# A Cellular Automata Simulation of Two-Phase Flow on the CM-2 Connection Machine Computer

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## Cellular Automata

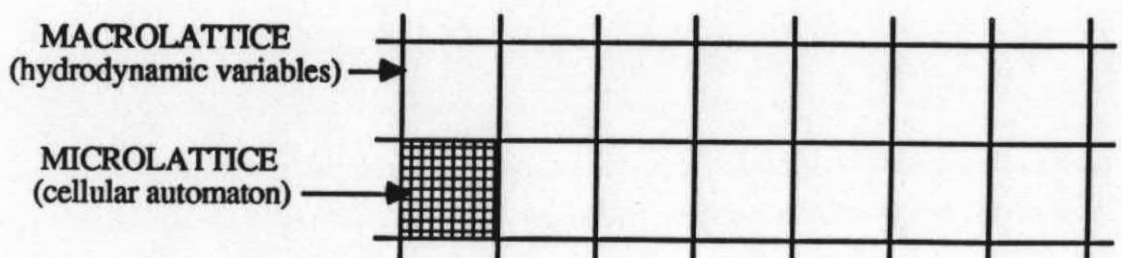
- Dynamical Systems

- Discretized in
  - space
  - time
  - dependent variable

- Deterministic Update Rule

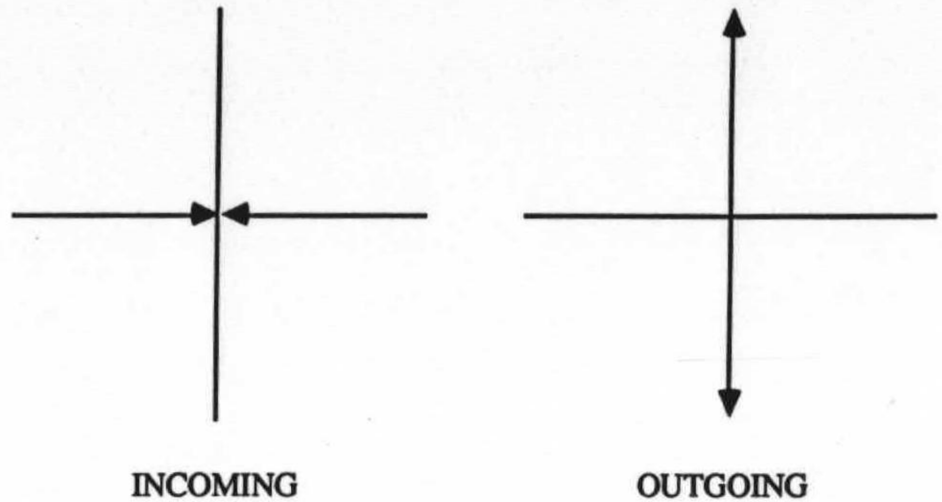
# Lattice Gases

- Cellular Automata
- Regular Lattice
- "Advect-Collide" type rule
- Conserve Particles and Momentum
- Coarse-Grain Averaged behavior is that of an incompressible Navier-Stokes fluid
- Microscopic (CA)
versus
Macroscopic (hydrodynamic variables)



## HPP Gas

- Square Lattice
- Collision Rules:

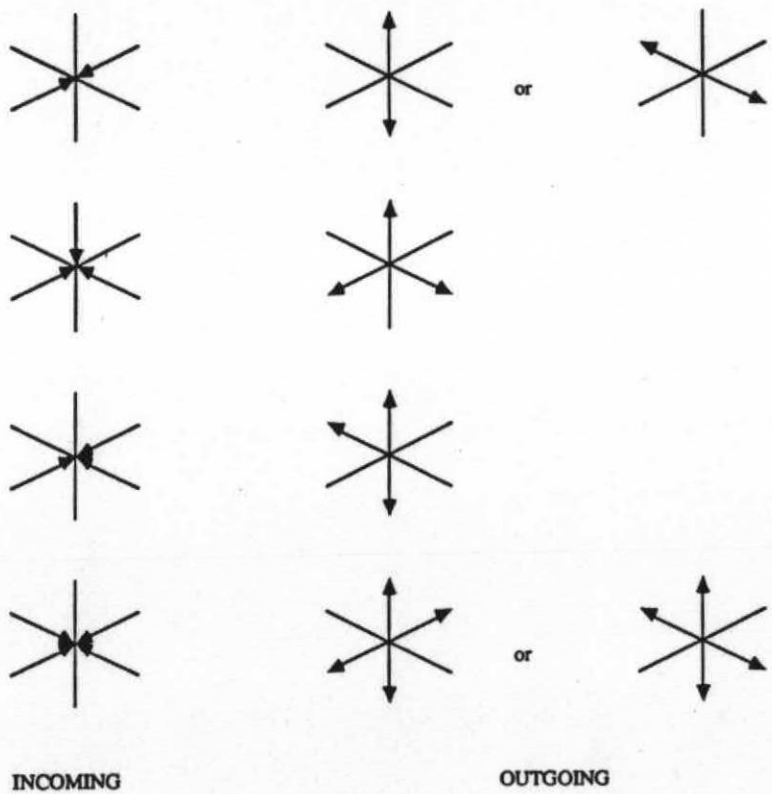



- Suffers from lack of macroscopic isotropy
- Reference:

1) *Hardy, J., de Pazzis, O., Pomeau, Y., Phys. Rev. A,* *__13__, 5 (1976)*

## FHP Gas I

- Triangular Lattice
- Collision Rules:



## FHP Gas II

- Extensive testing has verified macroscopic isotropy, and quantitative agreement with 2-D incompressible Navier-Stokes equations

•References:

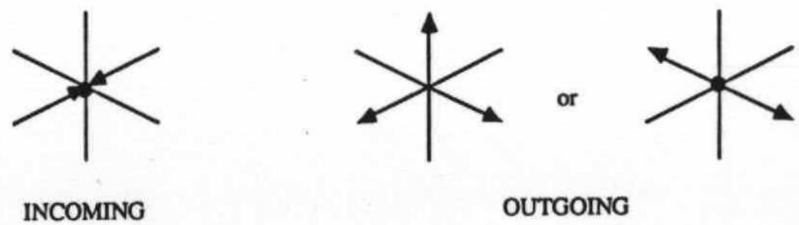

1) *Frisch, U. Hasslacher, B., Pomeau, Y. Phys. Rev. Lett., 56 (1986)*

2) *Kadanoff, L.P., McNamara, G.R., Zanetti, G., Complex Systems, 1 (1987)*

## Rothman-Keller Model for Two-Phase Flow I

Goal: To simulate the flow of two incompressible, immiscible, viscous fluids in two dimensions

- Triangular Lattice (like FHP)
- Rest Particle



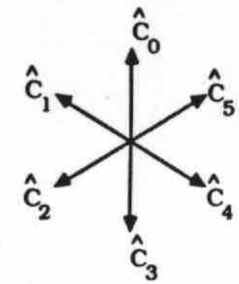

- Two Colors (red and blue)
- Collisions Conserve
  - number of red particles
  - number of blue particles
  - total momentum

## Rothman-Keller Model for Two-Phase Flow II

• Choose from among different possible outcomes to preferentially send particles toward particles of like color

• Lattice Vectors

$$\mathbf{c}_j \equiv -\hat{\mathbf{x}} \sin\left(\frac{2\pi j}{6}\right) + \hat{\mathbf{y}} \cos\left(\frac{2\pi j}{6}\right),$$




• Color Flux

$$\mathbf{q}[r(\mathbf{x}), b(\mathbf{x})] \equiv \sum_i \mathbf{c}_i [r_i(\mathbf{x}) - b_i(\mathbf{x})],$$

• Color Field

$$\mathbf{f}(\mathbf{x}) \equiv \sum_i \mathbf{c}_i \sum_j [r_j(\mathbf{x} + \mathbf{c}_i) - b_j(\mathbf{x} + \mathbf{c}_i)].$$

• Work...

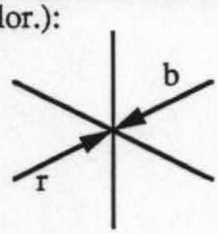
$$W(r, b) = -\mathbf{f} \cdot \mathbf{q}(r, b).$$

• ...must be minimized

$$W(r', b') = \min_{r'', b''} W(r'', b'')$$

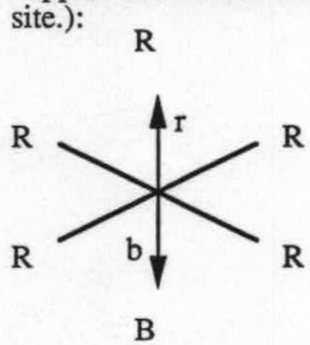
# Rothman-Keller Model for Two-Phase Flow III

INCOMING
(Lower case letters denote particle color.):

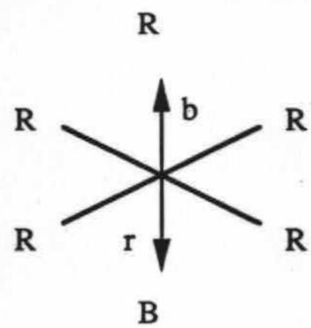



| Outcome | Color Flux | Work |
|---|---|---|
| I | $2\hat{y}$ | $-2$ |
| II | $-2\hat{y}$ | 2 |
| III | $-\sqrt{3}\hat{x} + \hat{y}$ | $-1$ |
| IV | $\sqrt{3}\hat{x} - \hat{y}$ | 1 |
| V | $\sqrt{3}\hat{x} + \hat{y}$ | $-1$ |
| VI | $-\sqrt{3}\hat{x} - \hat{y}$ | 1 |

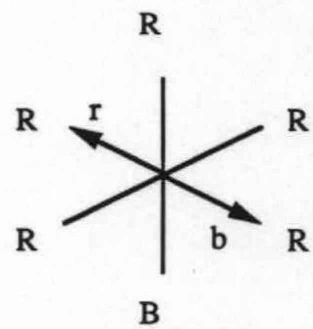
OUTGOING:
(Lower case letters denote particle color.
Upper case letters denote predominant color of surrounding site.):



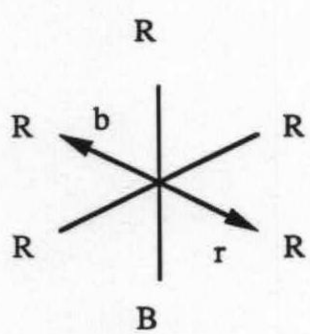

I



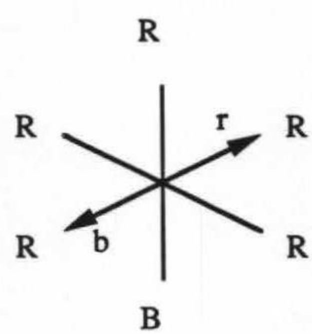

II



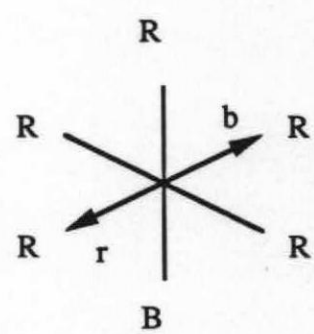

III




IV




V




VI

## Rothman-Keller Model for Two-Phase Flow IV

- Proof that model correctly simulates Navier-Stokes flow with interfacial surface tension:

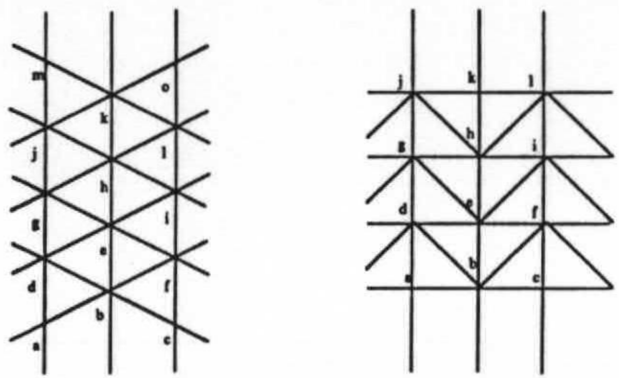
  - Reduces to FHP in regions of homogeneity

  - Separation does occur

  - Pressure differential between inside and outside of circular bubble of radius R is $\Delta P \propto R^{-1}$

# <u>Description of Algorithm I</u>

- Mapping triangular grid to Cartesian grid:



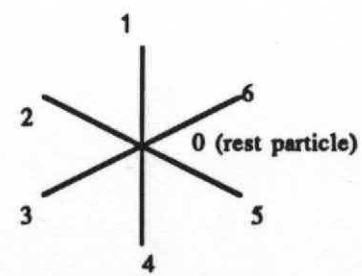

- Column parity bit needed

## Description of Algrithm II

- Encode configuration into 14 bit string:

| STATE | BIT PAIR |
|---|---|
| Empty<br>Red<br>Blue | 00<br>01<br>10 |

DIRECTIONS



## Description of Algorithm III

- Every site uses indirect lookup to get color density
  Table is $2^{14}$ entries long
  Output is 4 bits

- Every site gets color density from its six neighbors, and uses unsigned addition and subtraction to get $f_x$ and $f_y$ (a total of 13 bits of information).

## Description of Algorithm IV

- Dynamics are invariant under rotation by $\pi/6$

- Rotate by $l\pi/6$ so that color field $\underline{f}$ points at angle $0 \leq \theta_n < \pi/6$, get result for this "normal form," and rotate back by $-l\pi/6$

- This reduces size of resultant lookup table by a factor of 6

- Get $l$ and $\theta_n$ by lookup tables

  Tables are $2^{13}$ entries long

  Outputs are 3 bits each

- Barrel shift last 12 bits of 14 bit configuration to "normal form configuration"

## Description of Algorithm V

- Lookup normal form equivalence class

  Table is $2^{14}$ entries long

  Output is 9 bit equivalence class of result

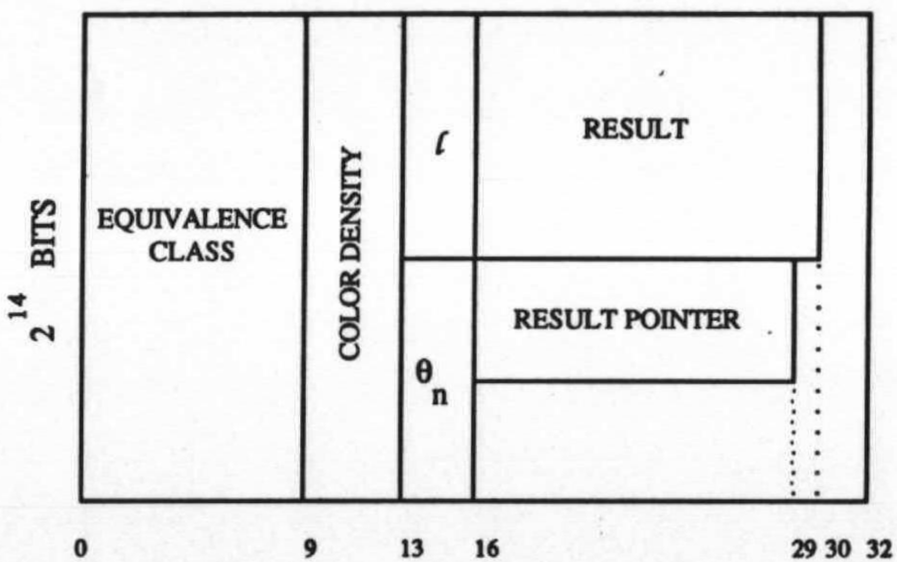
- Lookup normal form result

  Table is $2^{12}$ entries long

  Output is 14 bit normal form configuration

- Barrel shift last 12 bits back to get final result

## Storage of Lookup Tables



## Performance of CM-2

- One fourth of memory is used for lookup tables

- Remaining memory can accomodate ~$9.5 \times 10^7$ sites

- $10^8$ site updates per second

## Conclusions

- Rothman-Keller model for two incompressible, immiscible, viscous fluids

- Implementation on data-parallel CM-2 computer using six indirect lookup and numerous logical operations

- Future work

  [redacted]

  - Two-phase flow in porous media
  - Saffman-Taylor fingering
  - Kelvin-Helmholtz instability

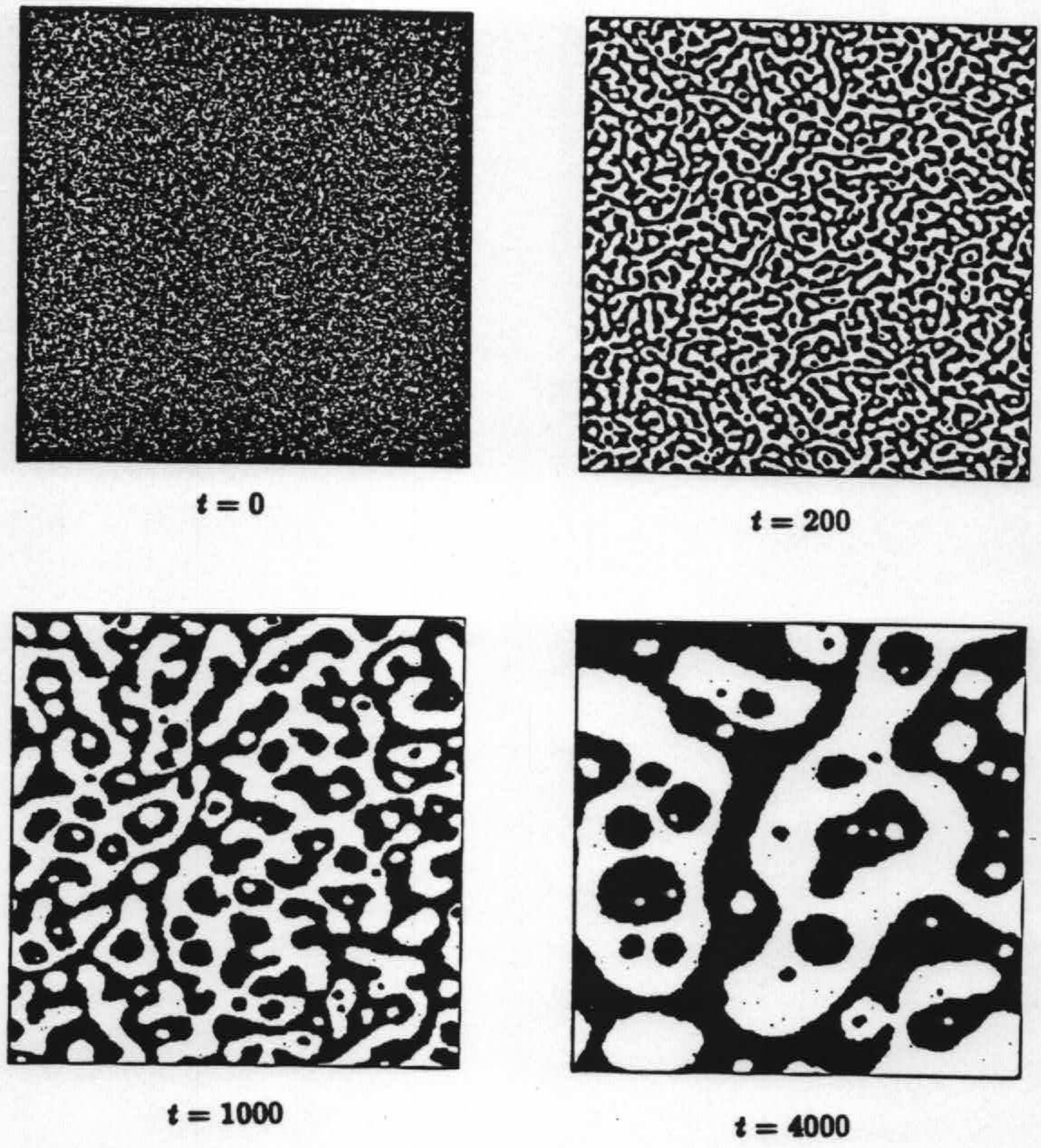
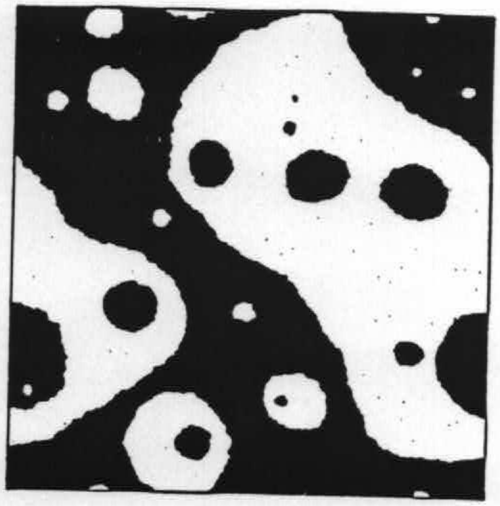
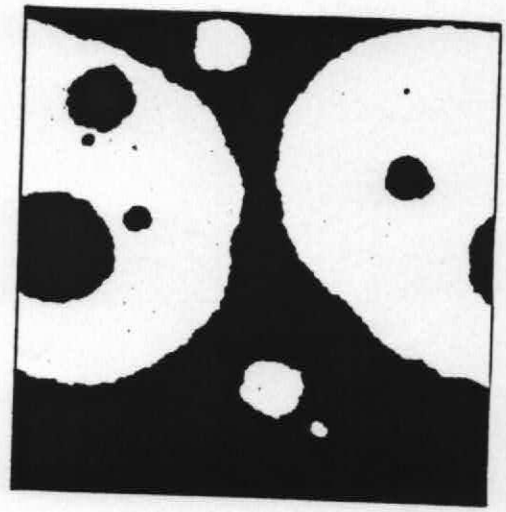


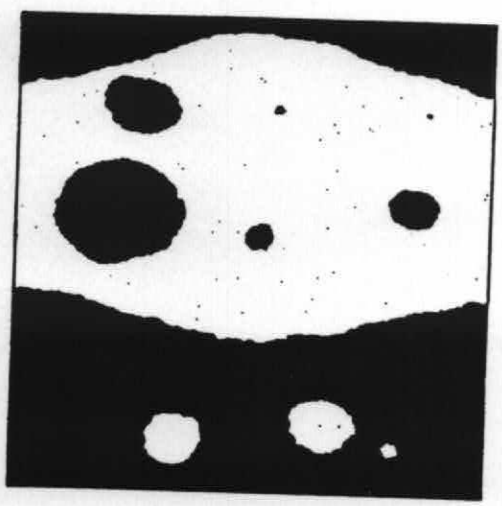

**Figure 9: Simulation Results**



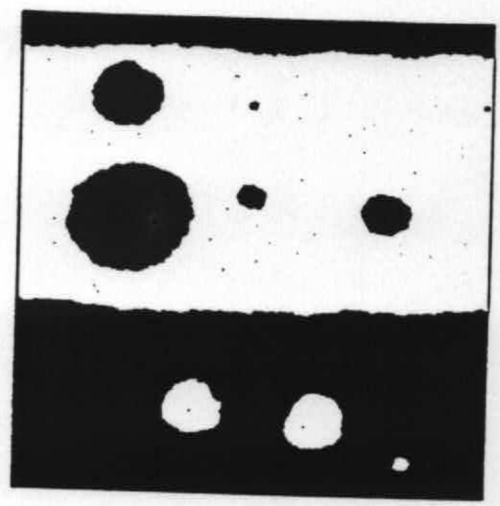
$t = 20000$



$t = 60000$



$t = 80000$



$t = 100000$

**Figure 10: Simulation Results**