Go With The Flow

Optical Flow-based Transport for Image Manifolds

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Concise Models

- Our interest in this talk: Ensembles of articulating images
  - translations of an object
    \( \theta \): x-offset and y-offset
  - wedgelets
    \( \theta \): orientation and offset
  - rotations of a 3D object
    \( \theta \): pitch, roll, yaw

- Image articulation manifold

\[ \mathcal{M} = \{ I_\theta : \theta \in \Theta \} \]
Image Articulation Manifold

- $N$-pixel images: $I \in \mathbb{R}^N$

- $K$-dimensional articulation space

- Then $\mathcal{M} = \{I_\theta : \theta \in \Theta\}$ is a $K$-dimensional “image articulation manifold” (IAM)

- Submanifold of the ambient space
Image Articulation Manifold

- $N$-pixel images: $I \in \mathbb{R}^N$

- **Local isometry:**
  image distance $\propto$ parameter space distance

- **Linear tangent spaces**
  are close approximation locally

articulation parameter space \( \Theta \)
Image Articulation Manifold

- $N$-pixel images: $I \in \mathbb{R}^N$

- Local isometry:
  image distance $\propto$
  parameter space distance

- **Linear tangent spaces**
  are close approximation locally
Theory/Practice Disconnect

• Practical image manifolds are **not smooth**

• If images have sharp edges, then manifold is everywhere **non-differentiable**

[Donoho, Grimes, 2003]
Theory/Practice Disconnect – 1

• Lack of isometry

• Local image distance on manifold should be proportional to articulation distance in parameter space

• But true only in toy examples

• Result: poor performance in classification, estimation, tracking, learning, ...
Theory/Practice Disconnect – 2

- **Lack of local linearity**
  - Local image neighborhoods assumed to form a **linear tangent subspace** on manifold
  - But true only for extremely small neighborhoods
  - Result: **cross-fading** when synthesizing images that should lie on manifold
A New Model for Image Manifolds

Key Idea: model the IAM in terms of 
Transport operators

\[ I_\theta = f \circ I_{\theta_{\text{ref}}} \]

For example:

\[ I_\theta(x) = I_{\theta_{\text{ref}}} (f(x)) \]
Optical Flow

- Given two images $I_1$ and $I_2$, we seek a displacement vector field $f(x, y) = [u(x, y), v(x, y)]$ such that

$$I_2(x, y) = I_1(x + u(x, y), y + v(x, y))$$

- **Linearized brightness constancy**

$$I_2(x, y) = I_1(x, y) + (\nabla_x I_1)u(x, y) + (\nabla_y I_1)v(x, y)$$
Optical Flow Manifold (OFM)

- Consider a reference image $I_{\theta_0}$ and a $K$-dimensional articulation.
- Collect optical flows from $I_{\theta_0}$ to all images reachable by a $K$-dimensional articulation. Call this the optical flow manifold (OFM).
- Provides a transport operator to propagate along manifold.
OFM: Example

Reference Image
OFM: Properties

- **Theorem:** Collection of OFs (OFM) is a **smooth** $K$-dimensional submanifold of $\mathbb{R}^{2N}$ [S,H,N,B,2011]

- **Theorem:** OFM is **isometric** to Euclidean space $\mathbb{R}^K$ for a large class of IAMs [S,H,N,B,2011]
OFM = ‘Nonlinear’ Tangent Space
App 1: Image Synthesis

Training Images

Value in Euclidean reference

Synthesized Images

Value in Euclidean reference
App 2: Manifold Learning

2D rotations

Reference image

Embedding of OFM
App 2: Manifold Learning

Data
196 images of two bears moving linearly and independently

Task
Find low-dimensional embedding

IAM

OFM
App 3: Karcher Mean Estimation

- Point on the manifold such that the sum of squared geodesic distances to every other point is minimized
- Important concept in nonlinear data modeling, compression, shape analysis

[Srivastava et al]
Summary

• Manifolds: **concise model** for many image processing problems involving **image collections** and multiple sensors/viewpoints

• But practical image manifolds are non-differentiable
  – manifold-based algorithms have not lived up to their promise

• **Optical flow manifolds** (OFMs)
  – smooth even when IAM is not
  – OFM ~ nonlinear tangent space
  – support accurate image synthesis, learning, charting, ...
Open Questions

• Our treatment is specific to **image manifolds** under brightness constancy

• What are the natural transport operators for **other data manifolds**?
Optical Flow

\[ I_2(x, y) = I_1(x + u(x, y), y + v(x, y)) \]

\[ I_2(x, y) = I_1(x, y) + (\nabla_x I_1)u(x, y) + (\nabla_y I_1)v(x, y) \]
Limitations

• Brightness constancy
  – Optical flow is no longer meaningful

• Occlusion
  – Undefined pixel flow in theory, arbitrary flow estimates in practice
  – Heuristics to deal with it

• Changing backgrounds etc.
  – Transport operator assumption too strict
  – Sparse correspondences?
Pairwise distances and embedding
Occlusion

- Detect occlusion using forward-backward flow reasoning

- Remove occluded pixel computations

- **Heuristic** --- formal occlusion handling is hard
History of Optical Flow

• Dark ages (<1985)
  – special cases solved
  – LBC an under-determined set of linear equations

• Horn and Schunk (1985)
  – Regularization term: smoothness prior on the flow

• Brox et al (2005)
  – shows that linearization of brightness constancy (BC) is a bad assumption
  – develops optimization framework to handle BC directly

  – practical systems with reliable code