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Exercise set 1

Continuity

Exercise 1.1

In Lecture 3 (27ix07) we defined the semantics $[\![\, \textbf{while}\ B\ \textbf{do}\ D\ \textbf{od}\,]\!]$ of a typical while-loop to be the *least fixed point* of the function

$$\mathcal{W}_{B,D}.f.s \quad := \quad f.([\![\, D\,]\!].s) \ \triangleleft [\![\, B\,]\!].s \triangleright \ s\ ,$$

where the bound variable f is of type "denotation of program" and s (also bound) is of type "program state." (Recall that $(\cdot \triangleleft \cdot \triangleright \cdot)$ is the "Hoare conditional.")

Before that, we had "installed" all the necessary apparatus to make that meaningful: partial orders, chains, least upper bounds… and continuity. Indeed we proved the general theorem that a continuous function F over a complete partial order (*cpo*) is guaranteed to have a least fixed point given by $(\sqcup_n F^n.\bot)$, and so concluded that in fact the meaning of the while loop above was equivalently $(\sqcup_n \mathcal{W}^n_{B,D}.\bot)$. This –as remarked in the lecture– supports the intuition that the effect of a loop can be approached by "unrolling" it n times, for higher and higher finite n. Here's the question:

How do we know (or ensure) that $\mathcal{W}_{B,D}$ is continuous?

Don't forget that *while*'s can be nested.

Exercise 1.2

Would it matter if $\mathcal{W}$ were *not* continuous? *Hint*: Check *Tarski* and *Boom* in the recommended reading.