A Geometric Approach for Encoding Demonstrations and Learning New Trajectories

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***Abstract*—We propose a novel learning approach based on differential geometry to extract and encode important characteristics of a set of trajectories captured through demonstrations. The proposed approach represents the trajectories using a surface in Euclidean space called *Canal Surface*. The surface is formed as the envelope of a family of regular implicit surfaces (e.g. spheres) whose centers lie on a space curve. Canal surfaces extract the essential aspects of the demonstrations and retrieve a generalized form of the trajectories while maintaining the extracted constraints. Given a random initial pose in task space, a new trajectory is reproduced by considering the relative ratio of the initial point with respect to the corresponding cross-section of the obtained canal surface. Our approach produces a continuous representation which is visually perceivable and easily understandable even by non-expert users. Preliminary experimental results using real-world data are presented.**

I. INTRODUCTION

The ultimate goal of Learning from Demonstration (LfD) approaches is to enable even non-expert users to teach new skills to robots through demonstration. The robot should be able to learn the skill and generalize it to novel situations autonomously. However, it has been shown that current robotic platforms are not good at being autonomous and need human assistance constantly[1]. Therefore, ignoring the presence of human teacher after the demonstration phase, which is a common case, becomes one of the main drawbacks of the existing LfD approaches. In fact, the existing representations are so complicated that make it almost impossible for non-expert users to diagnose a failure or an undesirable behavior and resolve the issue. A possible solution, however, is to employ an encoding process that is both powerful enough to learn and generalize the task robustly and is also readily comprehensible for non-experts. Such learning approach should take advantage of teacher's feedback not only for providing new demonstrations but also for updating and adjusting the learned model in the loop.

During the past two decades, several LfD approaches have been developed [1]. Many of them use regression-based techniques to represent the given set of demonstrations using a probabilistic representation. The approach proposed by Calinon *et al.,* uses Gaussian Mixture Model (GMM) to represent the demonstrations and Gaussian Mixture Regression (GMR) to retrieve a smooth trajectory [2]. Similarly, Gaussian Process Regression (GPR) was proposed to generalize over a set of demonstrated trajectories [4]. Later local Gaussian process regression was employed to extract the constraint of a demonstrated skill [9]. Another approach similar to Gaussian Process called *LfD by Averaging Trajectories* (LAT), was introduced in [8]. LAT is based on the product of normal distributions estimated from trajectories relative to the objects observed during the demonstrations. Both LAT and GPs cannot extract constraints of the demonstrated skills. In addition, the representations generated by such probabilistic approaches in most cases are not easily understandable by non-experts. Ijspreet *et al.,* showed that dynamical systems can also be used to encode and reproduce trajectories [6]. Dynamic Movement Primitives (DMPs) represent the demonstrated trajectories as movements of a particle subject to a set of damped linear spring systems perturbed by an external force. The shape of the movement is approximated using Gaussian basis functions and the weights are calculated using locally weighted regression. However, the boundaries and the state-space formed by DMPs representation are not visually perceivable. To reflect a human's intention, Dong and Williams proposed a representation of continuous actions (i.e. trajectories) by extracting covariance data which is called probabilistic flow tubes [3]. Using binary contact information and state of the environment during the demonstrations all the sequences are matched temporally. Their representation can be seen as a special case of our approach in which the cross-section is formed using covariance data. An approximation of a boundary around a trajectory, which can be visualized as a funnel, was used to tackle the problem of real-time motion planning [7]. A library of funnels and their corresponding open-loop controllers are computed off-line and a closed-loop system was used to generate trajectories from the library in real-time that can deal with obstacles. In this paper, we propose a novel geometric LfD approach to encoding a trajectory-based skill as a geometric model composed of a regular curve and a surface in 3D Cartesian space called a *Canal Surface*. The constructed canal surface represents the main features of the skill and its constraints. Since the encoded skills using canal surfaces are visually perceivable and easily understandable, they enable even non-expert users to provide feedback to improve the quality of the learned skill.

II. CANAL SURFACE

Canal Surfaces, also known as Generalized Cylinders [10], play a fundamental role in descriptive geometry. In the context of Computer Aided Graphic Design (CAGD), they are

[1]For instance, see results from the recent DARPA Robotic Challenge at http://www.theroboticschallenge.org/

used for the construction of smooth blending surfaces, shape reconstruction, and transition surfaces between pipes [5].

A canal surface, $\mathcal{C}_u$, is defined as an envelope[2] of the one-parameter pencil[3] of spheres and can be written as

$$\mathcal{C}_u : f(\mathbf{x}; u) := \{(\mathbf{c}(u), r(u)) \in \mathbb{R}^4 | u \in \mathbb{R}\}, \qquad (1)$$

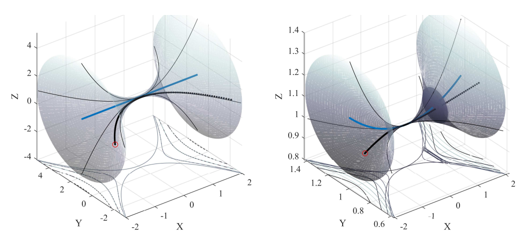
where the spheres are centered on a regular curve $\Gamma : \mathbf{x} = \mathbf{c}(u) \in \mathbb{R}^3$ in Cartesian space. The radius of the spheres are given by the function $r(u) \in \mathbb{R}$, which is a C^1-function. The non-degeneracy condition is satisfied by assuming $r > 0$ and $|\dot{r}| < \|\dot{\mathbf{c}}\|$ [5]. In Differential Geometry, Γ is known as the *spine curve* or *directrix* and $r(u)$ is called the *radii* function. For the one-parameter pencil of **spheres**, Equation 1 can be written as

$$\mathcal{C}_u : f(\mathbf{x}; u) := (\mathbf{x} - \mathbf{c}(u))^2 - r(u)^2 = 0. \qquad (2)$$

Parameterizing Equation 2 using an orthonormal frame such as Frenet-Serret (also called TNB frame), for each s value of the arc-length of the directrix, the cross-section is a circle orthogonal to the tangent of the directrix. Thus the canal surface can be represented as a set of circles

$$\mathcal{C}_s : f(s, v) = \mathbf{c}(s) + \qquad (3)$$
$$r(s)\left(-\mathbf{e}_T \frac{dr}{ds} + \sqrt{1 - (\frac{dr}{ds})^2}\,(\mathbf{e}_B \sin(v) - \mathbf{e}_N \cos(v))\right),$$

where v represents the arc-length of the circles and $\mathbf{e}_T, \mathbf{e}_N, \mathbf{e}_B$ denote the unit normal vectors of the TNB frame. In a more general form, the cross-section may vary in shape and size when the TNB frame translates along the directrix. Also, cross-sections can be represented using different shapes and techniques such as polygons, polynomials, and B-splines[10]. Such characteristics make canal surfaces a suitable candidate for encoding complicated constraints of trajectory-based skills captured through demonstrations.



(a) four demonstrations (b) three demonstrations

Fig. 1: Canal surfaces with circular and elliptical cross-sections.

III. Skill Learning using Canal Surfaces

We assume that n different demonstrations of the same task are performed and captured in task-space. Any demonstration technique such as kinesthetic teaching, teleoperation, and

[2]A surface tangent to each member of a family of surfaces in 3D space is called an envelope.

[3]A pencil is a family of geometric objects sharing a common property.

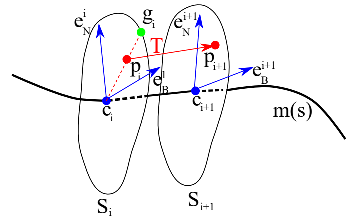


Fig. 2: A single step during reproduction.

shadowing can be employed. For each demonstration the 3D Cartesian position of the robot's end-effector is recorded over time as $\hat{\boldsymbol{\xi}}^j = \{\xi_1^j, \xi_2^j, \xi_3^j\} \in \mathbb{R}^{3 \times T^j}$, where $j = 1 \ldots n$ denotes the j^{th} demonstration including T^j points. To gain a frame-by-frame correspondence mapping among the recorded demonstrations and align them temporally, for each demonstration firstly a set of piecewise polynomials is obtained using cubic spline interpolation. Then a set of temporally aligned trajectories is generated by resampling from the obtained polynomials. Another advantage of this technique is that when the velocity and acceleration data are unavailable, the smoothed first and second derivatives of the obtained piecewise polynomials can be used instead. This process gives the set of n resampled demonstrations $\boldsymbol{\xi} \in \mathbb{R}^{3 \times N \times n}$ each of which consists of N data-points. An alternative widely used solution is employing the Dynamic Time Warping method.

Estimating the Directrix: To estimate the backbone curve or the directrix, we can simply calculate the directional mean value for the given set of demonstrations. Let $\mathbf{m} \in \mathbb{R}^{3 \times N}$ be the arithmetic mean of $\boldsymbol{\xi}$. Note that $\mathbf{m}$ is the space curve that all the spheres are centered on to form a canal surface. Alternatively, the directrix can be produced using GMR by sampling from a statistical model learned using GMM.

Estimating the Radii: In its simplest form, the radii function of a canal surface with a circular cross-section can be calculated by measuring the distance from each point on the directrix to the corresponding points on the demonstrated trajectories. The maximum distance is then used to make a circle that bounds other points as well. We could also use elliptical cross-sections instead. The advantage is that the obtained cross-section can cover a smaller yet more reasonable area while maintaining all the implicit local constraints of the task more efficiently. In a more general form, cross-sections can be represented by different shapes using B-splines that provide a powerful tool for fitting a smooth curve to a set of key-points. Given the set of demonstrations, at each arc-length, the cross-section can be estimated by fitting a closed B-spline to the data. In Figure 1a, a canal surfaces estimated from a set of four demonstrations is shown. The directrix and the demonstrations are depicted in blue and gray respectively. The main features of the movement are extracted. For instance, the narrow region in the middle of the canal indicates that all the reproduced trajectories must satisfy this constraint. After removing one of the demonstrations, the updated canal surface is shown in Figure 1b. It can be seen that the shape of the cross-section and the directrix are adapted to the new constraints. The cross-section is elliptical and smaller at the bottom and the directrix is not a straight line anymore.

Reproduction: During the reproduction phase, the initial position of the end-effector p_1 is used as input. Starting from the initial pose, we measure the ratio η by calculating the distance from the directrix $\overline{p_1 c_1}$ and its corresponding distance from the boundary of the canal surface at the corresponding cross-section $\overline{g_1 c_1}$. We then use η to generate the next pose of the end-effector by transforming the current point from the current TNB frame to the next. An illustration of a single-step reproduction process using this *ratio rule* can be seen in Figure 2. The ratio rule ensures that the essential characteristics of the demonstrated skill are applied to the reproduced time-independent trajectory. Figure 1 depicts two trajectories (in black) reproduced from arbitrary initial poses using the ratio rule.

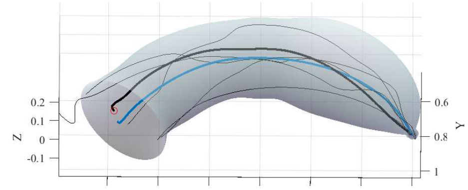
IV. Experiment and Comparison

To validate the capabilities and interpretability of the proposed approach, we performed a real-world experiment. First, we captured five demonstrations from a 6DOF Jaco^2 robotic arm through kinesthetic teaching (gray curves in Figure 3a). The obtained canal surface with elliptical cross-section is illustrated in Figure 3a. The reproduced trajectory (black curve) in Figure 3a is generated using the ratio rule. Then we applied GMM/GMR as presented in [2] with 4 Gaussian components to the set of demonstrations. Figure 3b shows the results. The comparison in Figure 3c reveals that the reproduced trajectory by GMR is similar to the directrix of the modeled canal surface. The obtained representation by using our approach is visually descriptive and easily understandable even for non-experts. This feature enables end-users to evaluate the given set of demonstrations and improve the learned model by providing proper feedback (e.g. verbal, physical). Therefore, unlike many existing LfD approaches including GMM/GMR, our approach is capable of keeping the user in the loop. Another advantage is that our approach reproduces time-independent trajectories from various initial poses. However, GMR reproduces time-based trajectories and requires an additional component to generate trajectories from an arbitrary initial pose.

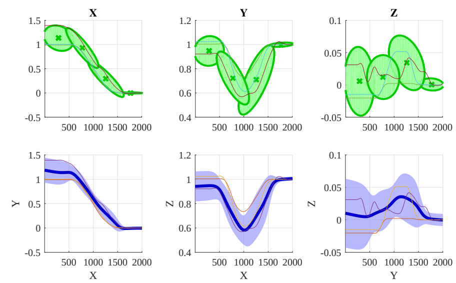
V. Discussion and Future Work

In order to use human feedback inside the loop for adjusting the learned model, our future work includes activating the robot in compliant control mode to enable users to interact and correct the robot's movements even during the reproduction phase. The learned model can be actively updated and the new reproduced trajectories will reflect the given corrections in the form of new constraints on the surface.

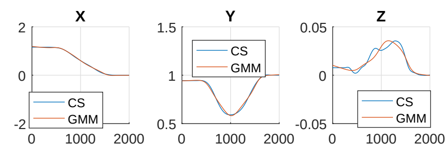
Furthermore, although the proposed approach is capable of encoding and reproducing trajectories on its own, it can be combined with other methods such as DMPs and keyframe-based LfD. Integration with DMPs can provide us with a different reproduction method while keyframe-based LfD enables the system to take advantage of smoother trajectories while retaining their important key-points.



(a) The canal surface, demonstrations (gray), directrix (blue), and the reproduced trajectory (black).



(b) result from GMM/GMR.



(c) The directrix vs. GMR reproduction.

Fig. 3: Comparing the proposed approach and GMM/GMR.

References

[1] Brenna D. Argall, Sonia Chernova, Manuela Veloso, and Brett Browning. A survey of robot learning from demonstration. *Robotics and autonomous systems*, 57(5):469–483, 2009.

[2] Sylvain Calinon, Florent Guenter, and Aude Billard. On learning, representing, and generalizing a task in a humanoid robot. *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on*, 37 (2):286–298, 2007.

[3] Shuonan Dong and Brian Williams. Learning and recognition of hybrid manipulation motions in variable environments using probabilistic flow tubes. *International Journal of Social Robotics*, 4(4):357–368, 2012.

[4] David B. Grimes, Rawichote Chalodhorn, and Rajesh PN Rao. Dynamic imitation in a humanoid robot through nonparametric probabilistic inference. In *Robotics: science and systems*, pages 199–206. Cambridge, MA, 2006.

[5] E Hartmann. Geometry and algorithms for computer aided design. *Darmstadt University of Technology*, 2003.

[6] Auke J. Ijspeert, Jun Nakanishi, Heiko Hoffmann, Peter Pastor, and Stefan Schaal. Dynamical movement primitives: learning attractor models for motor behaviors. *Neural computation*, 25(2):328–373, 2013.

[7] Anirudha Majumdar and Russ Tedrake. Funnel libraries for real-time robust feedback motion planning. *arXiv preprint: 1601.04037*, 2016.

[8] Benjamin Reiner, Wolfgang Ertel, Heiko Posenauer, and Markus Schneider. Lat: A simple learning from demonstration method. In *Intelligent Robots and Systems (IROS), 2014 IEEE/RSJ International Conference on*, pages 4436–4441. IEEE, 2014.

[9] Markus Schneider and Wolfgang Ertel. Robot learning by demonstration with local gaussian process regression. In *Intelligent Robots and Systems (IROS), 2010 IEEE/RSJ International Conference on*, pages 255–260. IEEE, 2010.

[10] Uri Shani and Dana H Ballard. Splines as embeddings for generalized cylinders. *Computer Vision, Graphics, and Image Processing*, 27(2): 129–156, 1984.