Ceilings and floors as Galois connections

Program

All variables are integers.

\[
\{N \geq 0 \land D \geq 1\} \quad \begin{align*}
n, d &:= N, D; \\
do \quad d \neq 0 \rightarrow \\
x: \lfloor n/d \rfloor \leq x \leq \lceil n/d \rceil; \quad // \text{Choose } x \text{ so that } \lfloor n/d \rfloor \leq x \leq \lceil n/d \rceil. \\
n, d &:= n-x, d-1; \\
o d \{N \text{ has been "evenly divided" into } D \text{ pieces of } x\}
\end{align*}
\]

Conjecture

\[d\lfloor N/D \rfloor \leq n \leq d\lceil N/D \rceil \text{ is invariant, whence every assignment to } x \text{ satisfies } \lfloor N/D \rfloor \leq x \leq \lceil N/D \rceil.\]

Proof

Working backwards, we have

\[d\lfloor N/D \rfloor \leq n \leq d\lceil N/D \rceil\]

through \(wp.(n,d := n-x,d-1)\) gives

\[d-1\lfloor N/D \rfloor \leq n-x \leq (d-1)\lceil N/D \rceil \equiv x + (d-1)\lfloor N/D \rfloor \leq n \leq x + (d-1)\lceil N/D \rceil\]

through \(wp.(x: \lfloor n/d \rfloor \leq x \leq \lceil n/d \rceil)\) gives

\[\lfloor n/d \rfloor + (d-1)\lfloor N/D \rfloor \leq n \leq \lfloor n/d \rfloor + (d-1)\lceil N/D \rceil \equiv n/d + (d-1)\lfloor N/D \rfloor \leq n \leq n/d + (d-1)\lceil N/D \rceil \]

“Galois property”

\[d \geq 1 \text{ from loop guard}^1\]

\[\equiv \lfloor N/D \rfloor \leq n/d \leq \lceil N/D \rceil \quad \text{"Galois property"}\]

\[\equiv \text{true}.\]

\[^1\text{Requires extra "housekeeping" invariant } d \geq 0.\]