Stop Wasting Time and Fuel in Traffic Jams

Berthold K.P. Horn & Liang Wang

"Try honking again."
Two thirds of the oil used around the world goes to power transportation vehicles. (of which half goes to passenger cars and light trucks).

In emerging Asia alone the total number of vehicles is expected to rise from 55 million in 2003 to 420 million in 2030.
The Cost of Traffic Flow Instabilities

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- This wastes 72 liters of fuel,
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- This wastes 11 billion liters of fuel per annum,
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- In the USA alone, the costs of congestion are $121 billion per annum (that is $820 per commuter).
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- also, traffic flow instabilities increases the risk of collisions.
Competing Explanations

Many different models predict traffic flow instabilities:

- Cellular automata;
- Differential equations;
- Feedback control models;
- Fluid flow models;
- Particle tracking models;
- Car-following simulation models;

... 

What is needed is a method for *suppressing* instabilities
Traffic flow instabilities waste energy:

- At high densities traffic flow becomes unstable
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- Kinetic energy wasted every time brakes are used
- Building more roads has high energy cost as well
- “Metering” reduces potential throughput
Source of Instabilities

- At high flow densities, traffic flow becomes unstable
- Travelling waves of velocity and density fluctuations
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- At high flow densities, traffic flow becomes unstable
- Travelling waves of velocity and density fluctuations
- Perturbations are amplified
- Effects propagate upstream
- Instabilities reduce average speed and throughput
- Instabilities limit the carrying capacity of a roadway
- Increase wear and tear on vehicles — and on nerves
- Stop-and-go traffic greatly reduces fuel efficiency
Control of car C depends on \( d_l \) and \( v_l - v_c \)

(*) Human drivers, as well as adaptive cruise control systems
Car-Following Feedback Control

Acceleration depends on $d_l$ and $v_l - v_c$
(and possibly $v_{des}$, $v_c$, and $v_{max}$)
Car-Following System Model

Overall transfer function \((H(s))^n\)
Traffic Jam without Bottleneck

Experimental evidence for the physical mechanism of forming a jam

Yuki Sugiyama, Minoru Fukui, Macoto Kikuchi, Katsuya Hasebe, Akihiro Nakayama, Katsuhiro Nishinari, Shin-ichi Tadaki and Satoshi Yukawa

Movie 1

The Mathematical Society of Traffic Flow
Need more than “adaptive cruise control”

- Many explanations for how flow instabilities arise
- For example: Simple car-following model
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- But, few ideas on what to do about it
- “Adaptive cruise control” does not solve the problem
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- Solution is to use bilateral information flow
- Cheap machine vision systems support bilateral control
Alternative Schemes

- Building more roads reduces density — for a while;
- “Metering” reduces instabilities by limiting density;
- Reduction in reaction time allows higher density;
- Platooning allows small inter-vehicle distances;
- ... not
Driver-Friendly Bilateral Control —
Suppressing Traffic Flow Instabilities

Simulation of the bilateral control model (space / meter)

Simulation of the car-following model (space / meter)

Toyota-CSAIL Project
Berthold K.P. Horn & Liang Wang
Smooth Flow Analysis

In the absence of instabilities:

- Safe separation — speed $\times$ reaction time: $d = vT$
- Density — inverse of length plus separation: $\rho = 1/(l + d)$
- Throughput — speed $\times$ density: $c = v\rho = v/(l + vT)$
- Approaches inverse of reaction time: $c \to 1/T$
- E.g. $T = 1$ sec — $c$ approaches 3600 vehicles per hour

- In practice, throughput is considerably lower — because flow is not smooth
Bilateral Control

Control of car C depends on \( d_l \) and \( v_l - v_c \)

and on \( d_f \) and \( v_c - v_f \)
(a) acceleration depends on $d_l$, $v_l - v_c$,

(b) acceleration depends on $d_l$, $v_l - v_c$,
   — as well as $d_f$ and $v_c - v_f$
Illustrative Bilateral Control System
System Model Comparison

... H(s) H(s) H(s) ...

... + + + + ...

... H_1(s) H_1(s) H_1(s) ...

... + + + + ...

... H_2(s) H_2(s) H_2(s) ...
Car Following Control
Bilateral Control
Physical Model of Bilateral Control

Chain of spring-damper-mass systems
Disturbances spread *and* decay
(Car following system cannot be modelled this way)
What’s New?

- Improved “safe headway” calculation ($\delta v$ dependence)
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\[
\frac{\partial^2 x}{\partial t^2} = k_d \frac{\partial^2 x}{\partial n^2} + k_v \frac{\partial^2}{\partial n^2} \frac{\partial x}{\partial t}
\]
Sensors

- Need sensors for distance and (relative) velocity
- Alternatives: radar, lidar, sonar, and machine vision
- Imaging chips are low cost — as is on-board processing
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- Distance: binocular stereo, trinocular stereo, ...
- Velocity: motion vision methods ...
- Distance/Velocity: time to contact (TTC)
Time To Contact
Problems to solve

- What is the business model?
- What sensors and algorithms? TTC + trinocular stereo?
- Full automation, “modulation” or merely advisory?
- Extend to mix of automated and legacy vehicles
- Extend to multiple lanes, exits and entrances etc.
- Explore use of inter-vehicle communication for sensing
- “Optimize” the control scheme
How to Spread the News?

- Physical Models
  
  e.g. magnets in a tube
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- Small scale implementations
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  - e.g. The Mathematical Society of Traffic Flow
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SELF-DRIVING CAR

Google

SELF-CONSCIOUS CAR

Google

would I look better in red?

Google

SELF-ACTUALIZED CAR

Google

I'm going back to school!
Time To Contact (time lapse sequence)
Time To Contact (real world sequence)
Car Following Control
Car following control (1 minute)
then bilateral control (1 minute)
Fig. 12. Unstable traffic flow using car following model.

Fig. 13. Stable traffic flow using bilateral control.
(1) CAR FOLLOWING:

Closeup of track:

- With “car following” control, disturbances move upstream (to the left) only, and increase in amplitude as they go.
- The disturbance near the initial cause dies down, but the wave travelling upstream does not.
- (In car following, acceleration of each vehicle depends on distance and relative velocity of the leading vehicle.)

(2) BILATERAL CONTROL:

Closeup of track:

- With “bilateral” control, disturbances travel in both directions and decrease in amplitude.
- The system soon returns to smooth flow.
- (In bilateral control, acceleration of each vehicle depends on distance and relative velocity of leading and following vehicle.)

For additional details:

- search for Suppressing Traffic Flow Instabilities
- or search for bkph traffic
Block Diagram of Bilateral Control System

1. Leading distance and speed sensors
2. Following distance and speed sensors
3. Vehicle controller system
4. Forward motion control of vehicle
5. Current speed sensor and driver input
Driver-Friendly Bilateral Control for Suppressing Traffic Flow Instabilities

Berthold K. P. Horn
Liang Wang
What is Bilateral Control?

• For human drivers:
  Input: Leading car
  Output: force
• For self-driving car
  Input: Neighboring cars
  Output: force
• More information
• Smart control strategy

(a) The bilateral control model
(b) The physical analog of bilateral control
Why does Bilateral Control work?

• Mathematically, we get the following ODE systems:

\[
\begin{align*}
\text{(CFM)} \quad \frac{d}{dt} Y &= AY \quad \text{and} \quad \text{(BCM)} \quad \frac{d}{dt} Y &= BY.
\end{align*}
\]

\(A\) and \(B\) are both block-constant-diagonal (or block-Toeplitz) matrices:

\[
A = \begin{pmatrix}
\ddots & M & M \\
N & M & M \\
N & N & M \\
\ddots & \ddots & \ddots
\end{pmatrix}, \quad B = \begin{pmatrix}
\ddots & \ddots & \ddots \\
M & N/2 & N/2 \\
N/2 & M & N/2 \\
\ddots & \ddots & \ddots
\end{pmatrix}
\]

\[
N = \begin{pmatrix}
0 & 0 \\
k_d & k_v
\end{pmatrix}, \quad M = \begin{pmatrix}
0 & 1 \\
-k_d & -k_v
\end{pmatrix}
\]

TOYOTA RESEARCH INSTITUTE

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CSAIL
Why does Bilateral Control work?

• The solutions are:
  
  \[(\text{CFM}) \quad Y(t) = e^{tA}Y(0)\]
  \[(\text{BCM}) \quad Y(t) = e^{tB}Y(0)\]

• The stability depends on the eigenvalues of \(A\) and \(B\).

• By tedious calculation, we find:
  
  Some eigenvalues of \(A\) have **positive** real part
  All eigenvalues of \(B\) have **non-positive** real part

• Bilateral control suppresses perturbations.
Why does Bilateral Control work?

• By car-following model

• By bilateral control model
Plans for the next 12 months

- Multi-node bilateral control:

- More information can be obtained through wireless communication.
- How do we best use this information?
- For example, what weights should be given to additional measurements?
Plans for the next 12 months

• Mathematically,

\[ a_n = k_d \left( \sum_{m=-k}^{k} g_m x_{n-m} \right) + k_v \left( \sum_{m=-k}^{k} h_m v_{n-m} \right) \]

• What should the coefficients \( \{g_m\} \) and \( \{h_m\} \) be?

• For the original bilateral control model (i.e. \( k = 1 \)), we chose:

\[ \{g_m\} = \{h_m\} = \{1, -2, 1\} \]

• But what is best when \( k = 2 \)? Or \( k = 3 \)?

• In the case of \( k = 2 \), there are 5 coefficients to be figured out.
Plans for the next 12 months

• First, note that the 3 weights \( \{g_m\} = \{h_m\} = \{1, -2, 1\} \) for the simplest bilateral control are exactly the coefficients of the polynomial:

\[
(x - 1)^2 = 1x^2 - 2x + 1
\]

• Thus, an intuitive way of generating the 5 weights might be:

\[
(x - 1)^4 = 1x^4 - 4x^3 + 6x^2 - 4x + 1
\]

• However, this approach doesn’t work. The traffic flow under such weights \( \{1, -4, 6, -4, 1\} \) goes to the equilibrium state much more slowly than the weights \( \{0, 1, -2, 1, 0\} \) (i.e. simplest bilateral control).

• We will continue studying this problem.
Plans for the next 12 months

• Collision-free condition for the simplest bilateral control:

• Note that “stability” does *not* imply that there is a guarantee that vehicle trajectories will not cross (i.e. that there will be no collisions)

• It is possible — during the decay of propagating waves caused by large enough perturbations — that vehicle trajectories may cross.
Plans for the next 12 months

• Under which initial condition, will there be no collisions?
• We can analyze this problem from the viewpoint of energy.
• We will continue studying this problem.
Plans for the next 12 months

• Improve the bilateral control model to prevent car-collisions.
• In the simplest bilateral control model, i.e.,
  \[ a_n = k_d(x_{n-1} - 2x_n + x_{n+1}) + k_v(v_{n-1} - 2v_n + v_{n+1}) \]
• The two feedback gains are both constants.
• This can be generalized to two functions, e.g.
  \[ k_d(x_{n-1}, x_n, x_{n+1}) \quad \text{and} \quad k_v(v_{n-1}, v_n, v_{n+1}) \]
• That is, non-linear springs and dampers.
Plans for the next 12 months

• Then, we have to face the following questions:
  How do we design $k_d(x_{n-1}, x_n, x_{n+1})$ and $k_v(v_{n-1}, v_n, v_{n+1})$?  
  Will the new design be effective in preventing car collisions?  
  Will the traffic flow under the new bilateral control model be stable?  
• We will study this interesting problem of great practical concern.  
• We also want to build a real demonstration of both car following model and bilateral control model (perhaps with help from the Duckietown team in MIT, or researchers at Univ. of Nagoya)
What’s Next?

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(5) Explore non-linear version with state dependent gain Can safety be improved at the cost of driver comfort?
And after that?

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   Implement plausible lane changing behavior
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(10) Further explore the use of inter-vehicle communication (Extend results in Baran & Horn paper)
And then?

(11) Study performance on large scale traffic models (such as PATH at USB, and ITTR at UCSD)
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- Collision-free initial conditions — “energy” based.
Driver-Friendly Bilateral Control

Toyota-CSAIL Project
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