6.977: Reasoning about sequential programs **CSAIL, MIT**
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Exercise set 4

Galois connections (again!) and deterministic probabilistic programs

Background

By Lecture 5 we should have defined both "operational" (forward, relational) and "logical" (backward, transformer-al) semantics for deterministic probabilistic programs:

$$\begin{array}{rll} & rp\colon & Syn \to S \to \overline{S} \\ \text{and} & wp\colon & Syn \to (\mathcal{E}S \leftarrow \mathcal{E}S)\ , \end{array}$$

where $\overline{S}$ is distributions over S and $\mathcal{E}S$ is *expectations* over S, that is non-negative real-valued bounded functions (random variables, actually). The definitions are inspired by Kozen's ground-breaking paper *A probabilistic PDL*.

In our usual overloading style, we will also have defined

$$\begin{array}{rlcl} & wp\colon & (S \to \overline{S}) & \to & (\mathcal{E}S \leftarrow \mathcal{E}S) \\ \text{and} & rp\colon & (\mathcal{E}S \leftarrow \mathcal{E}S) & \to & (S \to \overline{S})\ , \end{array} \tag{1}$$

for converting between the two semantics.[1]

Exercise 4.1

Refer to (1) above. What does $rp \circ wp$ do? If you try to prove it's the identity, are there any proof-generated healthiness conditions on the probabilistic deterministic relational space?

Exercise 4.2

Refer to (1) above. What does $wp \circ rp$ do? If you try to prove it's the identity, are there any proof-generated healthiness conditions on the probabilistic deterministic transformer space?

[1] Have a look at Fig. 5.9.1 on p.158 for a picture of this truly wanton re-use of the names *wp* and *rp*.