Coded Source Imaging for Neutrons and X-Rays

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***Abstract*– X-ray and neutron imaging techniques are powerful tools applied to a wide array of problems, like fuel cells imaging, residual stress analysis and detection of flaws in materials. These modern applications are increasingly demanding of high performing imaging devices.**

Coded Source Imaging (CSI) is a method applied both to X-ray and neutron imaging. It consists of a source composed of a pattern of many point-like sources. We present a description of the image formation process with coded sources, via encoding and decoding of data. The Signal-to-Noise Ratio (SNR) of the CSI imager system is discussed, as well as its resolution and Field-of-View.

CSI is here presented in a formalism that allows a transposition to Coded Aperture Imaging (CAI). The implications of the relationship of the two techniques in the design of a CSI system are addressed.

I. Introduction

Non-destructive investigation of materials has been conducted for many years using X-ray and neutron radiography. The characteristics of these two types of radiation suggest a complementary use: X-rays are strongly absorbed by heavy elements, while neutrons are preferentially scattered by light elements. In Fig. 1, the neutron radiograph shows detailed features of the plastic component of the sample, whereas the X-ray radiograph highlights the shape of the metallic parts.

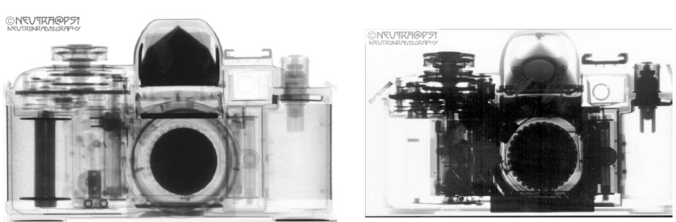

Fig. 1: Neutron radiograph of a camera (on the left). X-ray radiograph of a camera (on the right). [from http://neutra.web.psi.ch/What/index.html]

Radiation sources, in the form of X-ray tubes or neutrons from a nuclear reactor, are in general extended and not immediately suitable for imaging. A commonly used imaging technique is to obtain a point-like radiation source by using a pinhole. The pinhole, shielding the source except in a very small region, allows immediate image formation by establishing a geometrical correlation between the source and the object.

The resolution of the imager can be roughly described as its ability to distinguish between two close points in the object. The pinhole size is the limiting factor, and the smaller the pinhole, the higher the spatial resolution of the imager will be. In the extreme case of an imager with a pinhole with zero-radius, one would obtain perfect resolution, being able to distinguish points infinitely close to each other. It is immediately obvious that a system like the one just described would not work. A zero-size pinhole blocks all radiation from passing through, and no image would be collected by the detector. The increase in resolution comes with a price: a reduced throughput signal, which means longer exposures. The designer of an imaging system faces a trade-off between image resolution and exposure time.

While reducing the signal to increase resolution may be an affordable requirement for X-ray sources, it is particularly punishing for neutron sources. In general, neutrons are less readily available than X-rays and they come from sources that are less bright than X-ray sources. Nevertheless, neutron radiography should be the imaging technique of choice for problems where a sample containing light elements, such as plastic or water, needs to be inspected. Also, the inspection of a thick sample made of heavy elements, e.g. a plane wing, is not practical with X-rays.

The idea behind Coded Source Imaging (CSI) is to exchange the pinhole source with another, brighter one. The goal is to improve the imager by increasing the ratio of the signal to the noise. A simple idea can be to replace the single pinhole with two pinholes. It is straightforward to see that the number of photons or neutrons actually used in imaging doubles. Two images of the sample are cast on the detector. If the pinhole distance is chosen so that the two images are not overlapping, a reconstruction involving joining the two images will give better count statistics than a single pinhole would. Instead of two pinholes, a set of pinholes can be drilled to obtain a coded source.

The example of the two-pinhole coded source underlines two features of CSI: the importance of the pattern in the coded source, and the need for decoding. A wise choice of coded source pattern is paramount in the optimization of the signal-to-noise ratio (SNR) of the system. The decoding also depends on the pattern. As we will see, the proper design of the coded source mask and of the associated decoding array ensures perfect reconstruction.

II. Coded Source Imaging

Spatially coded sources first appear in literature with reference to X-ray imaging [1]. Coded source X-ray tube design comes from the need for high resolution.

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Conventionally, the ways to increase resolution in transmission imaging are to decrease the focal-spot size or to increase the source-to-object distance. These measures are not very effective or feasible, since the exposure time becomes too long: in medical applications, this translates into blurring from patient movement, whereas in material science this may result in an unbearable economic cost. A smaller focal-spot is indeed a way of obtaining higher frequencies, but it is not the only way. If we do not consider simple spots but detailed structures, it is possible to obtain high spatial frequencies despite the large physical dimension of the source. The way to do this is to build a coded source that is a distribution of pinholes following a pattern whose Fourier representation has high frequencies.

The idea of using a coded source had some momentum in the 1970s, and then was discarded in favor of more conventional methods. Both the increasing quality of the available X-ray tube, allowing high enough resolution with standard techniques, and the lack of fast computers to perform the reconstruction of the encoded image in a reasonable time explains the loss of interest for this idea. The success obtained by coded apertures, a method similar to coded source, and the huge increase in computing power in recent years justify a renewed interest in this field.

A. Encoding and Decoding

An encoded image is obtained through CSI. The description of the encoding of the image formation process is straightforward. With reference to Fig. 2, the recorded image on the detector can be written as:

$$R(x_r) = \int S(x_s) O(x_O) dx_s = \int S(x_s) O(x_r - \frac{x_r - x_s}{a+b} b) dx_s \quad (1)$$

where S is the function describing the coded source, O is the function describing the transmission of the object, x_s is the running variable along the source plane, x_r is the running variable along the detector plane, and x_o is the running variable along the object plane. In (1) the simple geometrical relation

$$x_O = x_r - \frac{x_r - x_s}{a+b} b \quad (2)$$

was used.

The recorded image can be written in a more useful form by the change of variable

$$x_s^{'} = -\frac{b}{a} x_s; \quad (3)$$

$$R(x_r) \propto \int S(-\frac{a}{b} x_S^{'}) O(\frac{a}{a+b}(x_r - x_S^{'})) dx_S^{'}. \quad (4)$$

This form is immediately recognizable as a correlation, giving the set of equations that are cardinal to the encoding process:

$$R(x_r) \propto S_{-}^{m_1} * O^{m_2} \quad (5)$$

$$S_{-}^{m_1}(\chi) = S(-\frac{a}{b}\chi) \quad (6)$$

$$O^{m_2}(\chi) = O(\frac{a}{a+b}\chi). \quad (7)$$

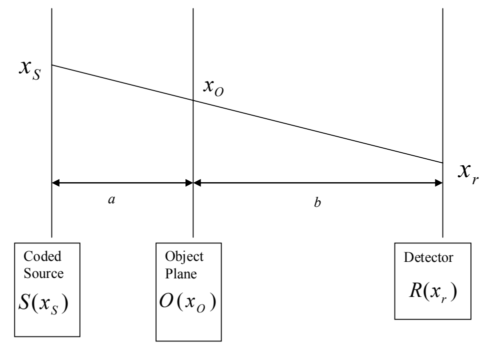

Fig. 2 : Outline of the geometry of a Coded Source Imaging system.

The recorded image is the correlation of the coded source and of the object. It is tempting to propose a decoding technique involving the fact that the Fourier transform of a correlation is the product of the Fourier transform of the two terms.

$$\hat{O} = F^{-1}(\frac{F(R)}{F(S)}) \quad (8)$$

Unfortunately, the presence of noise in the real data makes this technique impractical. In the real case, (8) would read

$$\hat{O} = F^{-1}(\frac{F(R)}{F(S)}) - F^{-1}(\frac{F(N)}{F(S)}), \quad (9)$$

where F(N) is constant at all frequencies, whereas F(S) typically has zeros. The noise term will then dominate the reconstructed image.

A better strategy for decoding is to use a decoding pattern G.

$$\hat{O} = (S_{-}^{m_1} * O^{m_2}) \times G = O^{m_2} * (S_{-}^{m_1} \times G) = O^{m_2} * PSF. \quad (10)$$

In (10) the recorded image is convoluted with the decoding pattern. The point spread function (PSF) is the response of the imager to a point-like object. It is easy to notice how the designer is free to set the PSF by choosing a couple (S, G) so that

$$S_{-}^{m_1} \times G = \delta. \quad (11)$$

This is the condition for CSI to achieve perfect reconstruction. The reader will notice that in the presence of noise, (11) reads

$$\hat{O} = (S_{-}^{m_1} * O^{m_2} + N) \times G = O^{m_2} * (S_{-}^{m_1} \times G) + N \times G, \quad (12)$$

where $N \times G$ is the convolution of a constant with a function, which is a constant no matter what the function is.

B. Signal-to-Noise Ratio advantage over a single pinhole.

The rationale for using a coded source in an imager is to improve the count statistics of the system. In this context, an accepted figure of merit is the ratio between throughput signal, i.e. the image of the object, and the noise associated with it. The ratio of the two gives a good indication of the quality of the imager and is called the Signal-to-Noise Ratio (SNR).

It is useful at this point to evaluate the SNR of a single pinhole and compare it to the SNR of a multiple pinhole system. Assuming ideal behavior of the detector and of the electronics associated with it, the source of noise is the radiation source itself. Without specifying if the source under investigation emits X-rays or neutrons, it is reasonable to assume that the emission process follows a Poisson distribution. Thus, for the properties of the distribution, if s is the transmitted signal, s is also its variance, and the noise associated to it is $\sqrt{s}$. The resulting SNR is $\sqrt{s}$.

The same calculation can be easily done for coded sources, with a limiting assumption. In the presence of N pinholes, the total signal is $N \cdot s$. Suppose that each pinhole casts an image of the object that does not overlap with the images cast by the other pinholes. Under this assumption, it is possible to distinguish on the detector the radiation coming from each of the sources, and each image is independent of the others. The variance of a sum of independent sources is the sum of the variances, i.e. $N \cdot s$, resulting in a noise term equal to $\sqrt{N \cdot s}$. The resulting SNR is $\sqrt{N \cdot s}$.

This simplistic argument, see Table 1, shows a SNR advantage of the coded source over a single pinhole of $\sqrt{N}$. This result suggests that the higher the number of pinholes in a coded source, the better the count statistics will be. The problem with this line of reasoning is that a higher N reduces the validity of the assumption that the counts coming from different pinholes are independent. Also, this assumption cannot be a requirement for the design of a coded source, since a non-overlapping property will depend not only on the coded source pattern but also on the geometry of the object.

	Pinhole	N Pinholes	Coded Sources
Signal	S	$N \cdot S$	$< N \cdot S$
SNR	$\sqrt{S}$	$\sqrt{N \cdot S}$	$\sqrt{N \cdot S}$
SNR/Pinhole	1	$\sqrt{N}$	$1 < SNR < \sqrt{N}$

Table 1: SNR advantage for a single pinhole, for an ideal system of N pinholes and for a real coded source with N pinholes.

The question of what happens to the SNR advantage for a real coded source will be addressed in section III. Suffice it here to say that the question depends on the pattern of the coded sources, on its open fraction, and on the characteristics of the object being imaged.

C. Field-of-View and Resolution

Fig. 3 demonstrates the geometry needed to calculate the Field-of-View (FoV) of a CSI imager. The calculation is straightforward and requires only simple geometrical considerations. The relationship between the FoV and the geometry of the imager system is

$$l_1 : d_m = (a - l_1) : FoV,$$

where l_1 is the focal distance to the source, d_m is the size of the coded source mask, and a is the source-to-detector distance. Considering also that

$$l_1 : d_m = l_2 : d_d,$$

with some algebra we obtain

$$FoV = \frac{a \cdot d_d - b \cdot d_m}{a + b}. \tag{13}$$

A simple geometrical approach is not enough to estimate the resolution of a CSI imager. Again, analogy with the single pinhole case gives insight into the problem. In the pinhole case, the resolution is set by the size of the pinhole. A point-like object casts on the detector a shadow the size of the magnified pinhole. If the shadows of two point-like objects overlap, resolution is corrupted and the image blurred. More specifically, it is convenient to describe the size of the shadow of the pinhole shape on the detector in a statistical sense as the full width at half maximum (FWHM).

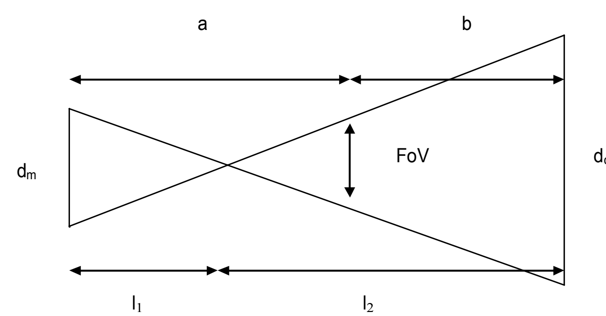

Fig. 3: Geometry for the calculation of the FoV of a CSI imager.

In the case of a coded source, it is necessary to establish a relationship between the shape of each pinhole and the projection of a point-like object on the detector. It is convenient to write the source S in terms of the shape h of each hole and of the pattern Sd.

$$S(x_s) = \int S_\delta(x_s - x) \cdot h(x) dx \tag{14}$$

For a point-like object, using the notation of (6) and (7), (4) becomes

$$R(x_r) \propto S_-^{m_1} * \delta = S_-^{m_1}(x_r), \tag{15}$$

and decodes according to (10) and (11) as

$$\begin{aligned}\hat{O}(x\ ) &\propto \int S_{-\ \delta}^{m_z}(x_S-\xi)\cdot h_{-}^{m_1}(\xi)\cdot G(x+x_S)\cdot dx_S\cdot d\xi = \\ &= \int h_{-}^{m_1}(\xi)\cdot d\xi \int S_{-\ \delta}^{m_z}(x_S-\xi)\cdot G(x+x_S)\cdot dx_S = \\ &= \int h_{-}^{m_1}(\xi)\cdot \delta(\xi+x)d\xi = h_{-}^{m_1}(-x) = h(\frac{a}{b}x)\end{aligned} \tag{16}$$

This is the proof that in CSI as well as in pinhole imaging, the size of the hole is the factor that establishes the resolution of the system. In the present case, assuming a circular hole with diameter dh, the relation between the geometrical parameters and the resolution

$$\lambda_g : a = \frac{a}{b} d_h : (a+b)$$

results in the expression

$$\lambda_g = \frac{a^2}{ab+b^2} d_h. \tag{17}$$

It is convenient to combine (13) and (17), since it is in general possible to change the geometry in order to change the resolution and the Field-of-View.

$$\frac{FoV}{\lambda_g} = \frac{b}{a^2}\frac{a\cdot d_d - b\cdot d_m}{d_h}. \tag{18}$$

III. Coded Sources and Coded Apertures

Coded source image formation is described by (5). The decoding strategy is summarized in (10) and the basic requirement for the design of a source / decoding pattern couple is given in (11). To some readers, these equations will look extremely familiar. Formally, the founding equations of CSI have been written to match the founding equations of Coded Aperture Imaging (CAI) [2].

CAI has a long tradition in astronomy, and more recently has been successfully used in near-field problems such as small animal imaging and medical applications. Being an emission imaging system, CAI relies on the encoding of a source (the object) via a mask (the coded aperture). Compared to CSI, the object and source exchange position.

Coded Apertures have been introduced in emission imaging for the same reason coded sources are being proposed in transmission imaging, i.e. to obtain a SNR advantage to achieve better resolution or shorter exposure. SNR and artifact reduction have been studied for CAI and results exist in literature [3, 4]. Those results can be ported to CSI thanks to the formal equivalence of the two techniques.

A short qualitative discussion of these two topics follows; a more rigorous analysis is under way.

A. SNR Advantage

The previous discussion on SNR pointed out that care should be put in the choice of the source pattern. Not only must it correlate with a suitable decoding pattern to result in a delta function in order to have perfect reconstruction according to (11), but we also need a pattern that ensures a sensitivity advantage of the system compared to the state-of-the-art. The signal-to-noise ratio must be studied in detail before choosing a specific design.

The main purpose of using a coded source is to increase the signal-to-noise ratio (SNR) of the imaging system. SNR calculation depends on the coded source pattern and on the characteristics of object and of the background. Similar calculations performed for CAI [2] suggest that, for objects with an opaque background and slit or pinhole features that have enhanced transmission, many types of coded source have a marked SNR advantage over the pinhole.

B. Artifacts

Every imaging device is affected by non-idealities. For CSI, since it involves sophisticated convolution and deconvolution of images, artifact reduction techniques are especially necessary. A common source of artifacts is the fact that the projection of the source onto the object will be modulated by a $\cos^3(\theta)$ factor. The use of a decoding pattern that is independent of such modulation will generate distortions in the final images. This is not the only source of artifacts that is expected. For example, the designer of a coded source should use care in selecting the thickness of the mask: a thin mask would transmit radiation and generate a high background; a thick one would collimate the radiation at the holes, which would no longer act like ideal pinholes.

The same artifacts arise in CAI imaging, and the same techniques of artifact reduction encountered there can be used in CSI. Specifically, it is possible to reduce artifacts drastically by using a mask / anti-mask technique. The technique involves taking two images of the object, where in the second image the transmitting spots have been exchanged with opaque ones and vice-versa. Patterns that allow the implementation of this technique are easily found. The MURA (Modified Uniform Redundant Array) pattern is orthogonal. A simple rotation of the coded source between the two images is then required.

IV. Conclusion

For the reader acquainted with the technique of Coded Aperture Imaging, Coded Source Imaging is a natural extension of CAI in transmission radiography. CSI provides an advantage in the signal-to-noise ratio (SNR) compared to the state of the art that can be traded off by the designer of the imager to achieve a desired feature. The most obvious such feature is better resolution, applicable, for example, to the detection of micron-size flaws in mechanical parts. It can also be traded for shorter exposures, a feature that is particularly desirable in medical imaging. Finally, a high SNR can be used to improve the spatial coherence of the radiation for phase contrast imaging.

Generally speaking, CSI improves on existing techniques by using an extended source more efficiently. In neutron imaging, where sources are typically few centimeters but are not very bright, it is essential to maximize the use of the available neutron flux.

ACKNOWLEDGMENT

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