Problem 1

(a) 
\[(\sqrt{3} + j)^5 e^{-j \frac{\pi}{2}} = (2e^{j \frac{\pi}{2}})^5 e^{-j \frac{\pi}{2}} = 2e^{j \frac{5\pi}{2}} e^{-j \frac{\pi}{2}} = 2e^{j \frac{\pi}{2}}.\]

See Figure 3

Problem 2

(a) 
\(x(1 - \frac{t}{3})\): Shift left 1, flip, expand by 3.

See Figure 4

(b) 
\(x(t - 2) [\delta(t - \frac{1}{2}) + u(3 - t)]\)
\(x(t - 2)\): Shift right 2
\(\delta(t - \frac{1}{2})\): Shift right \(\frac{1}{2}\)
u(3 - t): Shift left 3, flip

See Figure 5

Problem 3

(a) 
x[2 - n]: Shift left 2, flip.

See Figure 6

(b) 
x[2n + 1]: Shift left 1, compress by 2.

See Figure 7
Problem 4

Even part:
See Figure 8
Odd part:
See Figure 9

Problem 5

(a)
\[ x(t) = (\sin(4t - 1))^2 = (x_1(t))^2 \]
We see that \( x_1(t) \) oscillates equally above and below the x-axis. Since we square \( x_1(t) \), we flip the negative values above the x-axis. So the period will be cut in half. Now, calculate the period of \( x_1(t) \).
\[ x_1(t + T) = \sin(4(t + T) - 1) = \sin(4t - 1 + 4T) \]
For this to be periodic, we need:
\[ 4T = 2\pi k \]
for \( k = 0, \pm 1, \pm 2, \ldots \). Using our above analysis that the period will be cut in half, \( T_0 = \frac{\pi}{2} = \frac{\pi}{4} \).

(b)
Let us use the complex exponential:
\[ x_1[n] = e^{j(4n + \frac{\pi}{4})} \]
So,
\[ x_1[n + N] = e^{j(4(n+N) + \frac{\pi}{4})} = e^{j4N}e^{j(4n + \frac{\pi}{4})} \]
This is periodic if:
\[ e^{j4N} = 1 \iff 4N = 2\pi k \iff N = \frac{\pi}{2} k \]
for \( k = 1, \pm 1, \pm 2, \ldots \). However, notice that \( N \) can never be an integer. So, \( x[n] \) cannot be periodic.

(c)
Let us use complex exponentials:
\[ x_1[n] = (-1)^n e^{j\frac{2\pi n}{7}} = (-1)^n x_2[n] \]
Let us first look at \( x_2[n] \):
\[ x_2[n + N] = e^{j\frac{2\pi(n+N)}{7}} = e^{j\frac{2\pi N}{7}}e^{j\frac{2\pi n}{7}} \]
This is periodic if:
\[ e^{j\frac{2\pi N}{7}} = 1 \iff \frac{2\pi N}{7} = 2\pi k \iff N = 7k \]
for \( k = 1, \pm 1, \pm 2, \ldots \). We can easily observe that \((-1)^n\) has period of 2. So, \( N_0 = 14 \).

Problem 6

(a)
1. Memoryless? No—\( y \) depends on future and past values of \( x \).
2. TI? No
\[ x_2(t) = x_1(t - t_0) \]
\[ y_2(t) = x_2(t + 3) - x_2(1 - t) = x_1(t + 3 - t_0) - x_1(1 - t - t_0) \]
\[ y_1(t - t_0) = x_1(t - t_0 + 3) - x_1(1 - (t - t_0)) \]

3. Linear? Yes
\[ x_3(t) = ax_1(t) + bx_2(t) \]
\[ y_3(t) = x_3(t + 3) - x_3(1 - t) = ax_1(t + 3) + bx_2(t + 3) - ax_1(1 - t) - bx_2(1 - t) = a(x_1(t + 3) - x_1(1 - t)) + b(x_2(t + 3) - x_2(1 - t)) = ay_1(t) + by_2(t) \]

4. Causal? No. For \( t = -1 \), \( y(-1) \) depends on \( x(2) \).

5. Stable? Yes. Suppose \( |x(t)| \leq B \ \forall t \). Then \( |y(t)| \leq 2B \).

(b)

1. Memoryless? Yes, since \( y \) depends only on current values of \( x \).

2. TI? No. Let:
\[ x_1[n] = \delta[n] \rightarrow y_1[n] = \delta[n] \]
\[ x_2[n] = \delta[n + 1] \rightarrow y_2[n] = -\delta[n + 1] \]

3. Linear? No. Let:
\[ x_1[n] = \delta[n] \]
\[ x_2[n] = (-1)x_1[n] = -\delta[n] \rightarrow y_2[n] = -2\delta[n] \neq (-1)y_1[n] = -\delta[n] \]

4. Causal? Yes since it is memoryless.

5. Stable? Yes. If \( |x[n]| \leq B, |y[n]| \leq 2B \).

(e)

1. Memoryless? No since \( y[n] \) depends on future values of \( x[n] \).

2. TI? Yes. \( x_2[n] = x_1[n - n_0] \rightarrow y_2[n] = \sum_{k=n}^{\infty} x_2[k] = \sum_{k=n}^{\infty} x_1[k - n_0] = \sum_{k=n-n_0}^{\infty} x_1[k] \)
\[ y_1[n - n_0] = \sum_{k=n-n_0}^{\infty} x_1[k]. \]

3. Linear? Yes. \( x_3[n] = ax_1[n] + bx_2[n] \rightarrow y_3[n] = \sum_{k=n}^{\infty} x_3[k] = \sum_{k=n}^{\infty} ax_1[k] + bx_2[k] = a\sum_{k=n}^{\infty} x_1[k] + b\sum_{k=n}^{\infty} x_2[k] = ay_1[n] + by_2[n]. \)

4. Causal? No, see memoryless answer.

5. Stable? No, see memoryless answer.

Problem 7
\[ x_2(t) = -x_1(t - 1) - 2x_1(t - 2) \]
\[ y_2(t) = -y_1(t - 1) - 2y_1(t - 2) \]
See Figure 10
Problem 8

% (a)
xn = -3:7;
x = [0 0 0 2 0 1 -1 3 0 0 0];
figure(1);
stem(nx,x);
title('x');

% (b)
y1 = x;
y2 = x;
y3 = x;
y4 = x;
ny1 = -1:9;
ny2 = -4:6;
ny3 = 3:-1:-7;
ny4 = 4:-1:-6;

% (c)
% y1 - Delayed by 2.
% y2 - Advanced by 1.
% y3 - Flipped.
% y4 - Flipped and then delayed by 1.
figure(2);
subplot(2,2,1);
stem(ny1,y1);
title('y1');
subplot(2,2,2);
stem(ny2,y2);
title('y2');
subplot(2,2,3);
stem(ny3,y3);
title('y3');
subplot(2,2,4);
stem(ny4,y4);
title('y4');
Figure 1: Figure for part 8(a).
Figure 2: Figure for part 8(c).