

# Learning Collective Crowd Behaviors with Dynamic Pedestrian-Agents

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**Abstract** Collective behaviors characterize the intrinsic dynamics of the crowds. Automatically understanding collective crowd behaviors has important applications to video surveillance, traffic management and crowd control, while it is closely related to science fields such as statistical physics and biology. In this paper, a new Mixture model of Dynamic pedestrian-Agents (MDA) is proposed to learn the collective behavior patterns of pedestrians in crowded scenes from video sequences. From agent-based modeling, each pedestrian in the crowd is driven by a dynamic pedestrian-agent, which is a linear dynamic system with initial and termination states reflecting the pedestrian's belief of the starting point and the destination. The whole crowd is modeled as a mixture of dynamic pedestrian-agents. Once learned from the trajectories extracted from videos, MDA can simulate the crowd behaviors. It can also infer the past behaviors and predict the future behaviors of pedestrians given their partially observed trajectories, and classify them different pedestrian behaviors. The effectiveness of MDA and its applications are demonstrated by qualitative and quantitative experiments on various video surveillance sequences.

**Keywords** Crowd behavior analysis · Video surveillance · Motion analysis

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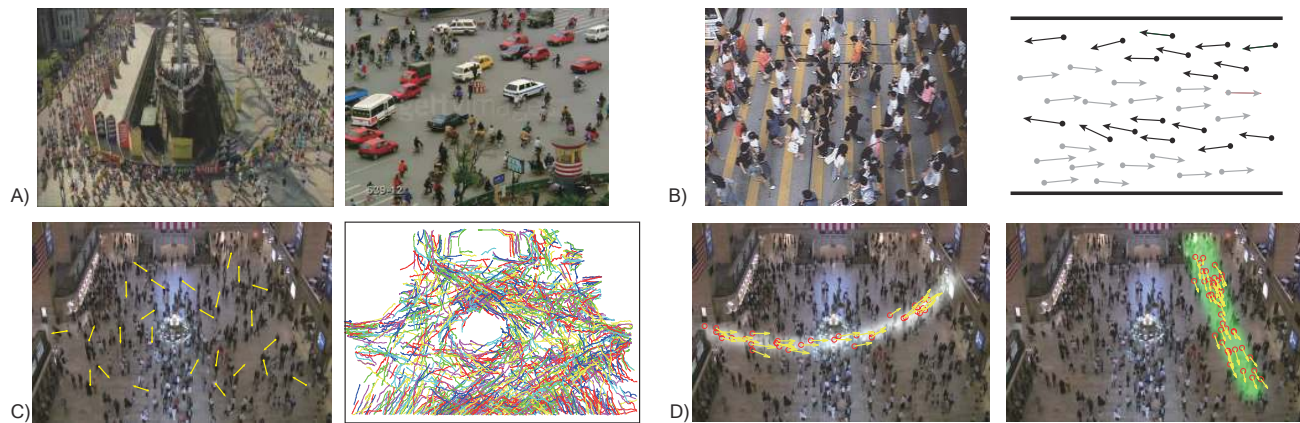
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## 1 Introduction

Automatically understanding the behaviors of pedestrians in crowd from video sequences is of great interest to the computer vision community, and has drawn more and more attentions in recent years (Zhou et al, 2010). It has important applications to event recognition (Hospedales et al, 2011), traffic flow estimation (Wang et al, 2008b), behavior prediction (Antonini et al, 2006), abnormality detection (Mehran et al, 2009), and crowd simulation (Treuille et al, 2006). For example, in video surveillance, many places of security interest, such as shopping malls, train stations, and street intersections, are very crowded. Automatically detecting dangerous and abnormal behaviors in such environments plays an important role to ensure public safety. However, conventional video surveillance systems do not work well in crowded environments. In crowd control and traffic management, recognizing traffic patterns and estimating traffic flows provide valuable information to avoid congestion and to prevent potential crowd disasters (Moussaid et al, 2011). In civil engineering, long-term statistical information from crowd behavior analysis provides guidelines for planning and designing crowded public areas to increase safety and to optimize traffic capacity. One of the underlying challenges of these problems is to model and analyze the collective dynamics of pedestrians in crowd. The collective behaviors of crowds show striking analogies with some self-organization phenomena observed in social processes, such as opinion formation (Helbing, 1994), and science fields such as physics and biology (Moussaid et al, 2009). Automatic crowd behavior analysis provides powerful tools for studying these related problems and leads to deep insight in these interdisciplinary fields.

Crowd behaviors have been studied in social science with a long history. French sociologist Le Bon (1841~1931) described collective crowd behaviors in his book *The Crowd*:



**Fig. 1** A) Marathon race at the street corner and traffic flow at the street intersection. B) Bidirectional traffic of pedestrians crossing the street. Collective behaviors of forming lanes spontaneously emerge from the bidirectional traffic of pedestrians. Black and gray arrows represent pedestrians walking in two opposite directions. C) Crowd of pedestrians walking in a train station scene and extracted trajectories of pedestrians. Since there are several entry and exit regions, the collective behaviors of the crowd in this scene become complicated. Pedestrians have different beliefs of the starting points and the destinations. These beliefs and other scene structures influence pedestrian behaviors. The shared beliefs and dynamics of movements also generate several major collective dynamic patterns. Yellow arrows indicate the moving directions of some exemplar pedestrians. Trajectories are highly fragmented because of frequent occlusions. D) display two collective dynamic patterns of the crowd learned with MDA from these fragmented trajectories. The colored densities indicate the spatial distributions of the collective behaviors. Some pedestrians are simulated from MDA. Red circles and yellow arrows represent the current positions of simulated pedestrians and their velocities.

A Study of the Popular Mind as, “*the crowd, an agglomeration of people, presents new characteristics very different from those of the individuals composing it, the sentiments and ideas of all the persons in the gathering take one and the same direction, and their conscious personality vanishes.*”

It leads to the motivation of this work: crowd has its intrinsic collective dynamics. Although individuals in crowd might not acquaint with each other, their shared movements and destinations make them coordinate collectively and follow the paths commonly taken by others.

Collective crowd behaviors are driven by both external and self organization. In Figure 1A, the collective behaviors of crowds are regularized by scene structures, such as athletic tracks, lane markers, and cross walks, and are controlled by traffic signals. Differently, in Figure 1B, pedestrians are self-organized into several lanes spontaneously. These collective behaviors emerge without external or centralized control (Moussaid et al, 2009). As the scene structure becomes complicated, there will be a variety of collective crowd behaviors happening at the same time. As shown in Figure 1E-H, since the train station has multiple entrances and exits, pedestrians have various destinations to reach, and the crowd forms multiple collective crowd behaviors with different dynamics and moving directions. The goal of this work is to statistically model and learn the collective dynamics of crowd from its observations. This is a fundamental problem for understanding the collective crowd behaviors. It is quite challenging since detecting and tracking pedestrians fails frequently in crowded environments. Crowd behaviors involve a large number of objects, which increase the complexity of this problem.

In this paper, a new Mixture model of Dynamic pedestrian-Agents (MDA) is proposed to learn the collective dynamics of pedestrians from a large amount of observations without supervision. MDA is an agent-based model (Bonabeau, 2002), which treats pedestrians as agents and models their process of deciding next actions based on current states. Therefore, it is suitable for simulating crowd behaviors once learned from real videos. Observations are trajectories of feature points on pedestrians obtained by a KLT tracker (Tomasi and Kanade, 1991). Because of frequent occlusions in crowded scenes, there are many tracking failures, and most trajectories are highly fragmented with large portions of missing observations. The movement of a pedestrian is driven by one of dynamic pedestrian-agents, which are modeled as linear dynamic systems with initial and termination states reflecting pedestrians’ beliefs of the starting points and the destinations. The timings of pedestrians entering the scene with different dynamic patterns are modeled as Poisson processes. Each dynamic pedestrian-agent represents one type of collective crowd behaviors. The collective dynamics of the whole crowd are modeled as a mixture of dynamic pedestrian-agents. The effectiveness of MDA is demonstrated by multiple applications: simulating collective crowd behaviors, detecting semantic regions, estimating transition probabilities of traffic flows between entrance and exit regions, classifying collective behaviors, detecting abnormal behaviors, and predicting pedestrian behaviors<sup>1</sup>.

The novelty and contributions of this work are summarized as follows. 1) Although there exist approaches (Hospedales

<sup>1</sup> Datasets, demo videos and related materials are available from <http://mmlab.ie.cuhk.edu.hk/project/dynamicagent/>

et al, 2009; Wang et al, 2008b; Lin et al, 2009; Zhou et al, 2011) of learning motion patterns in crowded scenes, they did not explicitly model the dynamics of pedestrians. Many of them only took local location-velocity pairs as input, while discarding the temporal order of trajectories, which is important for both classification and simulation. Instead, MDA takes trajectories as input, and models the temporal generative process of trajectories. Therefore, it is much more natural for MDA to simulate collective crowd behaviors and to predict pedestrians' future behaviors, once its parameters are learned from real data. 2) Under MDA, pedestrians' beliefs, which strongly regularize their behaviors, are explicitly modeled and inferred from observations. In order to be robust to tracking failures, the states of missing observations on trajectories are modeled and inferred. Because of these two facts, MDA can well infer the past behaviors and predict the future behaviors of pedestrians given their partially observed trajectories. It also leads to better accuracy of recognizing the behaviors of pedestrians. 3) MDA is the first agent-based model to learn the global collective dynamics of crowd from videos. Based on the conference version of this work (Zhou et al, 2012b), more technical details on the model derivation, applications and experimental evaluations on more crowded scenes, and limitations of our model are provided in this paper. The effectiveness and limitation of our approach are evaluated on three video sequences from different scenes: Grand Central Train Station Scene (Zhou et al, 2012b), MIT Traffic Scene (Wang et al, 2008b), and Marathon Race Scene (Ali and Shah, 2007).

## 2 Related Works

### 2.1 Crowd Behavior Analysis in Other Fields

Crowd behavior analysis is an interdisciplinary subject. Understanding the collective behaviors of crowd is a fundamental problem in social science. Social psychology research (Le Bon, 1897; Forsyth, 2009) shows that when an individual stays in crowd, he behaves differently than being alone. Other individuals in the crowd and the environment have a huge influence on his cognition and action. In biology, the collective behaviors of organisms such as fish school, flocking birds, and swarming ants have long attracted attentions over decades. People study the mechanism underlying the collective organization of individuals (Couzin, 2009), the evolutionary origin of animal aggregation (Parrish and Edelstein-Keshet, 1999) and the collective information processing in crowds (Moussaid et al, 2009) from both macroscopic and microscopic levels. Some important research topics such as self-organization, emergence, and phase transition in statistical physics have close relations with crowd behavior analysis. They study the physical laws governing

the ways in which objects behave and organize themselves (Ball, 2004).

In computer graphics, a number of models are proposed for crowd simulation. A compact survey could be found in (Zhou et al, 2010). Some simulation models come from the statistical fluid mechanics. For example, continuum-based pedestrian models (Hughes, 2003; Treuille et al, 2006) treat the crowd motion as fluid. Navigation fields are used to direct and control the virtual crowds (Patil et al, 2011). Another popular category is agent-based models (Bonabeau, 2002), such as the social force model (Helbing and Molnar, 1995), self-driven particle model (Vicsek et al, 1995), agent navigation model (Van den Berg et al, 2008), reciprocal velocity obstacles (Berg et al, 2008) and Couzin model (Couzin et al, 2002), which treat pedestrians as autonomous agents based on a set of defined rules and known scene structures. They require manually inputting parameters. Differently, under MDA the collective dynamics for crowd behavior simulation are automatically learned from the fragmented trajectories extracted from the real videos without knowing scene structures and without manually setting parameters.

### 2.2 Crowd Behavior Analysis in Computer Vision

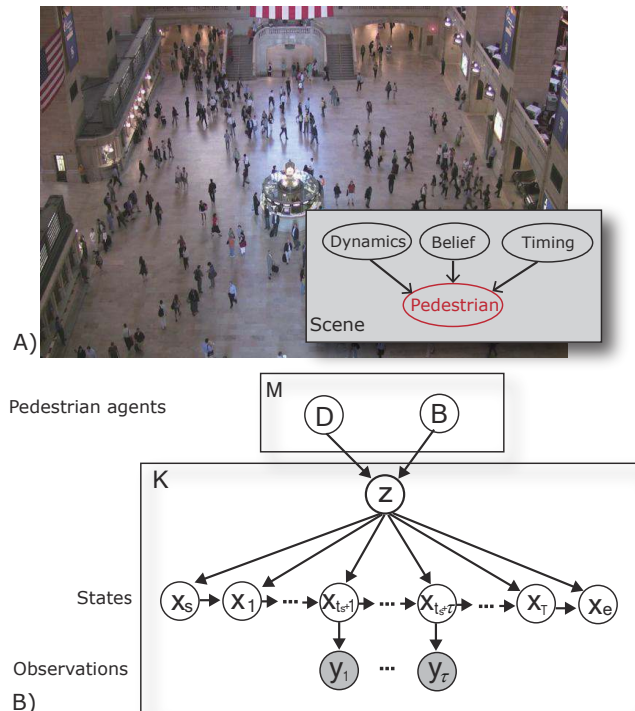
In computer vision, a lot of work focuses on learning global motion patterns (Ali and Shah, 2008; Lin et al, 2009, 2010; Mehran et al, 2010; Wang et al, 2008b; Li et al, 2008; Hospedales et al, 2009, 2011; Emonet et al, 2011; Loy et al, 2009; Yang et al, 2009; Kuettel et al, 2010; Makris and Ellis, 2005; Wang et al, 2008a, 2011; Kim et al, 2011; Zhou et al, 2011; Zen and Ricci, 2011; Saleemi et al, 2010), modeling local spatio-temporal variations (Mahadevan et al, 2010; Kratz and Nishino, 2009; Rodriguez et al, 2009; Wu et al, 2010; Saligrama and Chen, 2012), analyzing interactions among individuals (Pellegrini et al, 2009; Mehran et al, 2009; Scovanner and Tappen, 2009a), and detecting group behaviors (Zhou et al, 2012a; Ge et al, 2011; Moussaid et al, 2010; Choi et al, 2011; Yamaguchi et al, 2011; Pellegrini et al, 2010; Lan et al, 2011, 2012). The learned models of crowd behaviors are also used as priors to improve detection and tracking (Rodriguez et al, 2011; Chang et al, 2011; Zhao and Medioni, 2011; Yamaguchi et al, 2011). A brief review is given below.

There has been significant amount of work on learning the motion patterns of crowd. Ali *et al.* (Ali and Shah, 2007) and Lin *et al.* (Lin et al, 2009, 2010) computed flow fields and segmented crowd flows with Lagrangian coherent structures or Lie algebra. Mehran *et al.* (Mehran et al, 2010) proposed a streakline representation for crowd flows. With topic models, Wang *et al.* (Wang et al, 2008b) explored the co-occurrence of moving pixels without tracking objects to

learn the motion patterns in crowd. Topic models were augmented by adding spatio-temporal dependency among motion patterns (Hospedales et al, 2009, 2011; Emonet et al, 2011) or supervision (Kuettel et al, 2010). These approaches took the local location-velocity pairs as input while ignoring the temporal order of observations in order to be robust to tracking failures. The beliefs of pedestrians were not considered either. Some approaches (Makris and Ellis, 2005; Hu et al, 2007; Wang et al, 2008a, 2011; Morris and Trvedi, 2011; Kim et al, 2011; Zhou et al, 2011) learn motion patterns through clustering trajectories, and face the challenge of fragmentation of trajectories in crowded scenes. Different from MDA, none of the above methods used agent-based models, which could model the process of pedestrians making decisions based on the current states. It is also difficult for them to simulate or predict collective crowd behaviors.

Detecting collective motions and abnormal behaviors in crowd is of great interests for surveillance and crowd management. Zhou et al (Zhou et al, 2012a) proposed a graph-based method to detect coherent motions from tracklets. Collectiveness, defined as the degree of individuals acting as a union, was used to measure and detect collective motion patterns (Zhou et al, 2014, 2013). Some approaches were proposed to model local spatio-temporal variations for abnormality detection with dynamic texture (Mahadevan et al, 2010; Chan and Vasconcelos, 2008), HMM (Kratz and Nishino, 2009), distributions of spatio-temporal oriented energy (Rodriguez et al, 2009), chaotic invariants (Wu et al, 2010), and local motion descriptors (Saligrama and Chen, 2012).

To analyze interactions among pedestrians, the social force model, first proposed by Helbing *et al.* (Helbing and Molnar, 1995; Helbing et al, 2000) for crowd simulation, was introduced to the computer vision community recently and was applied to multi-target tracking (Scovanner and Tappen, 2009a; Pellegrini et al, 2009), abnormality detection (Mehran et al, 2009), and interaction analysis (Scovanner and Tappen, 2009a). It is also an agent-based model and assumes that pedestrians' movements for the next step are influenced by their destinations, the states of neighbors, and the borders of buildings, streets, and obstacles. It is complementary to MDA, since it models local interactive dynamics among pedestrians but requires the scene structures and the beliefs of pedestrians to be known in advance. MDA better models the global collective dynamics, automatically learns the regularization added by scene structures, and infers the beliefs of pedestrians. Both MDA and the social force model are agent-based models and have the potential to be well combined. It would be very interesting to integrate both collective dynamics and interactive dynamics which characterize the crowd behaviors from different perspectives into a single model in the future work. Some individuals with closer relationships form social groups in crowd. They have different interactions than individuals outside the groups. Ge



**Fig. 2** A) The behavior of a pedestrian in crowd is described with three key components, the dynamics of movements, the belief of the starting point and the destination, and the timing of entering the scene. B) Graphical representation of MDA. The shadowed variables are partial observations of the hidden states due to frequent tracking failures in crowded environment.

*et al.* (Ge et al, 2011) proposed a hierarchical clustering method to detect groups and Chang *et al.* (Chang et al, 2011) proposed a probabilistic strategy to softly assign individuals into groups. Moussaid *et al.* (Moussaid et al, 2010) modified the social force model to account for the influence of social groups. Lan *et al.* (Lan et al, 2011, 2012) analyzed individual behaviors considering the context of social groups with hierarchical models.

### 3 MDA Model

A crowd is an agglomeration of pedestrians. Although every pedestrian has his own dynamics of movement and belief of the starting point and the destination, some common statistical dynamic patterns would appear when enough pedestrians' behaviors are observed, because pedestrians in a specific scene share common dynamics and beliefs, and are regularized by the same scene structures. These shared movement patterns could be abstracted as different dynamic pedestrian-agents with various *dynamics* and *beliefs*. Each dynamic pedestrian agent represents one type of collective crowd behaviors. In a complex scene, there are multiple types of collective crowd behaviors happening simultaneously. Therefore, a mixture model of dynamic pedestrian-agents is needed. In our model, *dynamics* and *beliefs* of pedestrians are modeled

as two key modules  $D$  and  $B$  in the agent system. Since pedestrians of each dynamic pedestrian-agent emerge from an entrance with a certain frequency, we augment a dynamic pedestrian-agent with another module, *timing* of emerging. Thus, the crowd in a scene is formulated as a mixture model of dynamic pedestrian-agents as shown in Figure 2. In the following subsections, each module will be explained in details.

### 3.1 Modeling Collective Dynamics

Trajectories are time-series observations of pedestrian dynamics. If we treat a pedestrian as a dynamic agent system which actively senses the environment and makes decisions, the trajectory is a set of observations of the hidden dynamic states of this system. The dynamics of a pedestrian-agent is modeled as a linear dynamic system:

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + v_t, \quad (1)$$

$$\mathbf{y}_t = \mathbf{C}\mathbf{x}_t + w_t. \quad (2)$$

$\mathbf{x}_t = [x_t^1, x_t^2, 1]^\top$  is the current state of the agent and represents its position in homogeneous coordinates.  $\mathbf{y}_t \in \mathcal{R}^m$  is the observation of  $\mathbf{x}_t$ .  $\mathbf{A} \in \mathcal{R}^{3 \times 3}$  is the state transition matrix and  $\mathbf{C} \in \mathcal{R}^{m \times 3}$  is observation matrix.  $v_t$  is system noise and  $w_t$  is observation noise. Since observations are also positions of agents,  $m = 3$  and  $\mathbf{C}$  is a identity matrix. The conditional distributions of states and observations are

$$p(\mathbf{x}_t | \mathbf{x}_{t-1}) = \mathcal{N}(\mathbf{x}_t | \mathbf{A}\mathbf{x}_{t-1}, Q), \quad (3)$$

$$p(\mathbf{y}_t | \mathbf{x}_t) = \mathcal{N}(\mathbf{y}_t | \mathbf{C}\mathbf{x}_t, R), \quad (4)$$

where  $\mathcal{N}$  is a 3-dimensional multivariate Gaussian distribution,  $Q$  and  $R$  are covariance matrices. We denote  $D = (\mathbf{A}, Q, R)$  as the *dynamics* parameters to be learned.

Under homogeneous coordinates,  $\mathbf{A}$  can be expressed as,

$$\mathbf{A} = \begin{bmatrix} \mathbf{M} & \mathbf{b} \\ \mathbf{0} & 1 \end{bmatrix}, \quad (5)$$

and  $\mathbf{x}_t = \mathbf{M}\mathbf{x}_{t-1} + \mathbf{b}$ , where  $\mathbf{x}_t = [x_t^1, x_t^2]^\top$ ,  $\mathbf{M}$  is a linear transformation matrix, and  $\mathbf{b}$  is a translation vector. Therefore,  $\mathbf{A}$  is an affine transformation matrix and the dynamics of a dynamic pedestrian-agent is modeled as an affine transform. An important advantage of using homogeneous coordinates is that the multiplication of any two affine transform matrices is also an affine transform matrix. (Schneider and Eberly, 2003) shows that many important 2D geometric transforms such as translation, geometric contraction, expansion, dilation, rotation, shear, and their combinations are all affine transforms. Thus,  $\mathbf{A}$  in Eq (1) has good generalization capability of learning complex affine transforms from real data.

### 3.2 Modeling Pedestrian Beliefs

A pedestrian normally has a belief of the starting point and the destination when walking in a scene. This *belief* is a key factor driving the overall behavior of the pedestrian, and is also considered as the source and sink of the scene (Stauffer, 2003; Zhou et al, 2011). We model it as the initial state  $\mathbf{x}_s$  and the termination state  $\mathbf{x}_e$  of the agent system. For a trajectory  $k$ , the joint distribution of the system states and observations is

$$p(\mathbf{y}^k, \mathbf{x}^k, \mathbf{x}_e^k, t_s^k, t_e^k) = p(t_s^k) p(t_e^k) p(\mathbf{x}_s^k) p(\mathbf{x}_1^k | \mathbf{x}_s^k) p(\mathbf{x}_e^k | \mathbf{x}_{T^k}^k) \prod_{t=2}^{T^k} p(\mathbf{x}_t^k | \mathbf{x}_{t-1}^k) \prod_{t=1}^{\tau^k} p(\mathbf{y}_t^k | \mathbf{x}_{t_s^k+t}^k). \quad (6)$$

$\mathbf{x}^k = (\{\mathbf{x}_t^k\}_{t=1}^{T^k}, \mathbf{x}_s^k, \mathbf{x}_e^k)$  and  $\mathbf{y}^k = \{\mathbf{y}_t^k\}_{t=1}^{\tau^k}$  are the partial observations of the whole set of states  $\mathbf{x}^k$ . In crowd, the trajectories of objects are highly fragmented due to occlusions. Therefore, most trajectories are only partially observed. We assume that trajectory  $k$  is only observed from step  $t_s^k + 1$  to  $t_s^k + \tau^k$ .  $t_s^k$  is the number of steps with missing observations between the initial state  $\mathbf{x}_s^k$  and  $\mathbf{x}_{t_s^k+1}^k$ , and  $t_e^k$  is the number of steps with missing observations between  $\mathbf{x}_{t_s^k+\tau^k}^k$  and the termination state  $\mathbf{x}_e^k$  ( $T^k = t_e^k + t_s^k + \tau^k$ ). If  $t_s^k = 0$  and  $t_e^k = 0$ , the complete trajectory is observed. Here we assume the priors of  $p(t_s^k)$  and  $p(t_e^k)$  are uniform distributions over  $[0, H]$ , where  $H$  is the upper bound of  $t_s^k$  and  $t_e^k$  just to make their priors proper. Section 4 shows that the choice of  $H$  does not affect the learning and inference of MDA as long as it is large enough (e.g.  $H = 10,000$ ). We do not adopt other priors such as truncated Gaussian or exponential distributions, because of lack knowledge on typical distributions of  $t_s^k$  and  $t_e^k$ .

The initial state is sampled from a Gaussian distribution,

$$p(\mathbf{x}_s^k) = \mathcal{N}(\mathbf{x}_s^k | \mu_s, \Phi_s), \quad (7)$$

where  $\mu_s$  and  $\Phi_s$  are the mean and covariance matrix of the source region. The termination state  $\mathbf{x}_e$  conditioned on its previous state  $\mathbf{x}_{T^k}^k$  is sampled from

$$p(\mathbf{x}_e^k | \mathbf{x}_{T^k}^k) \propto \mathcal{N}(\mathbf{x}_e^k | \mathbf{A}\mathbf{x}_{T^k}^k, Q) \mathcal{N}(\mathbf{x}_e^k | \mu_e, \Phi_e), \quad (8)$$

where  $\mu_e$  and  $\Phi_e$  are the mean and covariance matrix of the sink region.  $p(\mathbf{x}_e^k | \mathbf{x}_{T^k}^k)$  is proportional to the product of two Gaussian distributions, and it is also a Gaussian distribution. The sampling of the termination state is regularized by  $\mathbf{x}_{T^k}^k$  and also the center of the sink region. We denote  $B = (\mu_s, \Phi_s, \mu_e, \Phi_e)$  as the *belief* parameters. Other conditional distributions such as  $p(\mathbf{x}_1^k | \mathbf{x}_s^k)$ ,  $p(\mathbf{x}_t^k | \mathbf{x}_{t-1}^k)$ , and  $p(\mathbf{y}_t^k | \mathbf{x}_{t_s^k+t}^k)$  are given by Eq (3) and (4).  $\mathbf{x}^k$ ,  $t_s^k$  and  $t_e^k$  are all hidden variables, to be inferred from the model.

### 3.3 Mixture of Dynamic Pedestrian-Agents

Numerous pedestrians in a scene have various *dynamics* and *beliefs*. To model the diversity, we extend the single agent system described above to a mixture system with  $M$  possible dynamics and beliefs  $(D_1, B_1), \dots, (D_M, B_M)$ . A hidden variable  $z^k = 1, \dots, M$  indicates the pedestrian-agent from which a trajectory  $k$  is sampled. The prior  $p(z^k)$  is a discrete distribution parameterized by  $(\pi_1, \dots, \pi_M)$ , i.e.  $p(z^k = m) = \pi_m$ . The joint distribution is

$$\begin{aligned} & p(\mathbf{x}^k, \mathbf{y}^k, t_s^k, t_e^k, z^k) \\ = & p(z^k)p(t_s^k)p(t_e^k)p(\mathbf{x}_1^k|z^k)p(\mathbf{x}_1^k|\mathbf{x}_s^k, z^k)p(\mathbf{x}_e^k|\mathbf{x}_{T^k}^k, z^k) \\ & \prod_{t=2}^{T^k} p(\mathbf{x}_t^k|\mathbf{x}_{t-1}^k, z^k) \prod_{t=1}^{\tau^k} p(\mathbf{y}_t^k|\mathbf{x}_{t_s^k+t}^k, z^k). \end{aligned} \quad (9)$$

### 3.4 Discussion

Linear Dynamic Systems (LDS) (Doretto and Chiuso, 2003; Oh et al, 2005) and mixture of LDS (Chan and Vasconcelos, 2008) have been successfully used to solve computer vision problems in literature. We are inspired by these works and apply mixture of LDS as an agent-based model for pedestrian behavior analysis. Besides the major difference on the targeting problems, our model is different than existing LDS in several other aspects. (1)  $\{\mathbf{y}^k\}$  are only partially observed in MDA, but fully observed in other LDS. (2) The temporal length is known in other LDS, while it is a hidden variable in MDA. (3) We model source and sinks regions which add regularization on the initial and termination states.

## 4 Model Learning and Inference

Given trajectories  $\{\mathbf{y}^k\}_{k=1}^K$ , we learn the model parameters  $\Theta = \{(D_1, B_1, \pi_1), \dots, (D_M, B_M, \pi_M)\}$  by maximizing the likelihood of observations,

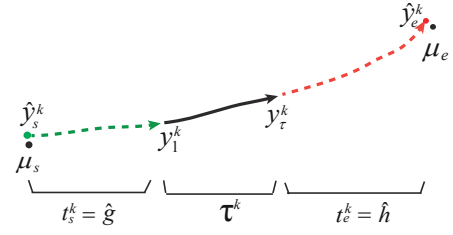
$$\Theta^* = \arg \max_{\Theta} \sum_{k=1}^K \log p(\mathbf{y}^k; \Theta). \quad (10)$$

There are three types of hidden variables: 1) the index  $z^k$  of assigning a trajectory  $k$  to a mixture component; 2) the complete sequence of states  $\mathbf{x}^k$  that produce the partial observation  $\mathbf{y}^k$ ; and 3) the numbers of steps with missing observations, i.e.  $t_s^k$  and  $t_e^k$ . We apply the EM algorithm to estimate parameters. Each iteration of EM consists of

$$\mathbf{E}\text{-step: } Q(\Theta; \hat{\Theta}) = E_{\mathbf{X}, \mathbf{T}, \mathbf{Z} | \mathbf{Y}; \hat{\Theta}}(\log p(\mathbf{X}, \mathbf{Y}, \mathbf{T}, \mathbf{Z}; \Theta)),$$

$$\mathbf{M}\text{-step: } \hat{\Theta}^* = \arg \max_{\Theta} Q(\Theta; \hat{\Theta}).$$

$p(\mathbf{X}, \mathbf{Y}, \mathbf{T}, \mathbf{Z}; \Theta)$  is the complete-data likelihood of the partial observations  $\mathbf{Y}$ , complete hidden states  $\mathbf{X}$  (including the



**Fig. 3** Estimate the most possible  $\hat{g}$  and  $\hat{h}$  from Eq (12) as the numbers of steps generating the nearest points  $\hat{\mathbf{y}}_s^k$  and  $\hat{\mathbf{y}}_e^k$  to  $\mu_s$  and  $\mu_e$  respectively. The black curve is a fragmented trajectory, the dashed curves are the estimation of missing states.

initial and termination states), the numbers of steps with missing observations  $\mathbf{T}$ , and hidden assignment variables  $\mathbf{Z}$ .

### 4.1 Initialization

To initialize the estimation of parameters, we roughly draw the boundaries of entry/exit regions in a scene as shown in Figure 4A. For every agent component  $m$ , its entry/exit region is randomly chosen from these regions (entry and exit regions cannot be the same). For initialization, we let points  $\mathbf{y}$  of trajectories which start/end within the source/sink regions of component  $m$  be equal to their hidden states  $\mathbf{x}$ , and then use  $\mathbf{x}$  to estimate the dynamics parameters  $\mathbf{A}_m$  and  $Q_m$  with maximum likelihood estimation.  $R_m$  is initialized as  $[0.1 \ 0 \ 0; 0 \ 0.1 \ 0; 0 \ 0 \ 0]$ . The starting/ending points of trajectories which start/end within the entry/exit regions are used to initialize the estimation of belief parameters  $(\mu_m^s, \Phi_m^s, \mu_m^e, \Phi_m^e)$ .  $\pi_m$  is initialized as  $1/M$ .

### 4.2 Expectation Step

The posterior probabilities and the expectation of complete-data likelihood under current estimated parameters  $\hat{\Theta}$  are,

$$\begin{aligned} Q &= E_{\mathbf{X}, \mathbf{T}, \mathbf{Z} | \mathbf{Y}; \hat{\Theta}}(\log p(\mathbf{X}, \mathbf{Y}, \mathbf{T}, \mathbf{Z}; \Theta)) \\ &= E_{\mathbf{Z}, \mathbf{T} | \mathbf{Y}; \hat{\Theta}}(E_{\mathbf{X} | \mathbf{Y}, \mathbf{Z}, \mathbf{T}; \hat{\Theta}}(\log p(\mathbf{X}, \mathbf{Y}, \mathbf{T}, \mathbf{Z}; \Theta))) \\ &= \sum_{k, m, g, h} \gamma_k(m, g, h) E_{\mathbf{x}^k | \mathbf{y}^k, z^k = m, t_s^k = g, t_e^k = h}(\log p(\mathbf{x}^k, \mathbf{y}^k, t_s^k, t_e^k, z^k)) \end{aligned}$$

where  $\gamma_k(m, g, h)$  is defined as

$$\begin{aligned} \gamma_k(m, g, h) &= p(z^k = m, t_s^k = g, t_e^k = h | \mathbf{y}^k) \\ &= \frac{\pi_m p(\mathbf{y}^k | z^k = m, t_s^k = g, t_e^k = h)}{\sum_{m'=1}^M \sum_{g', h'} \pi_{m'} p(\mathbf{y}^k | z^k = m', t_s^k = g', t_e^k = h')}. \end{aligned} \quad (11)$$

The priors of  $p(t_s^k)$  and  $p(t_e^k)$  are uniform. The likelihood of observations  $p(\mathbf{y}^k | z^k = m, t_s^k = g, t_e^k = h)$  is computed with the modified Kalman smoothing filter in Section 4.4.

$\gamma_k(m, g, h)$  has three discrete variables. It is time consuming to compute all their possible combinations in the range of  $[1, M] \times [0, H]^2$ . For most  $(g, h)$ ,  $\gamma_k(m, g, h)$  are approximately 0. We first estimate the most plausible  $\hat{g}$  and  $\hat{h}$  for fragmented trajectory  $k$  by optimization,

$$\begin{aligned}\hat{h} &= \arg \min_t \|\mu_e^m - \mathbf{A}_m^t \mathbf{y}_\tau^k\|, \\ \hat{g} &= \arg \min_t \|\mu_s^m - \mathbf{A}_m^{-t} \mathbf{y}_1^k\|,\end{aligned}\quad (12)$$

where  $\mathbf{y}_\tau^k$  and  $\mathbf{y}_1^k$  are the last and first points of  $\mathbf{y}^k$  and  $\mathbf{A}^t$  refer to  $t$  matrix power of  $\mathbf{A}$ . Eq. (12) is to find a candidate  $\hat{g}$  ( $\hat{h}$ ), the starting point  $\hat{\mathbf{y}}_s^k$  (ending point  $\hat{\mathbf{y}}_e^k$ ) predicted according to which is closest to the source center  $\mu_s^m$  (sink center  $\mu_e^m$ ). Since a starting (ending) point is regularized by the source (sink) with a Gaussian distribution, when  $g$  ( $h$ ) is largely different from  $\hat{g}$  ( $\hat{h}$ ), the predicted starting (ending) point is far away from  $\mu_s^m$  ( $\mu_e^m$ ). Then  $\gamma_k(m, g, h)$  is close to zero and can be ignored. The one-dimensional discrete search problems in Eq (12) can be solved efficiently. The illustration of this optimization is shown in Figure 3, we extend the trajectory and search the nearest points to the mean of initial state and termination state. Then we limit the plausible set of  $t_s^k$  as  $[\hat{g} - \Delta, \hat{g} - \Delta + 1, \dots, \hat{g}, \dots, \hat{g} + \Delta - 1, \hat{g} + \Delta]$ , and the plausible set of  $t_e^k$  as  $[\hat{h} - \Delta, \hat{h} - \Delta + 1, \dots, \hat{h}, \dots, \hat{h} + \Delta - 1, \hat{h} + \Delta]$ , where  $\Delta$  is an integer and empirically determined. When it is out of the plausible range,  $\gamma_k(m, g, h)$  is approximated as 0. So there are  $(2\Delta + 1)^2$  combinations of  $(t_s^k, t_e^k)$ .

### 4.3 Maximization Step

New parameters  $\Theta^*$  are estimated by maximizing  $\mathcal{Q}$ . We first recursively and efficiently estimate the expectations of hidden states and their products, *i.e.*

$$\hat{\mathbf{x}}^k = E_{\mathbf{x}^k | \mathbf{y}^k, z^k=m, t_s^k=g, t_e^k=h}(\mathbf{x}^k), \quad (13)$$

$$P_{t,t}^k = E_{\mathbf{x}^k | \mathbf{y}^k, z^k=m, t_s^k=g, t_e^k=h}(\mathbf{x}_t^k \mathbf{x}_t^{k\top}), \quad (14)$$

$$P_{t,t-1}^k = E_{\mathbf{x}^k | \mathbf{y}^k, z^k=m, t_s^k=g, t_e^k=h}(\mathbf{x}_t^k \mathbf{x}_{t-1}^{k\top}), \quad (15)$$

from partial observations with the modified Kalman smoothing filter (Palma, 2007; Shumway and Stoffer, 1982), whose details are summarized in Section 4.4. The values of  $\hat{\mathbf{x}}^k$ ,  $P_{t,t}^k$  and  $P_{t,t-1}^k$  depend on the choice of  $m$ ,  $g$ , and  $h$ . We do not include the indices of  $m$ ,  $g$  and  $h$  in notations to simplify the equations in Section 4.3 and 4.4. Then  $\Theta^*$  are updated are as follows,

$$\mathbf{A}_m^* = \left( \sum_{k,g,h} \gamma_k(m, g, h) \sum_{t=1}^{T^k+1} P_{t,t-1}^k \right) \left( \sum_{k,g,h} \gamma_k(m, g, h) \sum_{t=1}^{T^k+1} P_{t-1,t-1}^k \right)^{-1}, \quad (16)$$

$$Q_m^* = \frac{\sum_{k,g,h} \gamma_k(m, g, h) (\sum_{t=1}^{T^k+1} P_{t,t}^k - \mathbf{A}_m^* \sum_{t=1}^{T^k+1} P_{t,t-1}^k)}{\sum_{k,g,h} \gamma_k(m, g, h) (T^k + 1)}, \quad (17)$$

$$R_m^* = \frac{\sum_{k,g,h} \gamma_k(m, g, h) \sum_{t=1}^{T^k} (\mathbf{y}_t^k \mathbf{y}_t^{k\top} - \hat{\mathbf{x}}_t^k \mathbf{y}_t^{k\top} - \mathbf{y}_t^k \hat{\mathbf{x}}_t^{k\top} + P_{t,t}^k)}{\sum_{k,g,h} \gamma_k(m, g, h) \tau^k}, \quad (18)$$

$$\mu_s^{m*} = \frac{\sum_{k,g,h} \gamma_k(m, g, h) \hat{\mathbf{x}}_s^k}{\sum_{k,g,h} \gamma_k(m, g, h)}, \quad (19)$$

$$\Phi_s^{m*} = \frac{\sum_{k,g,h} \gamma_k(m, g, h) (\hat{\mathbf{x}}_s^k - \mu_s^{m*}) (\hat{\mathbf{x}}_s^k - \mu_s^{m*})^\top}{\sum_{k,g,h} \gamma_k(m, g, h)}, \quad (20)$$

$$\mu_e^{m*} = \frac{\sum_{k,g,h} \gamma_k(m, g, h) \hat{\mathbf{x}}_e^k}{\sum_{k,g,h} \gamma_k(m, g, h)}, \quad (21)$$

$$\Phi_e^{m*} = \frac{\sum_{k,g,h} \gamma_k(m, g, h) (\hat{\mathbf{x}}_e^k - \mu_e^{m*}) (\hat{\mathbf{x}}_e^k - \mu_e^{m*})^\top}{\sum_{k,g,h} \gamma_k(m, g, h)}, \quad (22)$$

$$\pi_m^* = \frac{\sum_{k,g,h} \gamma_k(m, g, h)}{\sum_{m'=1}^M \sum_{k,g,h} \gamma_k(m', g, h)}. \quad (23)$$

The proof in the supplementary material shows that  $\mathbf{A}_m^*$  is still an affine transform matrix during the EM update.

### 4.4 Modified Kalman Smoothing Filter

Kalman smoothing filter (Shumway and Stoffer, 1982; Palma, 2007) is used to estimate the means and covariances (in Eq (13)-(15)) of the states  $\mathbf{x}$  of a LDS conditioned on the observations  $\{\mathbf{y}_t\}_{t=1}^T$  ( $T = t_s + t_e + \tau$ ). It is assumed that  $\mathbf{y}_t$  is observed at steps  $t_s + 1$  to  $t_s + \tau$  and missed at steps 1 to  $t_s$  and  $t_s + \tau + 1$  to  $T$ . Kalman filter is also used to compute the likelihood of observations in Eq (11). Detailed discussion and proof on the modifications on the Kalman filter in order to account for the missing observations can be found in (Palma, 2007).

Denote the expectations conditioned on the observed sequence  $\mathbf{y}_1, \dots, \mathbf{y}_n$  as

$$\mathbf{x}_t^n = E_{\mathbf{x} | \mathbf{y}_1, \dots, \mathbf{y}_n}(\mathbf{x}_t), \quad (24)$$

$$V_t^n = E_{\mathbf{x} | \mathbf{y}_1, \dots, \mathbf{y}_n}((\mathbf{x}_t - \mathbf{x}_t^n)(\mathbf{x}_t - \mathbf{x}_t^n)^\top), \quad (25)$$

$$V_{t,t-1}^n = E_{\mathbf{x} | \mathbf{y}_1, \dots, \mathbf{y}_n}((\mathbf{x}_t - \mathbf{x}_t^n)(\mathbf{x}_{t-1} - \mathbf{x}_{t-1}^n)^\top). \quad (26)$$

For  $t = 1, \dots, T$ , we obtain the Kalman forward recursions:

$$\begin{aligned}\mathbf{x}_t^{t-1} &= \mathbf{A} \mathbf{x}_{t-1}^{t-1}, \\ V_t^{t-1} &= \mathbf{A} V_{t-1}^{t-1} \mathbf{A}^\top + Q, \\ K_t &= V_t^{t-1} (V_t^{t-1} + R)^{-1}, \\ \mathbf{x}_t^t &= \begin{cases} \mathbf{x}_t^{t-1} + K_t (\mathbf{y}_t - \mathbf{x}_t^{t-1}) & \text{if } \mathbf{y}_t \text{ observed,} \\ \mathbf{x}_t^{t-1} & \text{if } \mathbf{y}_t \text{ missed,} \end{cases} \\ V_t^t &= V_t^{t-1} - K_t V_t^{t-1},\end{aligned}$$

where  $\mathbf{x}_1^0 = \mu_s$  and  $V_1^0 = \Phi_s$ . When  $t = T+1$ , it reaches termination state, one more Kalman forward recursion with sink distribution constraint as

$$\begin{aligned}
\mathbf{x}_T^{T+1} &= \mathbf{A}\mathbf{x}_T^T, \\
V_T^{T+1} &= \mathbf{A}V_T^T\mathbf{A}^\top + Q, \\
K_{T+1} &= V_{T+1}^T(V_{T+1}^T + \Phi_e)^{-1}, \\
\mathbf{x}_{T+1}^{T+1} &= \mathbf{x}_T^{T+1} + K_{T+1}(\mu_e - \mathbf{x}_{T+1}^T), \\
V_{T+1}^{T+1} &= V_T^{T+1} - K_{T+1}V_{T+1}^T.
\end{aligned}$$

Then  $\hat{\mathbf{x}}_e = \mathbf{x}_{T+1}^{T+1}$ . To further compute  $\hat{\mathbf{x}}_t \equiv \mathbf{x}_t^{T+1}$  and  $P_{t,t} \equiv V_t^{T+1} + \mathbf{x}_t^{T+1}\mathbf{x}_t^{T+1\top}$ , we perform backward recursions from  $t = T + 1, \dots, 1$  using

$$\begin{aligned}
J_{t-1} &= V_{t-1}^{t-1}\mathbf{A}^\top(V_{t-1}^{t-1})^{-1}, \\
\mathbf{x}_{t-1}^{T+1} &= \mathbf{x}_{t-1}^{t-1} + J_{t-1}(\mathbf{x}_t^{T+1} - \mathbf{A}\mathbf{x}_{t-1}^{t-1}), \\
V_{t-1}^{T+1} &= V_{t-1}^{t-1} + J_{t-1}(V_t^{T+1} - V_{t-1}^{t-1})J_{t-1}^\top.
\end{aligned}$$

Here  $\hat{\mathbf{x}}_s = \mathbf{x}_0^{T+1}$ . To compute  $P_{t,t-1} \equiv V_{t,t-1}^{T+1} + \mathbf{x}_t^{T+1}\mathbf{x}_{t-1}^{T+1\top}$ , one performs the backward recursions from  $t = T + 1, \dots, 2$

$$V_{t-1,t-2}^{T+1} = V_{t-1}^{t-1}J_{t-2}^\top + J_{t-1}(V_{t,t-1}^{T+1} - \mathbf{A}V_{t-1}^{t-1})J_{t-2}^\top,$$

with initial condition  $V_{T+1,T}^{T+1} = (I - K_{T+1})\mathbf{A}V_T^T$ .

To compute the log-likelihood of observation  $\mathbf{y}$ , we use the innovations form (Shumway and Stoffer, 1982),

$$\begin{aligned}
\log p(\mathbf{y}) &= \sum_{t=1}^{\tau} \log p(\mathbf{y}_t | \mathbf{y}_1^{t-1}) \\
&= \sum_{t=1}^{\tau} \log \mathcal{N}(\mathbf{y}_t | \hat{\mathbf{x}}_t^{t-1}, V_t^{t-1} + R). \tag{27}
\end{aligned}$$

Then  $\gamma(m, g, h)$  can be computed from  $p(\mathbf{y}|z = m, t_s = g, t_e = h)$  in Eq (11).

## 5 Simulation and Prediction

### 5.1 Crowd Behavior Simulation

To simulate crowd behaviors by sampling trajectories, we also model the frequency of new pedestrians entering the scene over time, and integrate this module into MDA.

We assume the timings of pedestrians emerging in an entrance region follows a homogeneous Poisson process, whose underlying distribution is a Poisson distribution

$$p(N(t + \Delta t) - N(t) = n) = \frac{(\lambda\Delta t)^n e^{-\lambda\Delta t}}{n!}, \tag{28}$$

where  $n$  is the number of emerging pedestrians during time interval  $(t, t + \Delta t)$ .  $\lambda$  is the rate parameter and indicates the expected number of emerging pedestrians per time interval.

After  $\{(D_1, B_1), \dots, (D_M, B_M)\}$  are learned, every trajectory  $k$  has the most likely  $z^k$ , and its emerging time can also be estimated. Thus we can count the number of emerging pedestrians in each time interval  $\Delta t$  (here  $\Delta t$  is 5 seconds), and estimate  $\lambda$  for each dynamic pedestrian-agent by

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### Algorithm 1 Model sampling

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INPUT: time length  $H$ , resampling number  $N$ , pedestrian-agent  $m$ .  
OUTPUT: simulated trajectories.  
01: sample temporal order  $\delta_{1-H}$  from  $PoissonP(\lambda_m)$   
02: **for**  $\omega = 1 : H$   
03:   **if**  $\delta_\omega == 1$   
04:     **for**  $n = 1 : N$   
05:       sample  $\mathbf{x}_s^n$  from  $p_m(\mathbf{x}_s^n)$   
07:        $T^n = \arg \min_t \|\mu_m^e - \mathbf{A}_m^t \mathbf{x}_s^n\|$ .  
08:       generate trajectory  $\mathbf{y}^n = \{\mathbf{y}_t^n\}_{t=1}^{T^n}$  by sequentially sampling  $p_m(\mathbf{x}_t^n | \mathbf{x}_{t-1}^n)$  and  $p_m(\mathbf{y}_t^n | \mathbf{x}_t^n)$ .  
      sample  $\mathbf{x}_e^n$  from  $p_m(\mathbf{x}_e^n | \mathbf{x}_{T^n}^n)$ , then compute  $l_n = p_m(\mathbf{x}_e^n)$ .  
09:     **end for**  
10:     resample one trajectory  $\mathbf{y}$  out of the  $N$  simulated trajectories  $\{\mathbf{y}^n\}$  according to normalized distribution  $\{l_1, \dots, l_N\}$ .  
11:   **end if**  
12: **end for**

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### Algorithm 2 Model fitting

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INPUT: trajectory  $k$  from any tracker.  
OUTPUT: the optimal fitted  $z^*$ .  
01: **for**  $m = 1 : M$  **do**  
02:   compute  $\gamma(z^k = m) = \sum_{g,h} \gamma_k(m, g, h)$   
03: **end for**  
04:  $z^* = \arg \max_m \gamma(z^k = m)$   
05: compute the future states or past states with  $\mathbf{A}_{z^*}$ ;  
   predict the belief with  $B_{z^*}$ .

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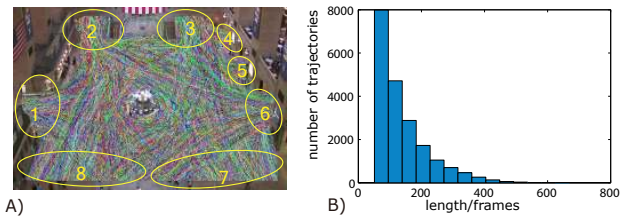
maximum likelihood estimation. Since a pedestrian may be lost and found again during the tracking process, it causes the multiple counting problem. Therefore a trajectory  $k$  is counted only if its first observation  $\mathbf{y}_1^i$  is within the source region, which is an ellipse specified by  $(\mu_s^{z^k}, 4\Phi_s^{z^k})$ , such that the bias can be reduced to some extent.

Once MDA is learned, crowd behaviors can be simulated as follows: firstly obtaining the temporal order of emerging by sampling from the Poisson process, and then at each time step of emerging generating the trajectory by sequentially sampling from the linear dynamic system with an initial state and a termination state. Since constraint on termination state is not considered during sequentially sampling, there is a resampling step in the end. The algorithm of sampling one trajectory from a pedestrian-agent is in Algorithm 1.

### 5.2 Pedestrian Behavior Prediction

After MDA is learned, given a fragmented trajectory of a pedestrian, our model can fit it to the optimal pedestrian-agent  $z^*$  and predict the pedestrian's past and future paths with the corresponding state transition matrix  $\mathbf{A}_{z^*}$ , as well as the starting point and the destination with the corresponding belief parameters  $B_{z^*}$ . The algorithm of fitting a pedestrian-agent is summarized in Algorithm 2.





**Fig. 4** A) Extracted trajectories and entry/exit regions indicated by yellow ellipses. The colors of trajectories are randomly assigned. B) Histogram of the lengths of trajectories. Most of them are short and fragmented.

## 6 Experiments and Applications

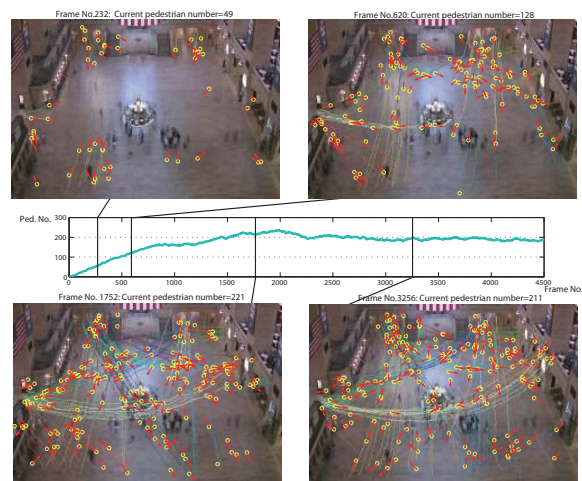
Most experimental results are reported on a 15-minutes video collected from the New York Grand Central Station at 24fps with a resolution of  $480 \times 720$ . A KLT keypoint tracker (Tomasi and Kanade, 1991) is used to extract trajectories. Tracking terminates when ambiguities caused by occlusions and clutters arise, and new tracks are initialized later. After filtering short and stationary trajectories, around 20,000 trajectories are extracted and shown in Figure 4A. Figure 4B shows that most trajectories are highly fragmented and short. More experimental results on the MIT traffic dataset (Wang et al, 2008b) and a Marathon race video (Ali and Shah, 2007) are reported in Section 6.8 and 6.9.

### 6.1 Model Learning

To initialize the parameters of MDA, we first roughly label 8 entry/exit regions with ellipses indexed by 1-8 in Figure 4A. Parameters are initialized according to Section 4.1. It takes around one hour for EM to converge, running on a computer with 3GHz Core Quad CPU and 4GB RAM with Matlab implementation. Totally  $M = 20$  agent components are learned. In this work,  $M$  is chosen empirically, but it also could be estimated with Dirichlet process (Wang et al, 2008b).

Figure 5A illustrates ten representative dynamic pedestrian-agents. Trajectories are sampled from each pedestrian-agent using the algorithm in Table 5.1. Results show that the learned dynamic pedestrian-agents have different dynamics, beliefs and timings of emerging, and they characterize various collective behaviors. The learned distributions of initial/termination states are more accurate than the initialized entry/exit regions. For example, region 8 in Figure 4A corresponds to multiple smaller initial/termination state distributions in Figure 5. The entry and exit regions of a dynamic pedestrian-agent are randomly selected in initialization. However, if there is no commonly taken path between them, the dynamic pedestrian-agent will diminish and switch to other entry/exit regions during EM learning. For example, there is

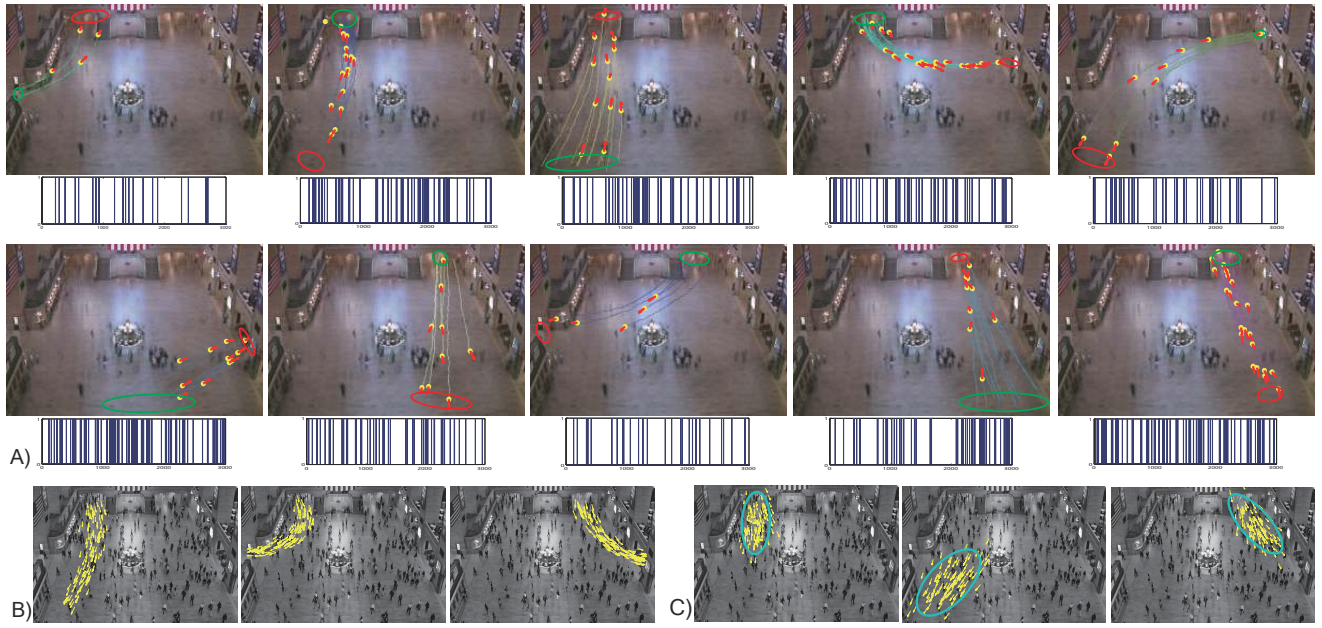
no path connecting regions 8 and 7, regions 2 and 3 among the learned dynamic pedestrian-agents. Some paths are deformed by the information booth at the center of the scene. By densely sampling, MDA can also estimate the velocity flow field for each pedestrian-agent as shown in Figure 5B. For comparison, the representative flow fields by LAB-FM (Lin et al, 2009), which learned motion patterns using Lie algebra, are shown in Figure 5C. MDA performs better in terms of capturing long-range collective behaviors and separating different collective behaviors. For example, some flow fields learned with LAB-FM are locally distributed, without covering the complete paths. The upper parts of the first two flow fields in Figure 5B, which represent two different collective behaviors, are merged by LAB-FM as shown in the first flow field in Figure 5C. This is due to the facts that 1) MDA better models the shared beliefs of pedestrians and states of missing observations, and takes the whole trajectories instead of local position-velocity pairs as input, and 2) LAB-FM assumes that the spatial distributions of the flow fields are Gaussian (indicated by cyan ellipses).



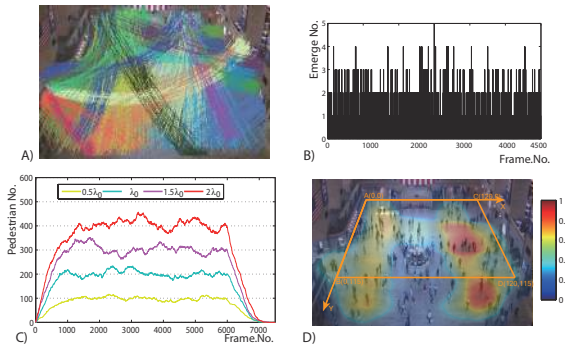
**Fig. 6** Four exemplar frames from crowd behavior simulation. Simulated trajectories are colored according to the indices of their dynamic pedestrian-agents. The middle plots the population of pedestrians over time.

### 6.2 Collective Crowd Behavior Simulation

Compared with other approaches (Hospedales et al, 2009; Wang et al, 2008b; Zhou et al, 2011) of modeling global motion patterns in crowded scenes, a distinctive feature of MDA is to simulate collective crowd behaviors once the model parameters are learned from observations. According to the superposition property of Poisson process (Kingman, 1993), the timings of overall pedestrians emerging in the scene also follow a Poisson process with  $\lambda = \sum_{m=1}^M \lambda_m$ .



**Fig. 5** A) Ten representative dynamic pedestrian-agents. Please find all the 20 dynamic pedestrian-agents from the supplementary material. Pedestrians are sampled from them. Green and red circles indicate the learned distributions of initial/termination states for each pedestrian-agent. Yellow circles indicate the current positions of sampled pedestrians along with their trajectories, and red arrows indicate current velocities. The timings of pedestrians entering the scene sampled from the Poisson process are shown below. One impulse indicates a new pedestrian entering the scene, who is driven by the corresponding dynamic pedestrian-agent. B) Flow fields generated from dynamic pedestrian-agents. C) Flow fields learned by LAB-FM (Lin et al, 2009).

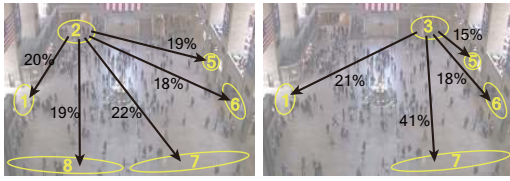


**Fig. 7** A) All the simulated trajectories. Colors of trajectories are assigned according to pedestrian-agent indices. B) The numbers of pedestrians entering the scene at different frames. C) The population of the scene with  $\lambda = 0.5\lambda_0, \lambda_0, 1.5\lambda_0, 2\lambda_0$  in simulation, where  $\lambda_0$  is the value learned from data. D) Relative population density map computed from simulation. It is normalized considering the perspective distortion.

To simulate the overall crowd, we first sample the temporal order series from the Poisson process with  $\lambda$ . Then for every newly emerging pedestrian, its pedestrian-agent index is first sampled from the discrete distribution  $(\pi_1, \dots, \pi_M)$ , and then its trajectory is sampled from the dynamic pedestrian-agent using the algorithm in Table 5.1.

Figure 6 shows four exemplar frames of the simulated crowd behaviors. At the first frame pedestrians begin to en-

ter the empty scene. After 1500 frames the crowd reaches the equilibrium population with around 200 pedestrians. Figure 7A plots all the simulated trajectories over 4500 frames. Figure 7B shows the numbers of new pedestrians entering the scene over time. The crowd simulation with MDA can provide valuable information about the dynamics of the crowd. For example, in Figure 7C, we investigate the relationship between the different rate parameter  $\lambda$  and the population of the scene, where pedestrians begin and stop to enter the scene at the Frame 1 and 6000 respectively. As pedestrians keep entering the scene with a constant birth rate, the scene reaches its equilibrium state. When  $\lambda = \lambda_0$ , which is learned from data, the system reaches its equilibrium state after 1500 frames with around 200 pedestrians in the scene. And the equilibrium state changes with different birth rates. In Figure 7D we compute the averaged population density map when  $\lambda = \lambda_0$ . In this scene, the crossing regions of multiple paths and the entrance/exit regions have higher population density. These populated areas deserve more attention of security since accidents would most likely happen there when panic or abnormal event strikes. These types of information are very useful for crowd management and public facility optimization.



**Fig. 8** The transition ratios of pedestrian flows from entries 2 and 3 to other exits.

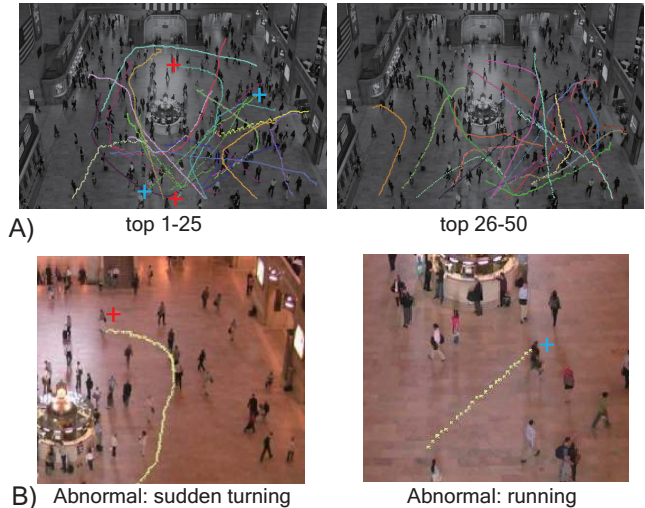
### 6.3 Flow Transitions between Sources and Sinks

We can compute the transition ratios of pedestrian flows from the simulation data. Figure 8 shows the transition probabilities from entries 2 and 6 to the other exits learned by MDA. For instance, we can observe that the pedestrian flow from source 2 goes roughly equally to the other five sinks; but differently 41% pedestrians from source 3 go to sink 7. Explaining the difference requires knowledge on the infrastructure and the transportation schedule of the train station. These statistics of the pedestrian flow provide useful information for crowd control and management.

### 6.4 Collective Behavior Classification

MDA can be used to cluster trajectories of pedestrians into different collective dynamics. We simply take the inferred  $z^k$  cluster index. A lot of works have been done on trajectory clustering (Hu et al, 2004; Morris and Trivedi, 2008). This problem is especially challenging in crowded scenes because trajectories are highly fragmented with many missing observations. Generally speaking, existing approaches are in two categories: distance-based (Wang et al, 2006; Hu et al, 2007) and model-based (Wang et al, 2008a; Morris and Trivedi, 2011). We choose one representative approach from each category for comparison: Hausdorff distance-based spectral clustering (Wang et al, 2006) and hierarchical Dirichlet processes (HDP) (Wang et al, 2008a). Figure 9A shows some representative clusters obtained by MDA. Even though most trajectories are fragmented and are far away from each other in space, they are still well grouped into one cluster because they share the same collective dynamics. Figure 9B and Figure 9C show the representative clusters obtained by spectral clustering (Wang et al, 2006) and HDP (Wang et al, 2008a). They are all in short spatial range and it is hard to interpret their semantic meanings, because they cannot well handle fragmentation of trajectories.

We use *correctness* and *completeness* introduced in (Mobert et al, 2005) to measure clustering accuracy. Correctness is the accuracy that two trajectories, which belong to different collective behaviors based on the ground truth, are also grouped into different clusters by the algorithm. Completeness is the accuracy that two trajectories, which belong to



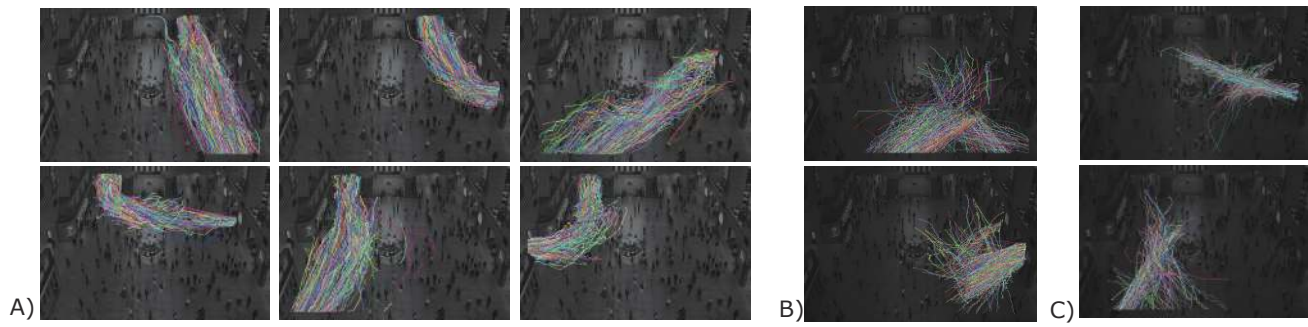
**Fig. 10** A) Top 50 abnormal trajectories. B) Examples in zoom-in views. Left: a pedestrian changes his mind and moves towards a different destination than his original plan. Right: a pedestrian is running, with dynamics quite different with other walking pedestrians.

the same collective behavior, are also grouped into the same cluster by the algorithm. If all the trajectories are grouped into one cluster, the completeness is 100% while the correctness is 0%; if every trajectories is put into a different cluster, the completeness is 0% while the correctness is 100%. A good clustering algorithm should have both high correctness and high completeness. Rand index (Rand, 1971) is another commonly used measure of similarity between clustering results, and can be viewed as a linear combination of correctness and completeness. We choose to report both correctness and completeness scores, such that readers can have a comprehensive understanding of clustering quality.

To measure correctness (completeness), we manually label 2000 (1500) pairs of trajectories and each pair of trajectories belong to different (the same) collective behavior categories (category) as ground truth. The accuracies of correctness and completeness for MDA, HDP (Wang et al, 2008a) and Spectral Clustering (Spectral) (Wang et al, 2006) are reported in Table 1. MDA achieves the best performance in terms of both correctness and completeness when the cluster number is chosen as 20, and outperforms HDP and spectral clustering. Note that the correctness is low when the cluster number is 2, since many trajectories of different collective behaviors have to be put into one cluster. The completeness is low when the cluster number is large, since trajectories of the same collective behaviors are divided into different clusters.

### 6.5 Abnormality Detection

We detect abnormal behaviors by measuring the likelihoods of trajectories with MDA, which are normalized by the lengths



**Fig. 9** Representative clusters of trajectories obtained with A)MDA, B)Spectral Clustering (Wang et al, 2006) and C)HDP (Wang et al, 2008a).

**Table 1** Completeness and correctness of MDA, and Spectral Clustering(Spectral) (Wang et al, 2006), with different numbers of clusters. The result of HDP(Wang et al, 2008a) is also shown. HDP automatically finds the number of clusters from data as 22.

Cluster Number		2	5	8	11	14	17	20	25	30
MDA	Completeness	0.82	0.71	0.73	0.52	0.51	0.61	<b>0.70</b>	0.62	0.59
	Correctness	0.21	0.75	0.80	0.92	0.92	0.91	<b>0.92</b>	0.95	0.97
Spectral	Completeness	0.83	0.60	0.50	0.43	0.42	0.39	0.36	0.27	0.26
	Correctness	0.51	0.80	0.87	0.89	0.90	0.91	0.91	0.92	0.95
HDP	Completeness	0.45(cluster number is 22)								
	Correctness	0.82(cluster number is 22)								

of trajectories. Figure 10A displays the top 50 abnormal trajectories with low normalized likelihoods. Two concrete examples of abnormality are shown in Figure 10B in zoom-in views. Their starting and ending points are marked with red and blue crosses in Figure 10A. The detected abnormal trajectories are mainly in two categories. (1) Pedestrians change their destinations in the middle way or loiter, such that their trajectories globally deviate from typical paths. (2) Pedestrians have abnormal speed. Our model has tolerance on the change of moving directions and speed in local regions, since linear dynamic systems allow Gaussian noise, whose covariance matrices are learned from data. Abnormality is detected only when significant global deviation happens. The abnormality detection results are reasonable given the proposed MDA. MDA does not model the interactions among pedestrians and therefore abnormal behaviors caused by interactions cannot be detected. On the other hand, the approaches (Pellegrini et al, 2009; Saligrama and Chen, 2012) of only modeling interactions of pedestrians cannot detect global abnormal behaviors. It is an interesting topic to integrate both types of models. Evaluating the usefulness of abnormality detection results aslo depends on applications scenarios.

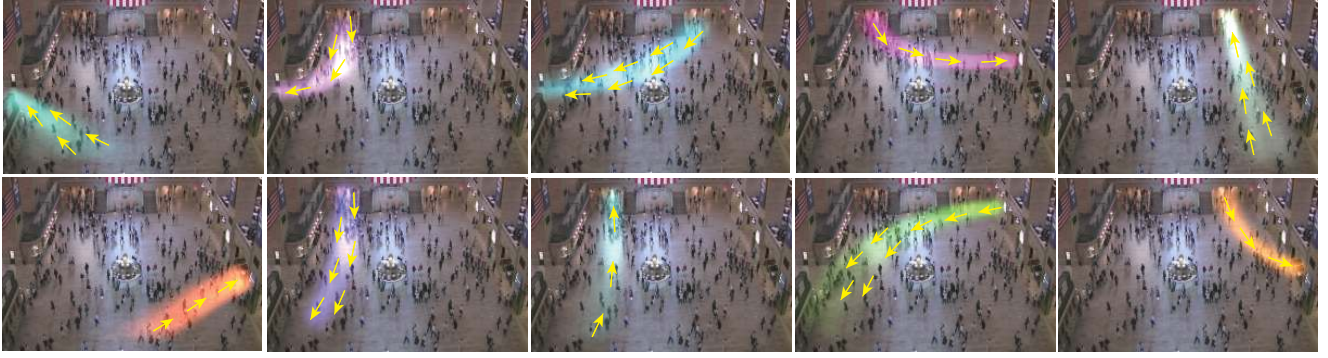
## 6.6 Semantic Region Generation

In video surveillance, there are a lot of works on learning semantic regions (Wang et al, 2008a; Makris and Ellis, 2005; Wang et al, 2008b; Zhou et al, 2011). Semantic regions correspond to paths commonly taken by objects, and the ac-

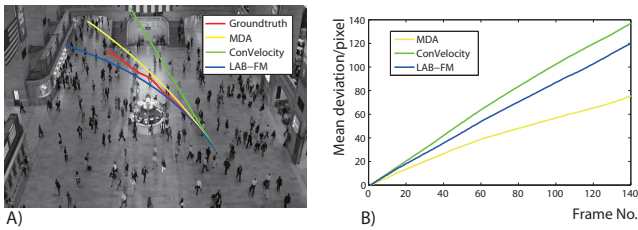
tivities observed in the same semantic region have similar semantic interpretation. Semantic regions could be used to improve object detection, classification and tracking (Kaucic et al, 2005; Wang and Wang, 2011). From the perspective of behavior analysis, semantic regions can be interpreted as *the temporal and spatial accumulation of trajectories generated by objects with shared belief and common movement dynamics in the scene*. MDA describes the generative process of semantic regions. Figure 11 shows the density distributions of ten semantic regions estimated from 1000 trajectories sampled from corresponding dynamic pedestrian-agents respectively. The distributions of paths converge and become denser towards the entry/exit regions in the back of the scene. They reflect the perspective distortion of the scene.

## 6.7 Behavior Prediction

MDA can predict pedestrians' behaviors given that their trajectories are only partially observed. We manually label 30 trajectories of pedestrians as ground-truth. For each ground-truth trajectory, we use the observations of the first 20 frames to estimate its pedestrian-agent index  $z$  with the algorithm in Table 5.2. Then, the model of the selected pedestrian-agent is used to recursively generate the following states as the predicted future trajectory. The performance is measured by the averaged prediction error, *i.e.* deviation between the predicted trajectories and the ground-truth trajectories. Two baseline methods are used for comparison. In the first comparison method(referred as ConVelocity), a constant veloc-



**Fig. 11** Density distributions of ten exemplar semantic regions estimated from trajectories sampled from MDA.

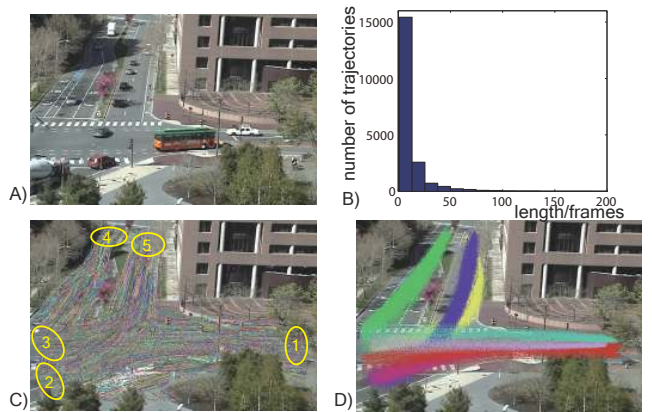


**Fig. 12** A) An example of predicting behaviors with different methods. B) The averaged prediction errors with different methods tested on 30 trajectories.

ity which is estimated as the averaged velocity of the past observations, is used to predict future positions. In the second comparison method LAB-FM (Lin et al, 2009), the learned flow field which best fit the first 20 frame observations, is used to predict future positions. Figure 12 show that MDA has better prediction performance.

## 6.8 Results on the MIT Traffic Dataset

We further test MDA on the MIT traffic Dataset (Wang et al, 2008b). The 25 minutes long video is 25 fps with a resolution of  $480 \times 720$ . 43,389 trajectories are extracted with the KLT tracker. Since majority of moving objects in the scene are vehicles, dynamic agents are vehicles moving on different roads in different directions. As shown in Figure 13B and C, most trajectories are short and highly fragmented due to occlusion and scene clutters. In Figure 13C, we label 5 entry/exit regions for initialization. Seven agent models are learned from data. Figure 13D shows trajectories simulated from the 7 agent components. They represent dominant motion patterns of vehicles. Figure 14A shows semantic regions estimated from MDA. Figure 14BCD show representative clusters obtained by MDA, spectral clustering (Wang et al, 2006), and HDP (Wang et al, 2008a). The clusters obtained by MDA better reflects the collective motion patterns in the scene.

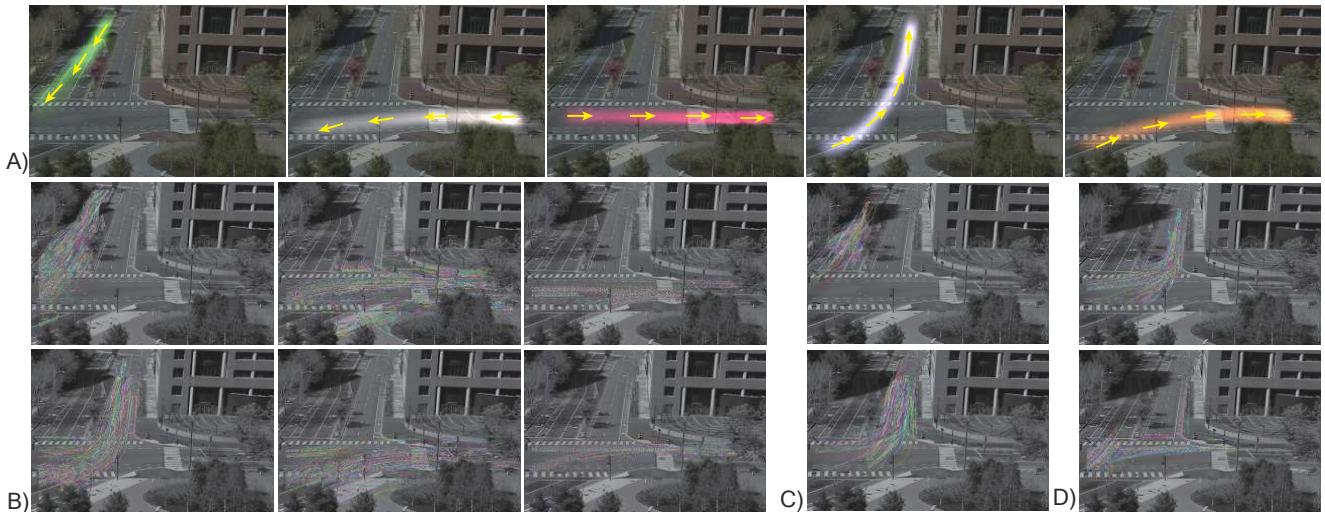


**Fig. 13** A) MIT Traffic Dataset. B) Histogram of trajectory lengths. C) Extracted trajectories and initialized entry/exit regions. D) Trajectories simulated from 7 agent models learned from fragmented trajectories.

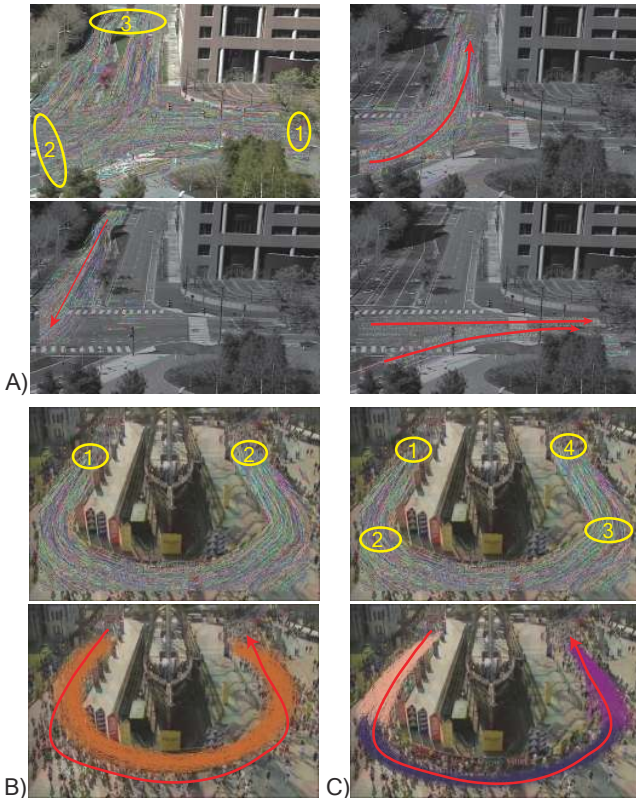
MDA has some robustness to the initialization for entry/exit regions. In Figure 15A we label 3 instead of 5 regions: regions 2 and 3 in Figure 15A are the superset of regions 2,3 and regions 4,5 in Figure 13C respectively. MDA can reasonably cluster trajectories into different dynamics as shown in the first two clusters of Figure 14A. However, rough labeling of entry/exit regions may also lead to merging similar motion patterns. The last cluster in Figure 13A shows that trajectories from two dynamic agents are clustered together with MDA.

## 6.9 Limitation and Extension of MDA

MDA assumes affine transform. Although it can represent many important geometric transforms as discussed in Section 3.1, MDA does have difficulty on some complex shapes such as u-turn or s-turn. In Figure 13B, we learn a single dynamic pedestrian-agent from the trajectories of a Marathon race video (Ali and Shah, 2007). The trajectories simulated from the learned MDA cannot well fit the real motion pattern indicated by the red trace, since this u-turn shape is not an



**Fig. 14** A) Density distributions of five semantic regions estimated from the learned MDA model. Representative clusters of trajectories obtained with B)MDA model, C)Spectral Clustering (Wang et al, 2006) and D)HDP (Wang et al, 2008a).



**Fig. 15** A) With only 3 initialized entry/exit regions in the MIT traffic scene, MDA still can reasonably cluster trajectories. Three representative clusters are shown. B) A single dynamic pedestrian-agent cannot well fit the real complex motion in the scene due to the limitation of affine transformation. C) 3 connected dynamic pedestrian-agents with different dynamics and shared starting and termination locations can well fit the real motion. Simulated trajectories from the same agent model are in the same color, and red trace indicates the real motion pattern.

affine motion. One possible extension is to decompose the complex motion pattern into multiple connected linear dynamic systems with affine motions. In Figure 13C, we label 4 entry/exit regions, and 3 dynamic pedestrian-agent components are learned with shared starting and terminating locations. The simulated trajectories from the three connected agent components well fit the real motion of Marathon race, if they can be connected. However, in order to use multiple agents to generate one trajectory, significant modification on MDA has to be made. This extension is related to switching linear dynamic models (Pavlovic et al, 1999), where multiple state transition parameters ( $\mathbf{A}$  and  $\mathbf{Q}$ ) are selected via a separate Markovian switching variable as time progresses in a single dynamic system. In our future work we would extend MDA to switching linear systems to model more complex motion patterns.

Here the number of agent components are empirically decided by the result in Table 1. We can also consider the number of agent components as one of the model parameters so that it can be automatically decided in the model inference. One way is to model the number of mixture as Dirichlet process then the joint distribution becomes the non-parametric bayesian model. The right number could be inferred by Markov Chain Monte Carlo or variational inference (Wang et al, 2008b, 2011; Hospedales et al, 2009).

MDA does not model local interactive dynamics among pedestrians, which is also an essential component of describing behaviors in crowd. MDA can be integrated with the social force models (Helbing and Molnar, 1995; Pellegrini et al, 2009) to characterize both the collective dynamics and interactive dynamics of crowd behaviors at both macroscopic and microscopic levels, since both are agent-based models. The local interactions from neighbors can be

added to the dynamic state transition process in Eq (1). It could lead to better accuracies on object tracking, behavior classification, simulation, and prediction.

MDA assumes that a pedestrian has a clear belief. Some pedestrians in the New York Grand Central Station simply wait or loiter in the scene without clear destination. Those behaviors cannot be well modeled with MDA.

## 7 Conclusion

In this paper, we propose a Mixture model of Dynamic Pedestrian-Agent to learn the collective dynamics from video sequences in crowded scenes. The collective dynamics of pedestrians are modeled as linear dynamic systems to capture long range moving patterns. Through modeling the beliefs of pedestrians and the missing states of observations, it can be well learned from highly fragmented trajectories caused by frequent tracking failures. Therefore, it is suitable for behavior analysis in crowded environments. By modeling the process of pedestrians making decisions on actions, it can not only classify collective behaviors, but also simulate and predict collective crowd behaviors. Various statistics valuable for traffic management and crowd control, such as flow fields, population density maps, flow transitions between sources and sinks, and semantic regions can be well estimated from simulation results.

MDA has more potential applications and extensions to be explored. In this work, we did not study the application of MDA to object tracking in crowd. Since MDA has the capability to predict future behaviors of objects based on partial observations, it can be used as prior for object tracking. Instead of being fixed, the dynamics parameters  $D$  and belief parameters  $B$  can also be dynamically updated over time by modeling their temporal dependency. It is known that variation of crowd density influences crowd behaviors. It is interesting to investigate how the dynamic and belief parameters change with crowd density.

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