Generalized Feldkamp Image Reconstruction from Equiangular Cone-Beam Projection Data

[a]Ben Wang
[ab]*Dept. of Elec. and Comp. Eng.*
Beijing Union Univ.
Beijing 100101, China
wbtl@publica.bj.cninfo.net

[b]Hong Liu
[b]*Dept. of Radiology*
Johns Hopkins Univ.
Baltimore, MD 21205
hliu@jhmi.edu

[c]Ge Wang
[c]*Dept. of Radiology*
Univ. of Iowa
Iowa City, Iowa 52242
ge-wang@uiowa.edu

Abstract

The cone-beam approach for image reconstruction attracts an increasing attention in various applications, especially medical imaging. Previously, the traditional practical cone-beam reconstruction method, the Feldkamp algorithm, was generalized into the case of spiral/helical scanning loci with equispatial cone-beam projection data. In this paper, we formulated the generalized Feldkamp algorithm in the case of equiangular cone-beam projection data. Because medical multi-slice/cone-beam CT scanners typically use equiangular projection data, our new formula may be useful in the area of medical X-ray imaging.

1. Introduction

Recently, spiral/helical CT began a transition from fan-beam to cone-beam geometry with the introduction of multi-slice systems [1,2]. These narrow-angle cone-beam spiral CT scanners, also referred to as multi-slice or multi-row detector scanners, are already commercially available. Cone-beam spiral CT uses a 2D-detector array, allows larger scanning range in shorter time with higher image resolution, and has important medical and other applications.

Despite progress in exact cone-beam reconstruction [3,4], approximate cone-beam algorithms, especially Feldkamp-type algorithms, remain valuable in practice [5-8]. The three advantages of approximate cone-beam reconstruction are as follows. First, incomplete scanning loci can be used with approximate reconstruction. The completeness condition for exact reconstruction requires that there exist at least a source position on any plane intersecting an object to be reconstructed. This condition may not be satisfied in many cases of X-ray CT. Second, computational efficiency of approximate reconstruction is high. Typically, approximate reconstruction involves less raw data than exact reconstruction. Also, the computational structure of approximate reconstruction is straightforward, highly parallel, hardware-supported, and particularly fast for reconstruction of a small region of interest (ROI). Third, image noise, blurring/ringing artifacts may be less with approximate reconstruction. With the direct Fourier method [3], it was found that the exact reconstruction produced more ringing as compared to the Feldkamp method. It appears that this type of ringing is inherent to all exact cone-beam reconstruction formulas that take the second derivative of data.

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Furthermore, Feldkamp-type algorithms are widely used to benchmark the performance of new cone-beam algorithms.

Traditionally, Feldkamp-type cone-beam reconstruction algorithms were formulated for equispatial cone-beam projection data [5-8]. However, in medical cone-beam X-ray CT, projection data are typically represented in an equiangular format. In this paper, we represent a generalized Feldkamp image reconstruction formula for equiangular cone-beam projection data.

2. Generalized Feldkamp reconstruction in the equispatial case

Feldkamp *et al.* adapted the conventional equispatial fan-beam algorithm for cone-beam reconstruction with a circular-scanning locus [5]. Using the same approach, a generalized Feldkamp algorithm for equispatial cone-beam data was derived based on a derivative-free, equispatial fan-beam reconstruction formula for a non-circular scanning locus. In the generalized Feldkamp reconstruction, cone-beam projection data from different orientations are filtered and back-projected along X-ray paths after voxel-to-source distance and angular differential are properly modified. The reconstructed value of a voxel is the sum of contributions from all horizontally tilted fan-beams passing through that voxel. The key formulas are briefly reviewed in this section, as a background for the derivation of a generalized Feldkamp formula for equiangular cone-beam projection data.

As shown in Figure 1, the equispatial fan-beam reconstruction formula for a non-circular scanning locus is expressed as [7]:

$$g(x,y)=\frac{1}{2}\int_0^{2\pi}\frac{\rho^2(\beta)}{[\rho(\beta)-s]^2}\int_{-\infty}^{\infty}R(p,\beta)f[\frac{\rho(\beta)t}{\rho(\beta)-s}-p]\frac{\rho(\beta)}{\sqrt{\rho^2(\beta)+p^2}}dpd\beta, \qquad (1)$$

where $g(x,y)$ is the reconstructed image, $\rho(\beta)$ is the distance from the X-ray source to the origin of the reconstruction system, $R(p,\beta)$ fan-beam projection data as a function of the detector linear position p and the X-ray source rotation angle β, f the reconstruction filter, and

$$t=x\cos\beta+y\sin\beta$$
$$s=-x\sin\beta+y\cos\beta\,.$$

As shown in Figure 2, the generalized Feldkamp algorithm for equispatial cone-beam data is formulated as [8]:

$$g(x,y,z)=\frac{1}{2}\int_0^{2\pi}\frac{\rho^2(\beta)}{[\rho(\beta)-s]^2}\int_{-\infty}^{\infty}R(p,\varsigma,\beta)f[\frac{\rho(\beta)t}{\rho(\beta)-s}-p]\frac{\rho(\beta)}{\sqrt{\rho^2(\beta)+p^2+\varsigma^2}}dpd\beta, \qquad (2)$$

where $g(x,y,z)$ is the reconstructed image, $\rho(\beta)$ is the distance from the X-ray source to the z axis of the reconstruction system, $R(p,\varsigma,\beta)$ cone-beam projection data as a function of the detector spatial position (p,ς) and the X-ray source rotation angle β, f the reconstruction filter, and

$$t=x\cos\beta+y\sin\beta$$
$$s=-x\sin\beta+y\cos\beta\quad.$$
$$\varsigma=\frac{\rho(\beta)[s-z+h(\beta)]}{\rho(\beta)-s}$$

3. Generalized Feldkamp reconstruction in the equiangular case

The essential step in the generalized Feldkamp reconstruction described in the preceding section may be considered as appropriate correction from cone-beam data to fan-beam data, so that exact fan-beam reconstruction of a 3D impulse function can be achieved in the transaxial plane containing the impulse. The correction is done by multiplying cone-beam data with the cosine of the X-ray tilting angle. In other words, the generalized Feldkamp reconstruction can be decomposed into two steps: (1) cone-beam to fan-beam data conversion, and (2) fan-beam reconstruction.

Recently, we derived and tested the derivative-free equiangular fan-beam reconstruction formula for non-circular scanning loci. The new formula is the same as the conventional equiangular fan-beam formula, except that the constant source-to-origin distance is made a function with respect to the X-ray source rotation angle [9]:

$$g(x,y)=\frac{1}{2}\int_0^{2\pi}\int_{-\infty}^{\infty}R(\gamma,\beta)f[r\cos(\beta+\gamma-\phi)-\rho(\beta)\sin\gamma]\rho(\beta)\cos\gamma d\gamma d\beta, \qquad (3)$$

where $g(x,y)$ is the reconstructed image, as shown in Figure 3, $\rho(\beta)$ is the distance from the X-ray source to the origin of the reconstruction system, $R(\gamma,\beta)$ fan-beam projection data as a function of the detector angular position γ and the X-ray source rotation angle β, f the reconstruction filter, and

$$r=x^2+y^2$$

$$\phi=\tan^{-1}\frac{x}{y}.$$

In order to derive a generalized Feldkamp image reconstruction formula for equiangular cone-beam data, we applying the fan-beam reconstruction formula (3) with cosine-corrected cone-beam data collected from a 3D-scanning locus, we obtain the main result of this paper:

$$g(x,y,z)=\frac{1}{2}\int_0^{2\pi}\int_{-\infty}^{\infty}R(\gamma,\tau,\beta)f[r\cos(\beta+\gamma-\phi)-\rho(\beta)\sin\gamma]\rho(\beta)\cos\gamma\cos\tau d\gamma d\beta, \qquad (4)$$

where $g(x,y,z)$ is the reconstructed image, as shown in Figure 4, $\rho(\beta)$ is the distance from the X-ray source to the z axis of the reconstruction system, $R(\gamma,\tau,\beta)$ cone-beam projection data as a function of the detector angular position (γ,τ) and the X-ray source rotation angle β, f the reconstruction filter, and

$$r=x^2+y^2$$

$$\phi=\tan^{-1}\frac{x}{y}$$

$$\tau=\tan^{-1}\frac{z-h(\beta)}{\rho(\beta)+x\sin\beta-y\cos\beta}$$

4. Discussion and conclusion

Although the generalized Feldkamp formula with equiangular cone-beam data is still approximate, several analytic properties of the formula can be established regarding exactness in reconstruction, similar to what was done for Feldkamp reconstruction with equispatial data. Firstly, the generalized Feldkamp reconstruction is exact on the mid-plane. In the 2D case, the generalized Feldkamp algorithm degrades to the derivative-free non-circular fan-beam reconstruction formula. If a planar scanning locus is differentiable almost everywhere, symmetric with respect to the origin of the reconstruction system, and the direction tangent to the locus stays outside of the fan delimited by the object support, reconstruction must be exact [7,9]. Secondly, the generalized Feldkamp reconstruction produces exact vertical integrals. The longitudinal integral of the spatially varying point spread function of the generalized cone-beam algorithm is a 2D impulse function. As a result, after longitudinal integral, an exact 2D parallel projection can be synthesized. Thirdly, the generalized Feldkamp reconstruction is exact reconstruction for longitudinal homogeneous specimens.

In conclusion, motivated by medical cone-beam X-ray applications, we have derived a generalized Feldkamp image reconstruction formula for equiangular cone-beam projection data. Our work is based on a newly published derivative-free, equiangular fan-beam reconstruction formula for a non-circular scanning locus [9]. In the case that projection data are in an equiangular format, our formula can be used to reconstruct the image without interpolation blurring. Image quality evaluation and refinement are underway.

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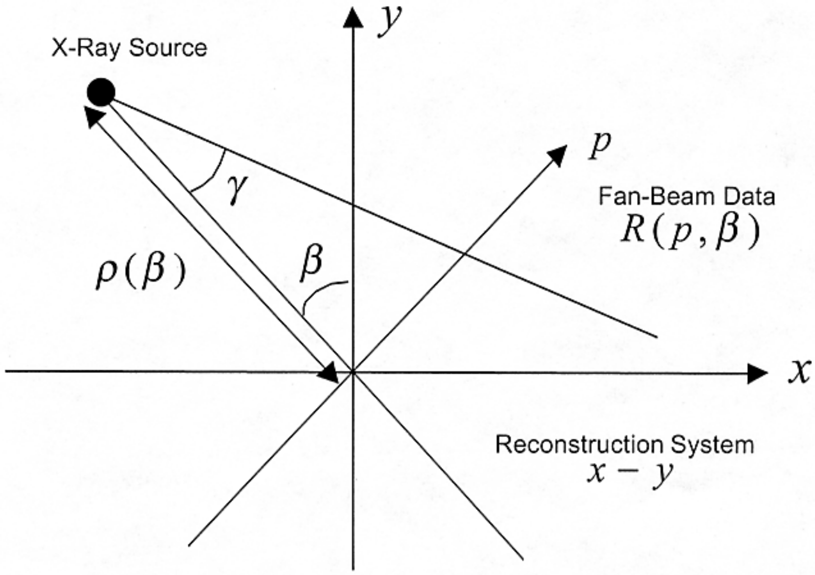

Figure 1. Geometry of equispatial fan-beam reconstruction.

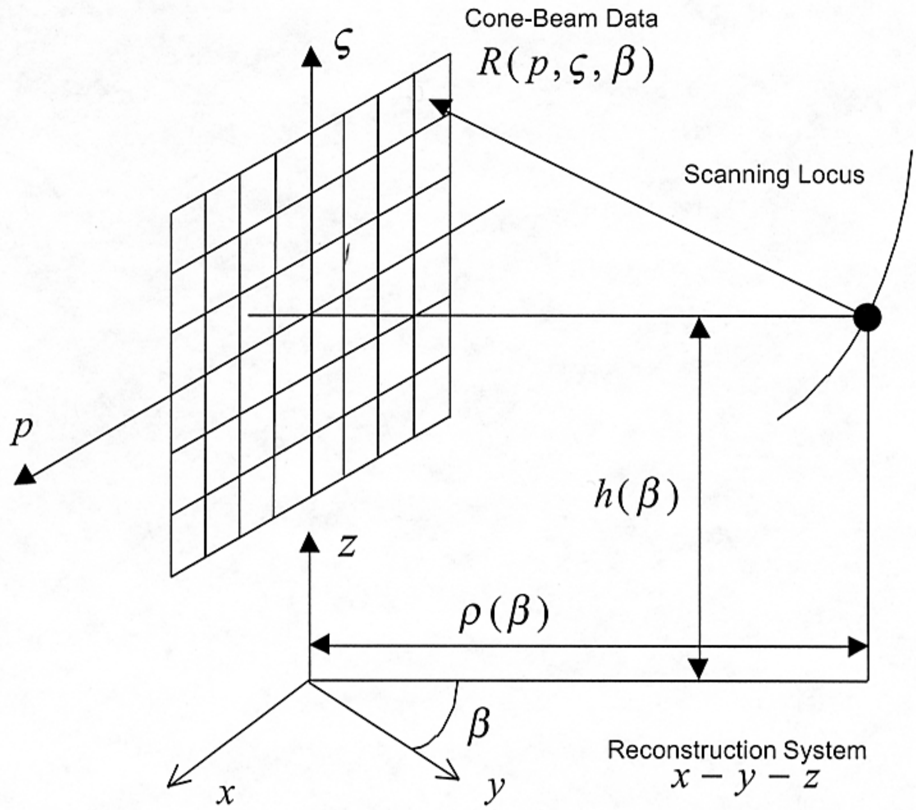

Figure 2. Geometry of equispatial cone-beam reconstruction.

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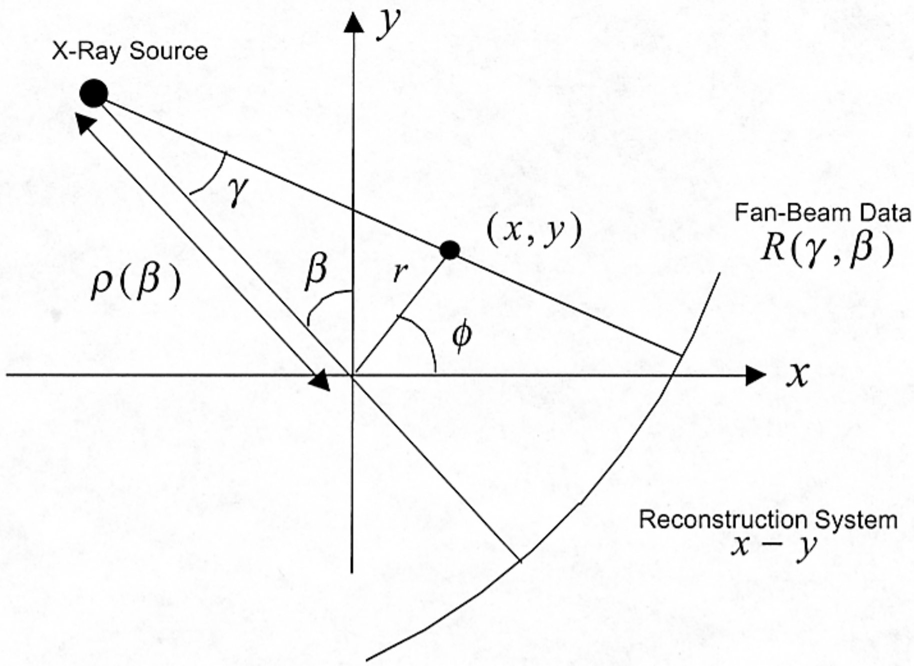

Figure 3. Geometry of equiangular fan-beam reconstruction.

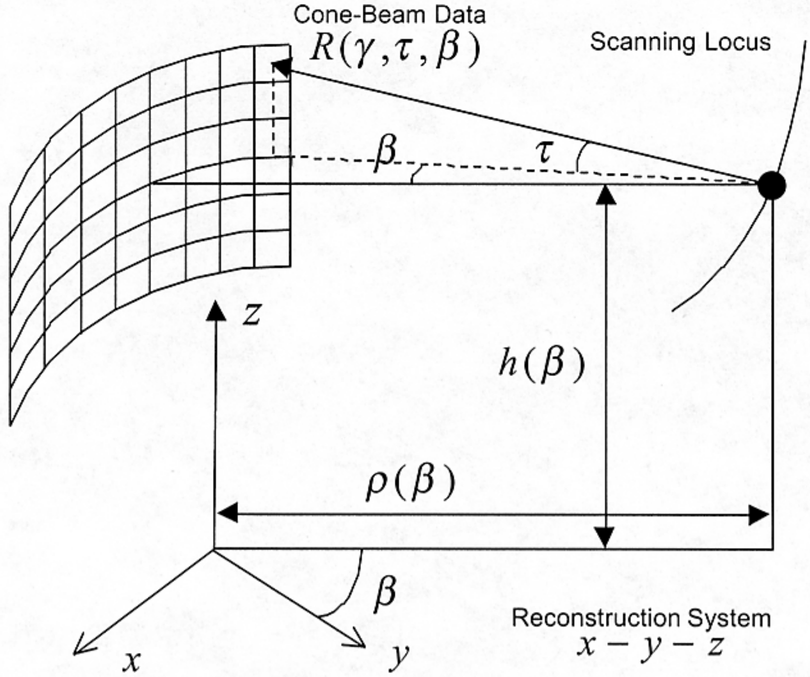

Figure 4. Geometry of equiangular cone-beam reconstruction.