

## Transformer semantics in your pocket

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The state space is  $S$ ; the type of program denotations is  $\mathbb{P}S \leftarrow \mathbb{P}S$ , where we write the function-space-forming arrow backwards to emphasise that it takes post-conditions to (weakest) preconditions.

Here is the transformer semantics of our simple imperative, nondeterministic language. It's not as direct as the relational, but (some would argue) is actually simpler.

$$\begin{aligned} wp.\llbracket \mathbf{abort} \rrbracket.post &:= \emptyset \\ wp.\llbracket \mathbf{skip} \rrbracket.post &:= post \\ s \in wp.\llbracket \mathbf{assign} \ E \rrbracket.post &:= \llbracket E \rrbracket.s \in post \\ wp.\llbracket S; T \rrbracket &:= wp.\llbracket S \rrbracket \circ wp.\llbracket T \rrbracket \\ wp.\llbracket S \triangleleft B \triangleright T \rrbracket.post &:= \\ &wp.\llbracket S \rrbracket.post \triangleleft \llbracket B \rrbracket \triangleright wp.\llbracket T \rrbracket.post \quad ^1 \\ wp.\llbracket S \sqcap T \rrbracket.post &:= wp.\llbracket S \rrbracket.post \cap wp.\llbracket T \rrbracket.post \\ wp.\llbracket \mathbf{while} \ B \ \mathbf{do} \ D \ \mathbf{od} \rrbracket.post &:= \\ &(\mu pre \cdot (wp.\llbracket D \rrbracket.pre \triangleleft \llbracket B \rrbracket \triangleright post)) \end{aligned}$$

- Command  $S \sqcap T$  is the pure nondeterministic choice between  $S$  and  $T$ .
- In general  $(\mu x \cdot E)$  means the least fixed point of function  $(\lambda x \cdot E)$ .

*Do all the definitions preserve healthiness, i.e. strictness, monotonicity and positive conjunctivity?*

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<sup>1</sup>Here for fun we use a set-theoretic conditional  $A \triangleleft B \triangleright C := (B \cap A) \cup (\overline{B} \cap C)$ .