Complementary Filtered Backprojection
- Implementation and Evaluation

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Based on the article
A New Approximate Algorithm for Image Reconstruction in Cone-Beam Spiral CT at Small Cone-Angles
Stefan Schaller, Thomas Flohr, Peter Steffen
Siemens / Erlangen-Nuremberg University

Evolution
- two-dimensional filtered backprojection
- three-dimensional FBP (Feldkamp)
- Feldkamp with helix trajectory (Wang)
- Wang with complementary rays (Schaller)

Two-Dimensional Filtered Backprojection

- Weighting and filtering
  \[ \hat{p}(\alpha, \beta) = g(\beta) \ast p(\alpha, \beta) \cos \beta \]

- Backprojection
  \[ f(x, y) = \frac{2}{\pi N} \sum_{\alpha=0}^{2\pi} \frac{R}{L^2} \hat{p}(\alpha, \beta) \]
Shortcomings

- acquisition of only one slice at a time
- most X-rays never hit the detector
- time consuming to acquire a volume

Feldkamp’s Algorithm

Shortcomings

- approximate method, exact only in the central plane
- changing image quality along z-axis

• Weighting and filtering
  \[ \hat{p}(\alpha, \beta, \zeta) = \frac{R}{\sqrt{R^2 + \zeta^2}} g(\beta) \ast p(\alpha, \beta, \zeta) \cos \beta \]

• Backprojection
  \[ f(x, y, z) = \frac{2}{\pi N} \sum_{\alpha=0}^{2\pi} \frac{R}{L^2} \hat{p}(\alpha, \beta, \zeta) \]
Wang’s Algorithm

- Weighting and filtering
  \[ \tilde{p}(\alpha, \beta, \zeta) = \frac{R}{\sqrt{R^2 + \zeta^2}} g(\beta) \ast p(\alpha, \beta, \zeta) \cos \beta \]

- Backprojection
  \[ f(x, y, z) = \frac{2}{\pi N} \sum_{n=0}^{2\pi} \frac{R}{L} \tilde{p}(\lambda 2\pi + \alpha_0, \hat{\beta}, \hat{\zeta}) \]

Shortcomings

- limited pitch
- not optimal interpolation

Complementary projections

\[ p^c(\alpha, \beta, \zeta) = p(\hat{\alpha}, \hat{\beta}, \hat{\zeta}) \]

where

\[ \hat{\alpha} = \alpha + 2\beta - \pi \]
\[ \hat{\beta} = -\beta \]
To be cut out and attached to the previous slide.

Slide 13

A complementary fan

Slide 14

A complementary cone

Slide 15

Direct and complementary cones

Slide 16
Schaller’s algorithm

- Weighting and filtering
  \[ \hat{p}(\alpha, \beta, \zeta) = \frac{R}{\sqrt{R^2 + \zeta^2}} \theta(\beta) * p(\alpha, \beta, \zeta) \cos \beta \]

- Backprojection
  \[ f(x, y, z) = \frac{2}{\pi N} \sum_{\alpha_\gamma=0}^{2\pi} \left( \frac{R}{L} \right)^2 \times \]
  \[ \sum_c \sum_\lambda \sum_\zeta \hat{p}(\lambda 2\pi + \alpha_\gamma, \beta, \zeta, c) h(d_z) \over \sum_c \sum_\lambda \sum_\zeta h(d_z) \]
Shortcomings

- only for small cone-angles ($\leq \pm 1.2^\circ$ according to Schaller)
- computationally expensive
- perhaps too complex interpolation