties with respect to the first condition (in fact it is a direct-imaging system), but delivers a poor signal-to-noise ratio, since the sensitive area is severely restricted by the size of the pinhole. The sensitivity may be increased by enlarging the pinhole, but at the expense of angular resolution. The idea behind the random pinhole camera is to increase the open area of the plate, while preserving the angular resolution, by placing many duplicate pinholes at random in the plate. The random character is necessary to meet the first condition as closely as possible.

Both Fresnel zone and random pinhole mask patterns are not ideal with respect to the first condition, the patterns possess autocorrelation functions whose sidelobes are not perfectly flat. Later work concentrated on finding patterns, based on the idea of the random pinhole pattern, that do have flat side-lobes. Ideal patterns were found that are based on cyclic difference sets (Gunson & Polychronopulos 1976).

A cyclic difference set D, characterized by the parameters n, k and z, is a collection of k integer numbers $\{I_1, I_2, ..., I_k\}$ with values $0 \leq I_i < n$, such that for any $J \neq 0 \pmod{n}$ the congruence

$I_i - I_j = J \pmod n$ has exactly z solution pairs (I_i, I_j) within D (Baumert 1971). An example of a cyclic difference set D with $n = 7$, $k = 4$ and $z = 2$ is the collection $\{0, 1, 2, 4\}$. Cyclic difference sets can be represented by a binary sequence a_i $(i = 0, ..., n-1)$ with $a_i = 1$ if i is a member of D and $a_i = 0$ otherwise. In the above example a_i is given by 1110100. a_i in turn can stand for the discretized mask pattern, assigning a transparent element to $a_i = 1$ and an opaque one to $a_i = 0$. The cyclic autocorrelation c_l of a_i is (Baumert 1971):

$$c_l = \sum_{i=0}^{n-1} a_i a_{\bmod (i+l,n)} = \begin{cases} k & \text{if } \bmod(l,n) = 0 \\ z = \frac{k(k-1)}{n-1} & \text{if } \bmod(l,n) \neq 0 \end{cases} \tag{3.1}$$

i.e. a single peak on a flat background. A mask pattern based on a_i consequently satisfies condition 1. a_i has the characteristic that every difference $i - j$ between a pair of $a_i, a_j = 1$ is equally sampled and therefore these arrays are also called Uniformly Redundant Arrays (URA, Fenimore & Cannon 1978).

From the autocorrelation it can be anticipated that it is advantageous with respect to condition 2 to have a difference between k and z that is as large as possible, for k determines the signal and z the background level (and its noise)[1]. The maximum difference is reached if $n = 4t - 1$, $k = 2t - 1$ and $z = t - 1$ if t is integer. These cyclic difference sets are called Hadamard difference sets (Hall 1967 and Baumert 1971) and can be classified in at least three types, according to the value of n:

1) Quadratic residue sets: n is prime (the members of this set are given by the squares, modulo n, of the first $(n + 1)/2$ integers);
2) Twin prime sets: $n = p(p + 2)$ for integer p, p and $p + 2$ being prime;
3) Pseudo-noise sets: $n = 2^m - 1$ ($m > 1$ is integer).

Some Hadamard difference sets may belong to more than one class, the existence of a set with a value for n given by a class is guaranteed. The above example is a quadratic residue set as well as a pseudo-noise set. Characteristic for Hadamard sets is that $k = (n - 1)/2$, i.e. for large n the mask pattern is about half open. The cyclic autocorrelation then is:

$$c_l = \begin{cases} \frac{n-1}{2} & \text{if } \bmod(l,n) = 0 \\ \frac{n-3}{4} & \text{if } \bmod(l,n) \neq 0 \end{cases}$$

Another collection of cyclic difference sets are the so-called Singer sets, that are characterized by $n = (t^{m+1} - 1)/(t - 1)$, $k = (t^m - 1)/(t - 1)$ and $z = (t^{m-1} - 1)/(t - 1)$, where t is a prime power. The equivalent mask pattern will have smaller open fractions than those based on Hadamard sets; for $t \gg 1$ the open fraction approximates $1/t$.

A way to construct a pseudo-noise Hadamard set is the following (Peterson 1961): if $p(0), ..., p(m-1)$ are the factors of an irreducible polynomial of order m ($p(i)$ is 0 or 1) then a_i is defined by a shift register algorithm:

$$a_{i+m} = \sum_{j=0}^{m-1} p(j) a_{i+j} \qquad (i = 0, ..., 2^m - 2) \qquad \pmod 2 \tag{1.2}$$

The first m values of this recursive relation, $a_0, ..., a_{m-1}$, can be chosen arbitrarily: a different choice merely results in a cyclic shift of a_i.

If n can be factorized in a product of two integers ($n = p \times q$), it is possible to construct a two-dimensional array $a_{i,j}$ $(i = 0, ..., p-1;\ j = 0, ..., q-1)$ from the URA a_i $(i = 0, ...n-1)$. The mask pattern thus arranged is called the 'basic pattern'. The ordering of a_i in two dimensions should be such, that the autocorrelation characteristic (Eq. 3.1) is preserved. This means that in a suitable extension of the basic $p \times q$ pattern, any $p \times q$ section should be orthogonal to any other $p \times q$ section. A characteristic of a URA a_i is that any array a_i^s, formed from a_i by applying a cyclic shift to its elements ($a_i^s = a_{\bmod (i+s,n)}$), is again a URA which is orthogonal to a_i. Therefore, the

[1]The argument followed here to meet condition 2 is simplified. In fact, the optimum open fraction of the mask pattern is also dependent on specific conditions concerning the observed sky. See e.g. Skinner (1984).

Figure 1.2: Schematic diagram, showing two methods of constructing a two-dimensional array from a one-dimensional URA, without loosing the autocorrelation property (Eq. 3.1): ordering by rows (left) and ordering along diagonals (right). The numbers indicate the element index of the one-dimensional array. The squares indicate the basic pattern (containing the array once). In these two examples $p = 7$, $q = 9$ and $n = 63$. The elements of the expansion to a 2×2 mosaic of the basic pattern are ordered in the same manner as the basic pattern. The last row and column of the expanded array are omitted, to avoid the autocorrelation to have more than one peak

autocorrelation characteristic of the expanded $a_{i,j}$ is fulfilled if every $p \times q$ section is a cyclic shift of the basic pattern. Two examples of valid ordering methods are shown in Fig. 1.2: ordering by rows (Miyamoto 1977) and ordering along extended diagonals (Proctor *et al.* 1979). In the latter case p and q should not have a common divisor (they should be mutually prime), otherwise one is not able to fill the basic pattern completely. Other valid orderings are possible.

The pseudo-noise arrays have the convenient property that they can easily be wrapped in almost a square of $n \gg 1$: if m is even, n can be written as $n = 2^m - 1 = (2^{\frac{m}{2}} - 1)(2^{\frac{m}{2}} + 1)$, so that p and q only differ by 2.

A problem that is encountered in the manufacturing of a two-dimensional mask plate for X-ray energies involves an opaque mask element that is completely surrounded by transparent elements. In the X-ray regime it is necessary to keep transparent elements completely open, because the use of any support material at open mask elements soon results in too much absorption of photons. Thus, an isolated opaque mask element will not have any support. Two methods may be applied to solve this problem:

1) Choose a mask pattern where no isolated opaque elements occur. Such patterns are called 'self-supporting'.
2) Include a support grid in the mask plate.

Generally the second option is chosen, because every opaque element is then supported completely along its sides while in the first option opaque elements might still be only weakly supported at some of its corners. This does not affect the ideal autocorrelation characteristic, because the pattern is not altered and remains two-valued. However, the open area will be decreased somewhat, resulting in less sensitive area.

The autocorrelation characteristic remains valid only if the coding is performed by the use of a complete cycle of a basic pattern. As soon as the coding is partial, noise will emerge in the side-lobes of the autocorrelation function. This noise can be interpreted as false peaks and thus deteriorates the imaging quality. In order to be able to record for every position in the observed

Figure 1.3: Schematic drawings of the two types of 'optimum' configurations discussed in the text. The left configuration is called 'cyclic'. Note the collimator, placed on top of the detector, necessary to confine the FOV to that part of the sky in which every position will be coded by one full basic pattern. From Hammersley (1986)

sky a full basic pattern, one needs a special optical configuration of mask and detector (see next section). Sometimes also a mask is needed that consists of more than 1 basic pattern. How such a mosaic mask is constructed has been discussed above (see Fig. 1.2).

1.4 Optical design

The optical design of a coded-mask camera is defined by the sizes of the mask, the mask elements and the detector, the number of basic patterns used in the mask, the distance between mask and detector and the size and place of an optional collimator. Apart from the imaging quality, the design determines the angular resolution and the FOV (the latter is usually expressed in the full-width at half maximum, FWHM, of the collecting area across the observed sky).

1.4.1 Optimum and simple configurations

As concluded in 1.3, for ideal imaging properties it is necessary to record for every position in the observed sky a complete cycle of the basic pattern. This can be accomplished by configuring the mask and detector in one of the following two ways[2] :

1) The mask consists of one $p \times q$ basic pattern, while the detector has a size of $(2p-1) \times (2q-1)$ mask elements (Fig. 1.3, right). By the implementation of a collimator in front of the detector, the observed sky is restricted to those positions in the sky from where the mask is completely projected on the detector.
2) The mask consists of a mosaic of almost 2×2 cycles of the basic pattern ($(2p-1) \times (2q-1)$ mask elements), while the detector is as large as one basic pattern (Fig. 1.3, left). A collimator is implemented, to restrict the observed sky, as seen from any position on the detector, to that

[2] Other configurations can be thought of (Proctor *et al.* 1979) that are extensions of the two mentioned here.

Figure 1.4: Schematic drawing of the 'simple' configuration. The sizes of the mask and detector are equal. Note that instead of a collimator, as in the optimum configurations, a shielding is used. The shielding prevents photons not modulated by the mask pattern to reach the detector. From Hammersley *et al.* (1992)

part of the sky seen through one complete cycle of the basic pattern. This type of configuration often is called 'cyclic' because of the nature of the mask.

In both types the collimator gives a pyramidical-to-zero response function to the collecting detector area over the observed sky and its use may be coupled to the need for a support structure for the entrance window of the detector (Gunson & Polychronopulos 1976). In practice the choice between both types may depend on which of the components is restricted most in size by exterior conditions: the mask or the detector. However, in astronomical applications, the argument of preservation of the collecting area prevails and the choice is usually for the cyclic type.

The above types of mask/detector configurations are called 'optimum systems' (Proctor *et al.* 1979) in the sense that the imaging property is optimum. An alternative configuration is the 'simple system'[3]. In this system the need for full coding is relaxed. The detector has the same size as the mask, which consists of one basic pattern (see Fig. 1.4). No collimator is then needed on the detector; instead a shielding is used to prevent photons that do not pass the mask from entering the detector. In a simple system only the on-axis position is coded with the full basic pattern, the remainder of the FOV is partially coded. Obviously, the off-axis sources will cause false peaks in the reconstruction. However, as will be discussed later on, this coding noise can be eliminated to a large extent in the data-processing, provided not too many sources are contained in the observed part of the sky.

If one assumes for the moment that coding noise is not relevant, the question arises how the simple system compares to the cyclic system. In order to do this comparison, it seems fair to impose on both systems the same FOV and sensitivity. This means that both have a detector of equal size, but in the cyclic system the 2×2 mosaic mask is two times closer to the detector than in

[3]Recently, also the designation 'box system' has become popular in the literature on this subject.

Figure 1.5: Schematic graphs for the comparison between the cyclic and the simple system, as described in the text. Plotted are the variances across the reconstructed sky, due to (**a**) the background and (**b**) an off-axis point source. Drawn curves refer to the cyclic system and dashed curves to the simple system. Sky positions and variances are given in arbitrary units. Some comments on both figures: **a.** Since both systems have the same sensitivity and FOV, the illumination by the sky background per detector element is the same. For an off-axis position, the reconstruction in the cyclic system involves all background photons, while that in the simple system only involves the photons not obscured by the shielding from that position. The variance due to the Poisson noise of background photons is therefore less in the simple system (except for the on-axis position). **b.** In a simple system an off-axis source only illuminates part of the detector, while in an cyclic system the total detector is illuminated. In both cases, the total number of detected photons from this source is the same. Reconstructing for another off-axis position in the simple system, only part (potentially even none) of the source photons give rise to Poisson noise. In the cyclic system ALL source photons give rise to Poisson noise

the simple system, with an appropriate adjustment of the collimator's dimensions. Therefore, the angular resolution in the cyclic system is two times worse in each dimension than in case of the simple system. Most important in the comparison is the following difference between the cyclic and the simple system, concerning the reconstruction of the flux from an arbitrary direction within the observed sky: in the cyclic system all detected photons on the complete detector may potentially come from that direction, while in the simple system only photons from the section of the detector not obscured by the shielding are relevant. Therefore, Poisson noise will affect the reconstruction in the cyclic system stronger than in the simple system. This is specified graphically in Fig. 1.5. Thus, regarding the Poisson noise, the simple system is superior in sensitivity to the cyclic system (except for the on-axis position where both systems have equal properties). This conclusion is in agreement with the findings of Sims *et al.* (1980), who have studied the performance of both systems via computer simulations.

1.4.2 Angular resolution

The angular resolution is given by the angle subtended by a mask element as seen from the detector. If the mask elements are square shaped, with a size of $s_m \times s_m$ mm^2, and the distance between mask and detector is F mm, the angular resolution ϕ (FWHM) along each axis at an off-axis angle θ in the observed sky is equal to:

$$\phi_{x,y} \quad = \quad \arctan\left(\frac{s_m \cos^2(\theta_{x,y})}{F}\right) \text{ rad} \tag{1.3}$$

The $\cos^2$-factor is due to projection effects.

1.4.3 Field of view

The size of the FOV is of the order of the angle that the mask subtends as seen from the detector along each axis. Its precise value depends on the characteristics of an optional collimator. If the mask and detector are square and r is the ratio of the one-dimensional detector size to the mask-detector separation, the FOV for a simple system is given by:

$$\Omega_{tot} \quad = \quad 2\pi - 8 \arcsin\left(\frac{1}{\sqrt{2(1+r^2)}}\right) \qquad \text{sr} \tag{1.4}$$

(Sims 1981). The same equation also applies to an optimum system, provided r is interpreted as the ratio of the horizontal collimator grid cell size to the cell height.

1.5 Detector requirements

The detector should make a proper recording of the projected mask pattern. Therefore, it should meet a number of requirements with respect to its spatial response.

1.5.1 Spatial resolution

Inherent to any detector system is a limited spatial resolution: the position of a detected photon is influenced by noise, emerging from various physical and electronic processes in the detector system. In general the spatial noise may be modeled by a two-dimensional Gaussian distribution function $P(x, y)$, describing the probability per unit area that the position of a count is measured within $(x - ...x + \mathrm{d}x, y - ...y + \mathrm{d}y)$ from the true position:

$$P(x, y)\mathrm{d}x\,\mathrm{d}y \quad \sim \quad \mathrm{e}^{-\frac{x^2}{2\sigma_x^2}}\,\mathrm{e}^{-\frac{y^2}{2\sigma_y^2}}\mathrm{d}x\,\mathrm{d}y \tag{1.5}$$

with σ_x and σ_y the standard deviations along x and y.

The projection of a mask hole is a square block function with a base size of $s_m \times s_m$. The convolution (or 'smearing') of this function with the Gaussian spatial detector resolution causes the sharp edges of the block function to be smoothed and, if the smearing dominates, the mask-hole projection becomes substantially larger than the mask hole. Fig. 1.6.a presents the FWHM of the convolution in one dimension, as a function of the FWHM of the Gaussian smearing. If the smearing is comparatively small, the extent of the mask-hole projection is not affected. Only if the FWHM of the spatial resolution is larger than about half the size of a mask hole, the projection will become seriously degraded.

Several parameters will be affected by the finite detector resolution. The angular resolution is one; it is defined by the ratio of FWHM of the mask-hole projection and the mask-detector distance F. Once the FWHM is degraded by the detector resolution, the angular resolution will also be degraded. As seen above, this will not happen as long as the detector resolution (FWHM) is limited to half the size of a mask hole.

Figure 1.6: Results on three parameters of modeling a non-ideal spatial detector resolution by a Gauss function, as a function of the FWHM of the Gauss function in units of one mask-element size (MES): **a.** the FWHM of the detected mask-hole projection; **b.** the signal-to-noise ratio, SNR, of the measured intensity; **c.** the required sampling rate prescribed by the sampling theorem. The instability in the latter curve is due to the fact that the Fourier transform of the mask-hole projection is a sinc-function

In how far the position of the edge of a mask-hole projection can be derived, determines the source location accuracy (SLA) of the imaging system. The limited spatial detector resolution will smooth this edge and limit the SLA according to the proportionality:

$$\mathrm{SLA}_{x,y} \quad \sim \quad \frac{\sigma_{x,y}}{F\,\mathrm{SNR}} \qquad \mathrm{rad} \tag{1.6}$$

with SNR the ratio of the signal (the number of photons from the source through the open mask hole) to the noise (as is present in signal and background). For a justification of the dependence of SLA on SNR the reader is referred to section 5.7.

Finally, the finite detector resolution limits the sensitivity of the imaging system. Because of the broadening of the mask-hole response, generally speaking, the flux extends over a larger portion of the detector and therefore meets a larger background. The associated SNR deterioration is indicated in Fig. 1.6.b. Deterioration of the sensitivity by the limited spatial resolution of the detector depends furthermore on the specific characteristics of the detector. For instance, in a proportional counter not only fluctuations along the x, y-plane exist, but also along the optical axis due to optical depth effects. This introduces an exponential smearing function on top of the lateral Gaussian spread. This effect will be described in section 6.5.2.

1.5.2 Sampling rate

The detector data will usually be discretized by digital encoding, introducing a pseudo sampling of the continuous data. The discretization interval should be chosen such that no information is lost. This can be illustrated as follows: imagine the detector resolution is infinite. If the detector data were to be sampled at intervals equal to the size of a mask element, the projection of that element may be spread in up to four detector bins. This would potentially degrade the SNR by a factor two.

The appropriate sampling rate may be inferred from the sampling theorem (see e.g. Bracewell 1986, chapter 10), which states that a function whose Fourier transform is zero for frequencies $\nu > \nu_c$ is fully specified by values spaced at equal intervals not exceeding $\frac{1}{2}\nu_c^{-1}$. If the finite detector resolution is included, the Fourier transform of the function under consideration (one-dimensional) is given by a sinc-function multiplied by a Gaussian. This transform never remains zero beyond a certain frequency. However, a convenient cut-off frequency ν_c may be defined by requiring that P % of all the power is taken into account. Fig. 1.6.c shows the values of the sampling rate for $P = 99\%$ as a function of the detector resolution (FWHM) and provides a tool for inferring the necessary sampling rate.

1.5.3 Flat-field response of the detector

A deviation from ideal flatness of the spatial response of the detector results in a reduction of the imaging quality of the camera as a whole. Several types of deviations are probable. Some of them are sketched in Fig. 1.7 for the one-dimensional case, including linear as well as non-linear deviations. The main impact of these deviations is, that they broaden the point spread function (PSF) in the reconstruction process: for one part of the detector the mask pattern will match the recorded detector at a different shift than for another part of the detector; combining all matches results in a broadened PSF. Fig. 1.7.b shows such PSF's for the types of deviations shown in Fig. 1.7.a. This broadening of the PSF degrades the sensitivity of the camera. In a simple-type camera, the PSF not only broadens, but may also become asymmetrical since often only part of the detector is used in the reconstruction of a particular source. Therefore, only part of the deviation plays a role in the reconstructed image. This is illustrated in Fig. 1.7.b.

The degradation in sensitivity may be kept within reasonable limits, if the absolute differences between recorded and true position do not exceed typically half the size of a mask element in the respective directions (this criterium is argued in 1.5.1).

1.5.4 Tolerance on relative positioning and alignment of the detector body and mask

The tolerance for the parallelism between mask plane and detector plane can be very liberal: the scale of the projection is not affected by more than 1% as long as both planes are parallel within 8°.

Of more importance is a possible misalignment angle between mask and detector about the optical axis. If the projected mask pattern is rotated with respect to the detector x, y-plane, sensitive area may be lost because parts of the projection fall outside the detector area (see Fig. 1.8). In case of a square shaped detector and mask, the fraction of lost sensitive area amounts to

$$\frac{1}{2}\left(1+\frac{\cos\alpha-1}{\sin\alpha}\right)^2\tan\alpha \qquad (0^\circ < \alpha < 90^\circ) \tag{1.7}$$

Figure 1.7: **a.** Three one-dimensional examples of a non-ideal spatial response of the detector. Plotted is the recorded position of any photon against its true position, both expressed in fractions of the detector size. An ideal detector would have a 1:1 response. **b.** Schematically, the resulting broadening of the PSF in the reconstructed image. The arrows indicate the center positions of the PSF for an ideal detector (not shown). Numbers in b correspond to the spatial responses indicated in a. The third plot in b shows for the first response the PSF if only half of the detector is used in the reconstruction. The sinusoidal response (3) results in a PSF with the form of the derivative of arcsin

with α the misalignment angle. This loss can be overcome if the sensitive detector area is slightly larger than the area of the mask.

The imaging process is very sensitive to a non-zero misalignment angle (see e.g. Charalambous *et al.* 1984). The point spread function (PSF) in the reconstructed sky will be considerably broadened if α exceeds roughly $\arctan(0.5\ s_m/d_l)$, with d_l the length of the detector. A post-observation correction may largely resolve this problem, provided α is known: the detector image may be rotated over $-\alpha$, back to the reference of the mask pattern. However, a slight PSF broadening,

Figure 1.8: If the mask is rotated with respect to the detector about the optical axis, parts of the mask-pattern projection are lost (the grey areas). This problem can be overcome if the usable sensitive area of the detector is larger than the mask projection

inherent to this procedure, will be introduced (see 5.5). α can be easily calibrated by optimizing the PSF as a function of the rotation angle $-\alpha$ applied to the data. The value for α thus found, will generally apply to all data during the instrument's life time, provided the tolerance on the instrument stiffness is strict.

1.6 Reconstruction methods

Coded-mask imaging is basically a two-step procedure. After the accumulation of spatially coded detector data, the second step involves the decoding of this data, in other words the reconstruction of the observed part of the sky[4]. Since a powerful computer is needed for the reconstruction process, this is usually done off-line (particularly if the number of mask elements is large). Several reconstruction algorithms are in use. The choice for a certain algorithm depends on the specific aim (e.g. search for detections of unexpected events or timing analysis of a restricted part of the sky), the available computer memory and the type of instrument configuration. Several types of algorithms may in fact be subsequently used on the same set of detector data. This section gives a short resume of various algorithms. For clearness the discussion is illustrated with one-dimensional examples and space is discretized in steps of a size equal to that of a mask element; the conclusions do not basically differ for two dimensions and smaller steps.

The basic problem to be solved concerns the following. Let the one-dimensional vector $\vec{d}$ describe the detector (in units of counts per detector-element area), $\vec{s}$ the sky (in counts per mask element area) and $\vec{b}$ the detector background (this includes all flux which is not modulated by the mask

[4] In literature, a number of verbs are used for the same action: decode, reconstruct and deconvolve.

Figure 1.9: Schematic sketch of the formalism used in the text for the detection process in **a.** an optimum configuration and **b.** a simple configuration. The basic mask pattern here consists of 4 elements

pattern, in counts per detector element area). Suppose the detector elements are as large as a mask element. The detection process can then be described by:

$$\vec{d} \;=\; \mathbf{C}\vec{s} + \vec{b} \tag{1.8}$$

where $\mathbf{C}$ is a matrix whose rows contain cyclic shifts of the basic mask pattern (see Fig. 1.9). An element of $\mathbf{C}$ is 1 if it corresponds to an open mask element and 0 otherwise. In the case of an optimum system with a basic mask pattern of n elements, $\vec{d}$, $\vec{s}$ and $\vec{b}$ contain n elements and $\mathbf{C}$ $n \times n$ elements. In the case of a simple system with a mask of n elements, $\vec{d}$ and $\vec{b}$ contain n elements, the sky vector $\vec{s}$ contains $2n-1$ elements and the matrix $\mathbf{C}$ contains $(2n-1)\times n$ elements (see Fig. 1.9). The problem to be solved is to reconstruct $\vec{s}$ out of this set of linear equations. Although $\vec{b}$ is unknown also, one is in principle not interested in it. An approximation for $\vec{b}$ is: $\vec{b}$ is homogeneous over the detector plane, i.e.

$$\vec{b} \;=\; b\ \vec{i} \tag{1.9}$$

where $\vec{i}$ is the unity vector (consisting of only ones), b is now a single unknown scalar.

1.6.1 Linear methods

<u>Inversion.</u> This is a straightforward method to find $\vec{s}$ from Eq. 1.8: multiply $\vec{d}$ with the inverse of $\mathbf{C}$, $\mathbf{C}^{-1}$, yielding:

$$\mathbf{C}^{-1}\vec{d} \;=\; \vec{s} + b\mathbf{C}^{-1}\vec{i} \tag{1.10}$$

Obviously $\mathbf{C}$ should be non-singular. This is not the case for a simple system, since $\mathbf{C}$ is not square. However, even if $\mathbf{C}$ is non-singular, the possibility exists that entries of $\mathbf{C}^{-1}$ are so large that the term $b\mathbf{C}^{-1}\vec{i}$ dominates the reconstruction. This will happen if rows of $\mathbf{C}$ are almost dependent on other rows, meaning that change of a few elements in a row might make it linearly dependent on other rows. $\mathbf{C}$ is then said to be 'ill-conditioned'. Fenimore and Cannon (1978) have shown that this is quite a common feature, especially for random mask patterns. This makes inversion an unfavorable reconstruction method.

Cross correlation. This is another obvious method for the reconstruction, cross correlating the detector image $\vec{d}$ with the mask pattern via a multiplication with a matrix. The mask pattern may be given by $\mathbf{C}$, but in practice a modified matrix is used: the so-called reconstruction matrix $\mathbf{M}$. $\mathbf{M}$ is constructed in such a way, that $\mathbf{M}\vec{d}$ evaluates directly $\vec{s}$ and cancels contributions from $\vec{b}$. Fenimore and Cannon (1978) introduced this method and called it 'balanced cross correlation'. Specifically, $\mathbf{M}$ is defined as[5]:

$$\mathbf{M} = (1+\Psi)\mathbf{C}^T - \Psi\mathbf{U} \tag{1.11}$$

where $\mathbf{C}^T$ is the transposed[6] of $\mathbf{C}$ and $\mathbf{U}$ is the unity matrix (consisting of only 1's and having the same dimensions as $\mathbf{C}^T$). Ψ is a constant and is determined from an analysis of the predicted cross correlation value: a prediction of the reconstruction can be easily evaluated if one assumes that the mask pattern is based on a cyclic difference set and the camera configuration is optimum. The expected value of the cross correlation is then (using the autocorrelation value, Eq. 3.1, for cyclic difference sets):

$$\begin{aligned}\mathbf{M}\vec{d} &= \mathbf{M}(\mathbf{C}\vec{s}+b\vec{i}) \\ &= (1+\Psi)(k-z)\vec{s} + \left[\{(1+\Psi)z-\Psi k\}\sum_i s_i + \{(1+\Psi)k-\Psi n\}\,b\right]\vec{i} \end{aligned} \tag{1.12}$$

Apart from a scaled value of $\vec{s}$, which is the desired answer, the result also includes a bias term (the $\vec{i}$-term). It is not possible to eliminate this bias by a single value of Ψ. Rather, the $\sum_i s_i$ factor or the b factor can be canceled separately. Canceling the $\sum_i s_i$ factor involves a value Ψ of

$$\begin{aligned}\Psi_1 &= \frac{z}{k-z} \\ &= \frac{k-1}{n-k}\,, \end{aligned} \tag{1.13}$$

canceling the b-factor involves Ψ to be[7]

$$\Psi_2 = \frac{k}{n-k} \tag{1.14}$$

Since k/n is the open fraction, t, of the mask pattern, Ψ_2 can be written as:

$$\Psi_2 = \frac{t}{1-t} \tag{1.15}$$

Ψ_1 approximates Ψ_2 if $k >> 1$. The reconstruction value then reduces to:

$$\mathbf{M}\vec{d} = k\vec{s} \tag{1.16}$$

Normalizing $\mathbf{M}$ results in:

$$\mathbf{M}/k = \frac{n/k}{n-k}\left(\mathbf{C}^T - \frac{k}{n}\mathbf{U}\right) \tag{1.17}$$

[5] An alternative description of $\mathbf{M}$ is: $M_{ij} = 1$ if $C_{ji} = 1$ and $M_{ij} = -\Psi$ if $C_{ji} = 0$.

[6] $\mathbf{C}^T$ instead of $\mathbf{C}$ is used to make this formulation applicable to the simple system; for the optimum system $\mathbf{C}^T = \mathbf{C}$.

[7] An interesting characteristic of the reconstructed sky is the sum of the reconstructed values. In case $\Psi = \Psi_1$ this is $\sum \mathbf{M}\vec{d} = k\sum_i s_i + nb$, i.e. the sum of all detected counts. If $\Psi = \Psi_2$ this sum is equal to 0.

In the case of a simple system, Eq. 1.12 would also apply if the 'hard' zeros in $\mathbf{C}$ (i.e. zeros that do not arise from zero a_i-values, see Fig. 1.9) were replaced by cyclically shifted a_i's. The result of Eq. 1.16 would then imply: $(\mathbf{M}\vec{d})_i = (\mathbf{M}\vec{d})_{\bmod (i+n,2n-1)}$; this is a consequence of the fact that $2n-1$ unknowns are to be determined from a set of only n linear equations, which is under-determined in general. In this formulation a source at position i causes a false peak of the same strength at position $\bmod(i+n, 2n-1)$. If the 'hard' zeros in $\mathbf{C}$ are not replaced, this does not apply directly, but an interdependence in the solution for the reconstruction remains. This interdependence is not so strong: one real peak will cause many small ghost peaks rather than a single false peak which is just as strong as the real peak. It is then possible to find a unique solution for $\vec{s}$ as long as it does not have more non-zero values than n. This search is accomplished by testing reconstructed peaks on their authenticity, the reconstruction process is then necessarily iterative. A detailed discussion of the cross correlation for a simple system is given in chapter 5.

Photon tagging (or URA-tagging). This reconstruction method is very similar to cross correlation and has been introduced by Fenimore (1987). It involves back-projecting every detected photon through the mask, towards a particular (possibly all) position in the sky field from which it could originate. If a closed mask element is 'encountered' in the back-projection, the photon is accumulated in the background contribution. If it 'encounters' an open mask element, it is accumulated in the source contribution for that position in the sky. Once all photons have been processed, the subtraction of the background from the source contributions (after proper normalization) completes the reconstruction. It is clear that this method is advantageous, in terms of computation time relative to a cross correlation, if the number of detected photons is very small with respect to the number of mask elements. The advantage may also hold if only a restricted part of the observed sky needs to be reconstructed. This latter situation may be applicable if all point sources in the sky have already been found in an analysis of the same data (e.g. by a total cross correlation) and one intends to analyze each point source in more detail, i.e. extract spectra and/or lightcurves. Skinner & Nottingham (1992) improved the method by extending it, taking into account imperfections of the detector (limited spatial resolution, 'dead spots' etc.), the support grid of the mask plate and a telescope motion.

Wiener filtering. Sims *et al.* (1980), Sims (1981) and Willingale *et al.* (1984) introduced Wiener filtering for use as a reconstruction method of coded-mask-system images. This filtering can be regarded as a weighted cross correlation (Sims 1981), weighing the Fourier transform components of the detector image with the inverse power density of the mask pattern. Thus, fluctuations in the modulation transfer function[8] of the mask pattern are smoothed. If $\frac{S}{N}(\omega)$ is the ratio of the signal power density (due to spatial fluctuations by sky sources) to the noise power density at spatial frequency ω, $C(\omega)$ the Fourier transform of the mask pattern, the Wiener filter $W(\omega)$ is defined as:

$$W(\omega) = \frac{\overline{C(\omega)}}{|C(\omega)|^2 + \frac{S}{N}(\omega)^{-1}} \qquad (1.18)$$

Because $\frac{S}{N}$ is not known before the reconstruction is completed, a frequency-independent expression is used for it.

It is clear that Wiener filtering is especially helpful if the mask pattern is not ideal, which is the case for random and Fresnel zone patterns. However, ideal patterns such as those based on cyclic difference sets are characterized by flat modulation transfer functions (all spatial frequencies are equally present for URA-patterns, which is apparent from the definition of URAs, see 1.3). Sims *et al.* (1980) confirmed this via computer simulations and found this also to be the case if an ideal pattern is used in partial coding, such as in a simple system.

[8]The modulation transfer function is defined as the square root of the Fourier power spectrum.

1.6.2 Iterative methods, Maximum Entropy Method

A separate class of reconstruction methods are formed by the iterative methods. These try to solve the sky vector $\vec{s}$ from Eq. 1.8 by an iterative search for the solution that is most consistent with the detector data. Three of such methods have been investigated for use in coded-mask imaging. One of these is the MAXIMUM ENTROPY METHOD (MEM) and was introduced in this field of work by Willingale (1979). Despite the good results that can be obtained with this method, a major drawback is the large amount of computer effort required, as compared to linear methods such as cross correlation. Another method, ITERATIVE REMOVAL OF SOURCES (IROS), does not have this disadvantage. IROS is in fact an extension of the cross correlation method and was introduced by Hammersley (1986) as a procedure to eliminate problems due to incomplete coding (also called 'missing data') in simple systems.

In this section MEM will shortly be addressed. IROS will be detailed in chapter 5.

MEM has gained widespread favor in different areas as a tool to restore degraded data. Introductions to the theory behind MEM as applied to image restoration can be found in Frieden (1972) and Daniell (1984), while a review is given by Narayan & Nityananda (1986). Examples of applications of MEM are given by Gull and Daniell (1978), Bryan and Skilling (1980) and Willingale (1981), while the application specifically to images from coded-mask systems are described by Sims *et al.* (1980) and Willingale *et al.* (1984). Here, a summary of the principles of MEM is presented.

MEM involves finding the one solution with the least amount of information from the set prescribed by:

$$F(\vec{s}) \equiv \chi^2 - \chi^2_{\rm expected} = 0 \tag{1.19}$$

with

$$\chi^2 = \sum_{i=1}^{N_d} \frac{(d_i - \widehat{d}_i(\vec{s}))^2}{\sigma_i^2} \tag{1.20}$$

where $\widehat{d}_i(\vec{s})$ is a prediction of the data in the detector domain, N_d the number of detector elements, σ_i^2 the variance in the measured value of d_i and $\chi^2_{\rm expected}$ is usually equal to N_d. The amount of information is measured through the entropy S. The minimum amount of information is equivalent to the maximum entropy. Regarding a 'correct' measure for the entropy, there is some confusion in the literature (Willingale *et al.* 1984). Ponman (1984) argues that a single correct expression for S does not exist. In view of the current basic discussion of MEM, assume S is straightforwardly given by:

$$S = -\sum_{i=1}^{N_s} p_i \log(p_i) \tag{1.21}$$

with

$$p_i = \frac{s_i}{\sum_{l=1}^{N_s} s_l}$$

where N_s is the number of sky elements in $\vec{s}$. p_i represents the probability that a given photon has arrived from sky element i rather than another. Finding the maximum of S within the set of solutions defined by Eq. 1.19 involves the solution of the Lagrange multiplier λ from:

$$\frac{\partial S}{\partial s_k} = \lambda \frac{\partial F(\vec{s})}{\partial s_k} \tag{1.22}$$

or, using Eq. 1.8:

$$s_k = {\rm e}^{\left(\frac{\sum_{i=1}^{N_s} s_i \log s_i}{\sum_{i=1}^{N_s} s_i} - 2\lambda \sum_{i=1}^{N_s} s_i \sum_{i=1}^{N_d} \frac{(\widehat{d}_i(\vec{s}) - d_i) c_{ik}}{\sigma_i^2} \right)} \tag{1.23}$$

under Eq. 1.19. Unfortunately Eq. 1.23 is transcendental in $\vec{s}$ and can only be solved using an iterative search algorithm.

Bryan and Skilling (1980) have shown that care should be taken in using Eq. 1.19 as a definition for allowable solutions. They showed that the use of Eq. 1.19, to fit the variance of the residuals $\widehat{d_i} - d_i$ to the expected value, may actually resolve a MEM solution whose residuals do not follow a Gaussian distribution. This may manifest itself as fluxes of confined sky sources that are not fully recovered and a biased background level. They proposed the use of another statistical definition of allowable solutions. Recent discussion of MEM, as applied on data of the Hubble Space Telescope (see e.g. Weir 1991) indicates that this problem is non-trivial and generally not yet fully solved. These discussions also make clear that no straightforward recipe exists yet to extract estimates for the flux accuracy from MEM solutions. However, MEM solutions do seem to provide a powerful tool to recognize extended structures.

Regarding the computational effort involved in solving Eq. 1.23 in relation to the cross correlation discussed in section 1.6.2: for every iteration two convolutions need to be calculated. The first convolution is necessary to evaluate $\widehat{d_i}$ for a given sky $\vec{s}$ (via Eq. 1.8). The second one needs to be done to convolve the difference between predicted detector and measured detector $\widehat{d_i} - d_i$ with the instrument response **C**. Therefore, every iteration will roughly take two times as much computing as a cross correlation. Sims *et al.* (1980) reports a total disadvantage in computational effort of MEM relative to cross correlation of about a factor 20. It should be noted that Sims *et al.* used MEM in simulations that did not take into account the response of a window support structure on the detector (see chapters 4 and 5). Although MEM was not actually employed by the author, it is suspected that, due to this complication, the quoted factor of 20 becomes considerably higher.

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