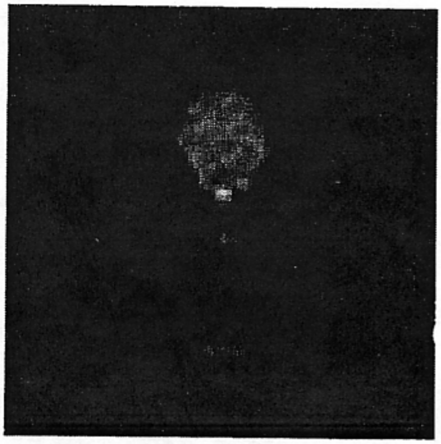

Fig. 7. The best reconstruction from noisy data using filtered backprojection and a spatially invariant filter.

Fig. 8. The best reconstruction from noisy data using a t-f mask.

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A Derivative-Free Noncircular Fan-Beam Reconstruction Formula

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Abstract—**In order to perform fan-beam reconstruction using projection data collected from a noncircular scanning locus, existing noncircular fan-beam formulas require a derivative of the scanning locus with respect to the rotation angle. In this paper, a derivative-free noncircular fan-beam reconstruction formula is obtained based on a geometrical explanation of the circular equispatial fan-beam reconstruction formula. A mathematical proof is then provided under the conditions that the source-to-origin distance is symmetric with respect to the origin of the reconstruction coordinate system, is differentiable almost everywhere and changes not too fast with respect to the rotation angle. The derivative-free noncircular fan-beam reconstruction formula is the same as the circular one, except that the source-to-origin distance is a function of the rotation angle. A typical simulation result of the noncircular fan-beam formula is given.**

I. Introduction

Fan-beam reconstruction was first studied in the case of a circular scanning locus [1], [2], and then extended into the case of

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noncircular scanning loci [3]–[8]. All existing noncircular fan-beam formulas require a derivative of the scanning locus with respect to the rotation angle. In this paper, a derivative-free noncircular fan-beam reconstruction formula is presented and proved under some conditions.

II. Intuitive Derivation

The circular equispatial fan-beam reconstruction formula is as below (formula (169) on page 105 of [9]):

$$g(x,y)=\frac{1}{2}\int_0^{2\pi}\frac{D^2}{(D-s)^2}\int_{-\infty}^{\infty}R_\beta(p)h\left(\frac{Dt}{D-s}-p\right)\frac{D}{\sqrt{D^2+p^2}}dpd\beta \tag{1}$$

where D is a constant source-to-origin distance, $R_\beta(p)$ represents equispatial fan-beam projection data,

$$t=x\cos\beta+y\sin\beta \quad\text{and}\quad s=-x\sin\beta+y\cos\beta$$

where β is the rotation angle, and

$$h(p)=\int_{-\infty}^{\infty}|\,w\,|\,e^{j2\pi wp}dw$$

where the integral limits can be set from $-W$ to W in practice, W is a sufficient large constant, corresponding to the bandwidth of the projection data.

In the noncircular scanning case, the source-to-origin distance is a function of the rotation angle, which can be described as $D(\beta), \beta\in[0,2\pi)$. Note that the definition range of β means that an object is irradiated from all directions. Summing up differential contributions from each source position based on the circular equispatial fan-beam reconstruction formula, we *intuitively* obtain a noncircular fan-beam reconstruction formula as follows:

$$g(x,y)=\frac{1}{2}\int_0^{2\pi}\frac{D^2(\beta)}{(D(\beta)-s)^2}\cdot\int_{-\infty}^{\infty}R_\beta(p)h\left(\frac{D(\beta)t}{D(\beta)-s}-p\right)\frac{D(\beta)}{\sqrt{D^2(\beta)+p^2}}dpd\beta \tag{2}$$

The difference between our noncircular fan-beam formula and the circular one lies in the definition of D. D is a function in the former, while a constant in the latter. Note that our formula is for an imaginary detector array that passes through the origin. In reality, an angular-dependent scale transform is needed to map the projection data onto the imaginary detector array. Alternatively, a formula for an actual detector array can be derived.

III. Mathematical Proof

Our noncircular fan-beam formula intuitively obtained is exact under the conditions that

1) $D(\beta)=D(\beta+\pi)$;
2) $D'(\beta)$ exists almost everywhere;
3) $D^2(\beta)>D'(\beta)p_m$, where p_m is the minimum value such that $R_\beta(p)=0$, if $|\,p\,|>p_m$.

Actually, the third condition is easily satisfied in practice, because in general $D(\beta)$ is much greater than p_m and $D'(\beta)$ is not very large. Fig. 1 shows several scanning loci which satisfy the above three conditions.

Let us start with the parallel-beam reconstruction formula (formula (74) on page 87 of [9]):

$$g(r,\phi)=\frac{1}{2}\int_0^{2\pi}\int_{-t_m}^{t_m}P_\theta(t)h(r\cos(\theta-\phi)-t)dtd\theta \tag{3}$$

where the polar coordinates (r,ϕ) is used,

$$x=r\cos\phi \quad\text{and}\quad y=r\sin\phi,$$

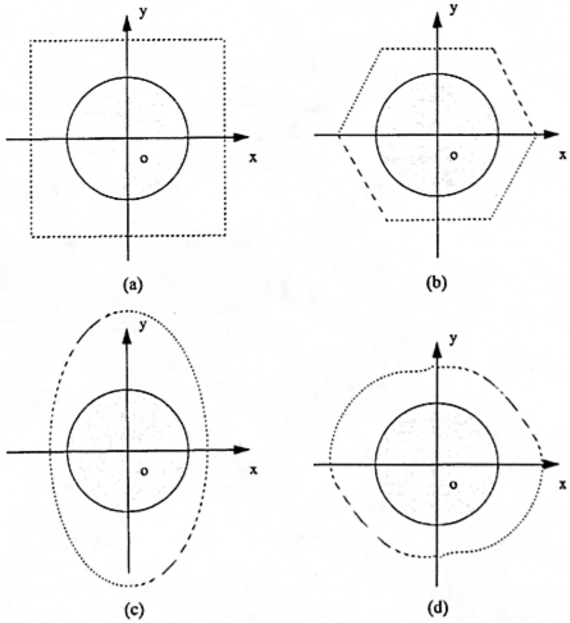

Fig. 1. Scanning loci (dotted lines) which are symmetric with respect to the origin of the reconstruction system x-y, differentiable almost everywhere and do not change too fast. The reconstruction region is represented by the shaded circle.

$P_\theta(t)$ represents parallel-beam projection data, and t_m is the counterpart of p_m. The polar coordinates (r,ϕ) are related to the rotated coordinates (t,s) by

$$t=r\cos(\beta-\phi) \quad\text{and}\quad s=-r\sin(\beta-\phi).$$

Taking advantage of Jacobi's transformation, the variables t and θ can be converted to the parameters p and β describing equispatial fan-beam projection data $R_\beta(p)$ (Fig. 2), based on the following relationship

$$t=p\cos\gamma \quad\text{and}\quad \theta=\beta+\gamma$$

where $\gamma=\tan^{-1}\frac{p}{D(\beta)}$. That is,

$$t=\frac{pD(\beta)}{\sqrt{D^2(\beta)+p^2}} \quad\text{and}\quad \theta=\beta+\tan^{-1}\left(\frac{p}{D(\beta)}\right). \tag{4}$$

It can be shown that

$$\frac{\partial t}{\partial p}=\frac{D\sqrt{D^2+p^2}-pD(D^2+p^2)^{-\frac{1}{2}}p}{D^2+p^2}$$
$$\frac{\partial t}{\partial \beta}=\frac{pD'\sqrt{D^2+p^2}-pD^2(D^2+p^2)^{-\frac{1}{2}}D'}{D^2+p^2}$$
$$\frac{\partial \theta}{\partial p}=\frac{1}{1+(\frac{p}{D})^2}\frac{1}{D}$$
$$\frac{\partial \theta}{\partial \beta}=1+\frac{1}{1+(\frac{p}{D})^2}\left(-\frac{p}{D^2}D'\right)$$

and hence,

$$\frac{dtd\theta}{dpd\beta}=\left\|\frac{D^3-DD'p}{(D^2+p^2)^{\frac{3}{2}}}\right\|. \tag{5}$$

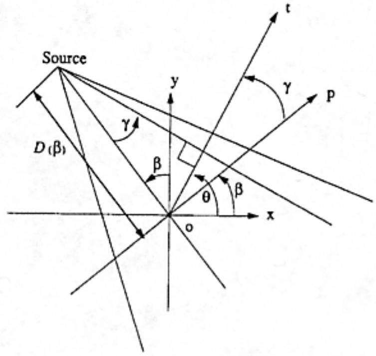

Fig. 2. Several parameters used in the derivation of the derivative-free noncircular fan-beam reconstruction formula.

If the third condition is satisfied,

$$dtd\theta = \left[\frac{D^3}{(D^2+p^2)^{\frac{3}{2}}} - \frac{DD'p}{(D^2+p^2)^{\frac{3}{2}}}\right]dpd\beta. \tag{6}$$

Consequently,

$$\begin{aligned} g(r,\phi) =& \frac{1}{2}\int_0^{2\pi}\int_{-p_m}^{p_m} P_{\beta+\gamma}\left(\frac{pD(\beta)}{\sqrt{D^2(\beta)+p^2}}\right) \\ &\cdot h\left[r\cos(\beta+\gamma-\phi) - \frac{pD(\beta)}{\sqrt{D^2(\beta)+p^2}}\right] \\ &\cdot \left[\frac{D^3(\beta)}{(D^2(\beta)+p^2)^{\frac{3}{2}}} - \frac{D(\beta)D'(\beta)p}{(D^2(\beta)+p^2)^{\frac{3}{2}}}\right]dpd\beta. \end{aligned} \tag{7}$$

The above equation can be reduced to

$$\begin{aligned} g(r,\phi) =& \frac{1}{2}\int_0^{2\pi}\int_{-p_m}^{p_m} P_{\beta+\gamma}\left(\frac{pD(\beta)}{\sqrt{D^2(\beta)+p^2}}\right) \\ &\cdot h\left[r\cos(\beta+\gamma-\phi) - \frac{pD(\beta)}{\sqrt{D^2(\beta)+p^2}}\right] \\ &\cdot \frac{D^3(\beta)}{(D^2(\beta)+p^2)^{\frac{3}{2}}}dpd\beta \end{aligned} \tag{8}$$

because

$$\begin{aligned} &\int_0^{2\pi}\int_{-p_m}^{p_m} P_{\beta+\gamma}\left(\frac{pD}{\sqrt{D^2+p^2}}\right) \\ &\cdot h\left[r\cos(\beta+\gamma-\phi) - \frac{pD}{\sqrt{D^2+p^2}}\right] \\ &\cdot \frac{DD'p}{(D^2+p^2)^{\frac{3}{2}}}dpd\beta = I_1 + I_2 = 0 \end{aligned} \tag{9}$$

where

$$\begin{aligned} I_1 =& \int_0^{\pi}\int_{-p_m}^{p_m} P_{\beta+\gamma}\left(\frac{pD}{\sqrt{D^2+p^2}}\right) \\ &\cdot h\left[r\cos(\beta+\gamma-\phi) - \frac{pD}{\sqrt{D^2+p^2}}\right] \\ &\cdot \frac{DD'p}{(D^2+p^2)^{\frac{3}{2}}}dpd\beta \end{aligned} \tag{10}$$

$$\begin{aligned} I_2 =& \int_{\pi}^{2\pi}\int_{-p_m}^{p_m} P_{\beta+\gamma}\left(\frac{pD}{\sqrt{D^2+p^2}}\right) \\ &\cdot h\left[r\cos(\beta+\gamma-\phi) - \frac{pD}{\sqrt{D^2+p^2}}\right] \\ &\cdot \frac{DD'p}{(D^2+p^2)^{\frac{3}{2}}}dpd\beta \\ =& \int_0^{\pi}\int_{-p_m}^{p_m} P_{\beta+\gamma}\left(\frac{-pD}{\sqrt{D^2+p^2}}\right) \\ &\cdot h\left[-r\cos(\beta+\gamma-\phi) - \frac{pD}{\sqrt{D^2+p^2}}\right] \\ &\cdot \frac{DD'p}{(D^2+p^2)^{\frac{3}{2}}}dpd\beta \\ =& \int_0^{\pi}\int_{p_m}^{-p_m} P_{\beta+\gamma}\left(\frac{pD}{\sqrt{D^2+p^2}}\right) \\ &\cdot h\left[r\cos(\beta+\gamma-\phi) - \frac{pD}{\sqrt{D^2+p^2}}\right] \\ &\cdot \frac{DD'p}{(D^2+p^2)^{\frac{3}{2}}}dpd\beta = -I_1 \end{aligned} \tag{11}$$

making use of the facts that $P_{\theta+\pi}(-t) = P_\theta(t)$, $D(\beta+\pi) = D(\beta)$, $D'(\beta+\pi) = D'(\beta)$ and $h(-p) = h(p)$. Note that

$$P_{\beta+\gamma}\left(\frac{pD(\beta)}{\sqrt{D^2(\beta)+p^2}}\right) = R_\beta(p)$$

and

$$\begin{aligned} r\cos(\beta+\gamma-\phi) - \frac{pD(\beta)}{\sqrt{D^2(\beta)+p^2}} &= r\cos(\beta-\phi)\frac{D(\beta)}{\sqrt{D^2(\beta)+p^2}} \\ &\quad - (D(\beta) + r\sin(\beta-\phi))\frac{p}{\sqrt{D^2(\beta)+p^2}} \\ &= (p'-p)\frac{UD(\beta)}{\sqrt{D^2(\beta)+p^2}} \end{aligned}$$

where

$$U = \frac{D(\beta)-s}{D(\beta)} \quad \text{and} \quad p' = \frac{D(\beta)t}{D(\beta)-s}$$

it follows that

$$\begin{aligned} g(r,\phi) =& \frac{1}{2}\int_0^{2\pi}\int_{-p_m}^{p_m} R_\beta(p) \\ &\cdot h\left[(p'-p)\frac{UD(\beta)}{\sqrt{D^2(\beta)+p^2}}\right]\frac{D^3(\beta)}{(D^2(\beta)+p^2)^{\frac{3}{2}}}dpd\beta. \end{aligned} \tag{12}$$

Furthermore,

$$\begin{aligned} h\left[(p'-p)\frac{UD(\beta)}{\sqrt{D^2(\beta)+p^2}}\right] &= \int_{-\infty}^{\infty}|w|e^{j2\pi w(p'-p)\frac{UD(\beta)}{\sqrt{D^2(\beta)+p^2}}}dw \\ &= \frac{D^2(\beta)+p^2}{U^2D^2(\beta)}\int_{-\infty}^{\infty}|w'|e^{j2\pi(p'-p)w'}dw' \\ &= \frac{D^2(\beta)+p^2}{U^2D^2(\beta)}h(p'-p) \end{aligned} \tag{13}$$

where $w = w'\frac{\sqrt{D^2(\beta)+p^2}}{UD(\beta)}$. Substituting (13) into (12),

$$g(r,\phi) = \frac{1}{2}\int_0^{2\pi}\frac{1}{U^2}\int_{-p_m}^{p_m} R_\beta(p)h(p'-p)\frac{D(\beta)}{\sqrt{D^2(\beta)+p^2}}dpd\beta. \tag{14}$$

It can be seen directly that (14) is equivalent to (2), which reduces to the conventional equispatial fan-beam formula if $D(\beta)$ is

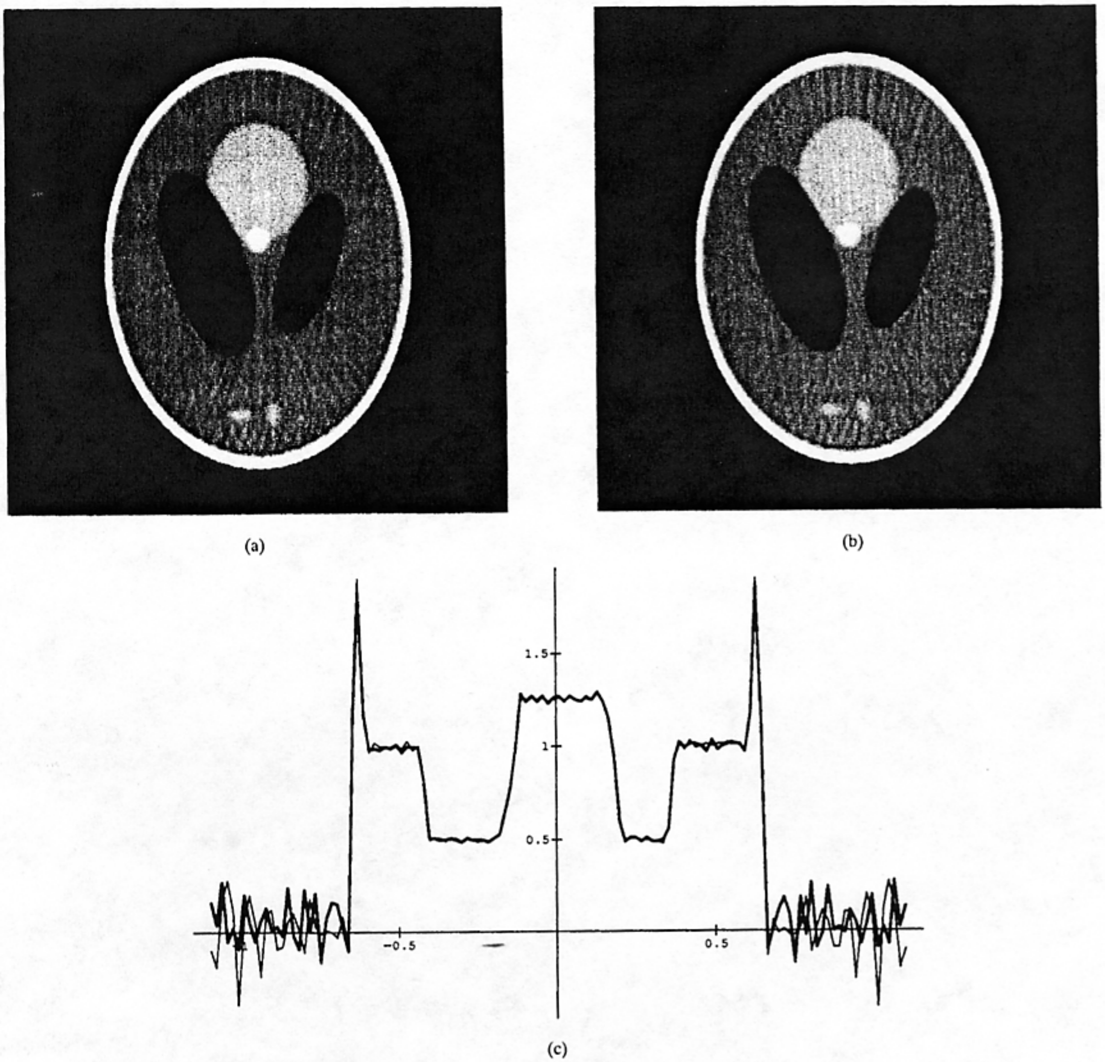

Fig. 3. Comparison between circular and noncircular fan-beam reconstruction. (a) Reconstructed Shepp and Logan's head phantom image using the circular fan-beam formula with a scanning circle of diameter 6. (b) Corresponding reconstructed image using the derivative-free noncircular fan-beam formula with a scanning square of side length 6. (c) Plots ($y = 0.198$) of images (a) (the thin curve) and (b) (the thick curve).

a constant (formula (116) on page 90 of [9]). If an arbitrary scanning locus does not meet all the three conditions, our formula will give an approximate reconstruction.

IV. SIMULATION

In the simulation, the length of the detector line is 2.2 with 128 cells and is so placed for the ease of computation that its center is at the origin of the reconstruction coordinate system x-y. Shepp and Logan's head phantom was used. The head phantom parameters are listed in Table I. There are eight ellipses in the phantom. x_0, y_0 are the center of an ellipse. a, b are the x and y semi-axes respectively. α is the rotation angle of an ellipse. τ is a relative parameter such that the x-ray absorption coefficient of any point is the sum of the relative parameters of ellipses which contain the point. One hundred projections with a 3.6° angular interval were generated for the reconstruction.

TABLE I
PARAMETERS OF SHEPP AND LOGAN'S HEAD PHANTOM USED IN THE SIMULATION.

No.	x_0	y_0	a	b	α	τ
1	0.00	0.000	0.663	0.884	0	2.00
2	0.00	0.000	0.635	0.838	0	-1.00
3	-0.22	0.000	0.410	0.160	108	-0.50
4	0.22	0.000	0.310	0.110	72	-0.50
5	0.00	0.350	0.210	0.250	0	0.25
6	0.00	0.100	0.046	0.046	0	0.50
7	-0.08	-0.650	0.046	0.023	0	0.25
8	0.06	-0.650	0.046	0.023	90	0.25

Fig. 3(a) shows reconstructed Shepp and Logan's head phantom image using the circular fan-beam formula with a scanning circle of diameter 6. Fig. 3(b) shows the reconstructed image corresponding to Fig. 2(a) using our noncircular fan-beam formula with a scanning square of side-length 6. Fig. 3(c) shows the plots of the $y = 0.198$ line for Fig. 3(a) and 3(b). It can be observed that the reconstructed results obtained using different fan-beam formulas are almost the same in the reconstruction region. More simulation results were presented in [10]. The parameters of the above simulation are typical in our x-ray microtomographic system [11]. It can be verified that the square scanning locus used indeed meets all three conditions.

V. Conclusion

As far as the mathematical form is concerned, the proposed formula is the simplest among noncircular fan-beam formulas, as it is the same as the circular fan-beam formula [9] except that the source-to-origin distance depends on the rotation angle. However, we would like to emphasize that our formula is exact only with a symmetric scanning locus. If this symmetry condition is violated, other noncircular fan-beam formulas should be used for exact reconstruction, provided that the relevant conditions required are satisfied.

The main difference between Weinstein's formula [3] or Gullberg's formula [5] and the proposed formula lies in that the new formula requires no derivative of the scanning locus with respect to the rotation angle. Smith's extended fan-beam formula also contains a derivative of the scanning locus [4]. For an irregular scanning locus, it may not be trivial to estimate accurately its derivative. For example, in our x-ray microtomographic study, the scanning locus is subject to random interferences introduced by the mechanical rotation of the specimen stage [11]–[13]. As a result, a precise estimation of the derivative of the scanning locus is particularly difficult. On the other hand, the practical scanning locus used in x-ray microtomography can be made to meet our three conditions [11]–[13]. Without using the derivative of the scanning locus, the proposed formula will not be affected by the error in estimating the derivative.

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A Fast Algorithm for Backprojection with Linear Interpolation

Berkman Sahiner and Andrew E. Yagle

Abstract—**In the filtered backprojection procedure for image reconstruction from projections, backprojection dominates the computation time. We propose a simple algorithm that reduces the number of multiplications in linear interpolation and backprojection stage by 50%, with a small increase in the number of additions. The algorithm performs the interpolation and backprojection of four views together. Examples of implementation are given and extension to interpolation of more than four views is discussed.**

I. Introduction

The basic problem of x-ray tomography is to reconstruct an image $\mu(x, y)$ from its projections $p(r, \theta)$ where

$$p(r,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu(x,y)\delta(r - x\cos\theta - y\sin\theta)dxdy \quad (1)$$

is the Radon transform of $\mu(x, y)$.

The most common procedure for image reconstruction from projections is filtered backprojection (FBP), in which the reconstruction is carried out in two stages. The first stage is filtering, in which the projections are filtered by a modified ramp filter to yield the filtered projections $q(r, \theta)$ [1]. The second stage is backprojection, in which the desired image $\mu(x, y)$ is obtained from filtered projections by backprojection.

In practical problems, we have only samples of $p(r, \theta)$. Let us denote

$$P_i(R) = p(r, \theta_i)\Big|_{r=Rd,\theta_i=\frac{\pi}{N}(i-1)}, R = 0, \cdots, M-1, i = 1, \cdots, N \quad (2)$$

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