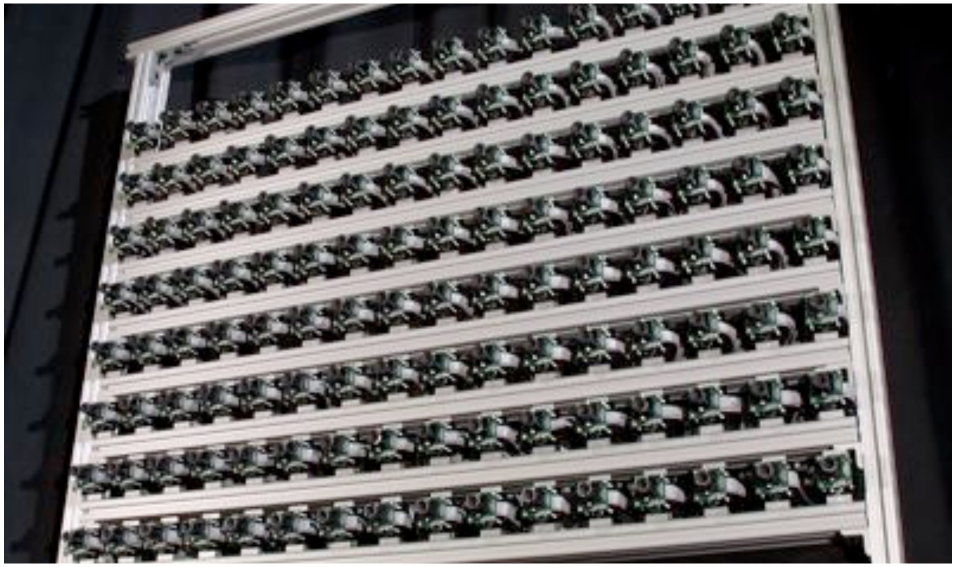
Go With The Flow

Optical Flow-based Transport
for Image Manifolds

Chinmay Hegde
Rice University

Richard G. Baraniuk **Aswin Sankaranarayanan** **Sriram Nagaraj**

Sensor Data Deluge

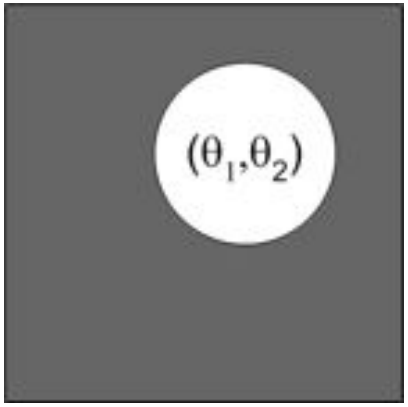


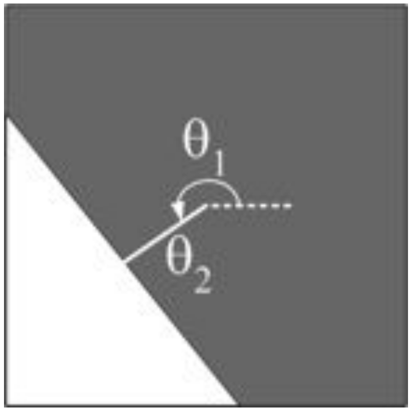
Concise Models

- Our interest in this talk:

  Ensembles of **articulating** images

  - translations of an object
    θ: x-offset and y-offset
  - wedgelets
    θ: orientation and offset
  - rotations of a 3D object
    θ: pitch, roll, yaw





- **Image articulation manifold**

$$\mathcal{M} = \{I_\theta \,:\, \theta \in \Theta\}$$

Image Articulation Manifold

- *N*-pixel images: $I \in \mathbf{R}^N$

- *K*-dimensional articulation space

- Then $\mathcal{M} = \{I_\theta \, : \, \theta \in \Theta\}$ is a *K*-dimensional "image articulation manifold" (**IAM**)

- Submanifold of the ambient space

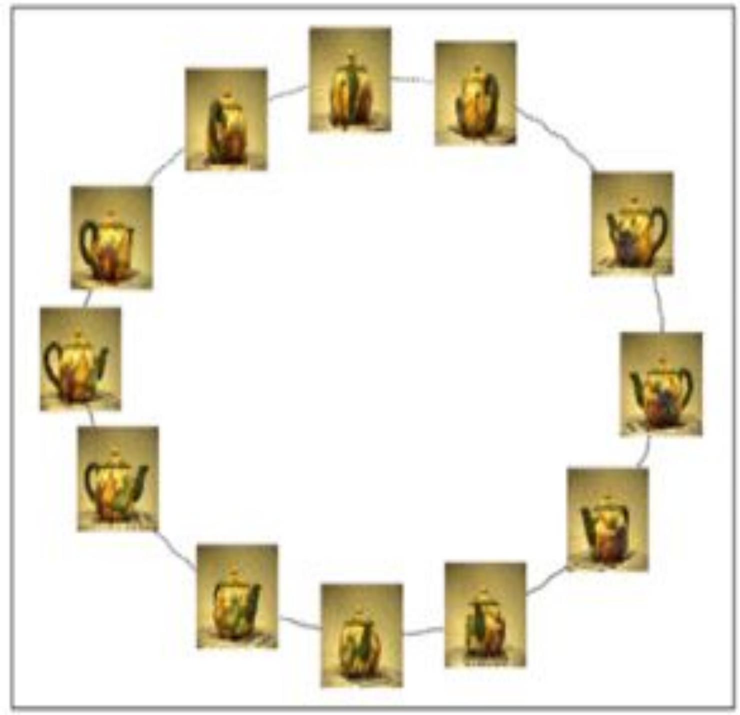
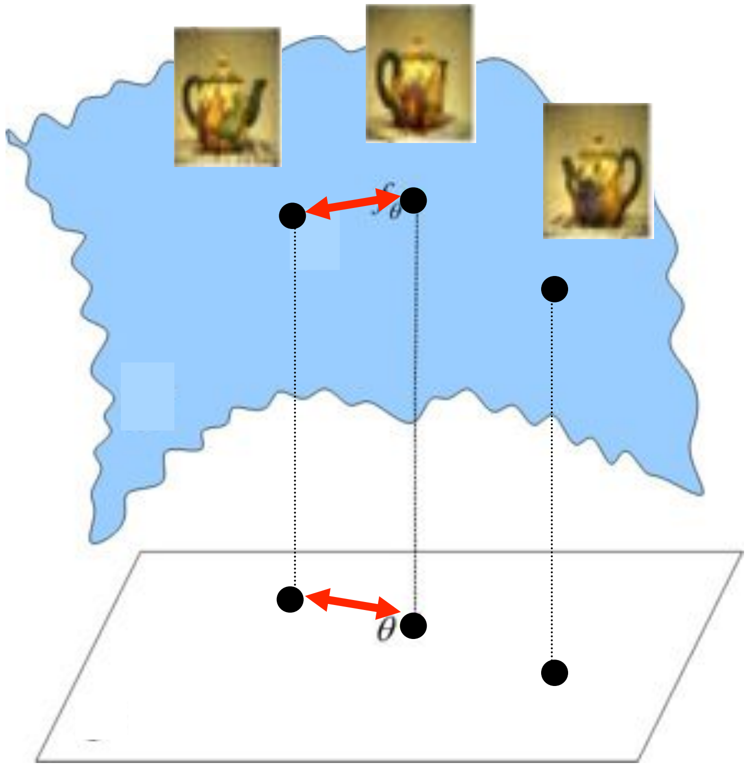


Image Articulation Manifold

- *N*-pixel images: $I \in \mathbf{R}^N$

- **Local isometry**:
  image distance $\propto$
  parameter space distance

- Linear tangent spaces
  are close approximation
  locally



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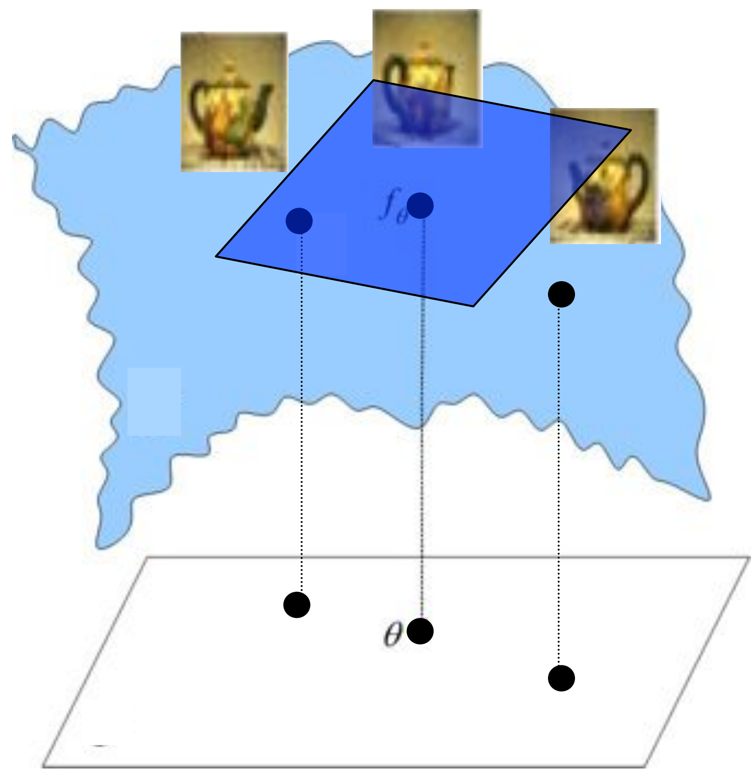


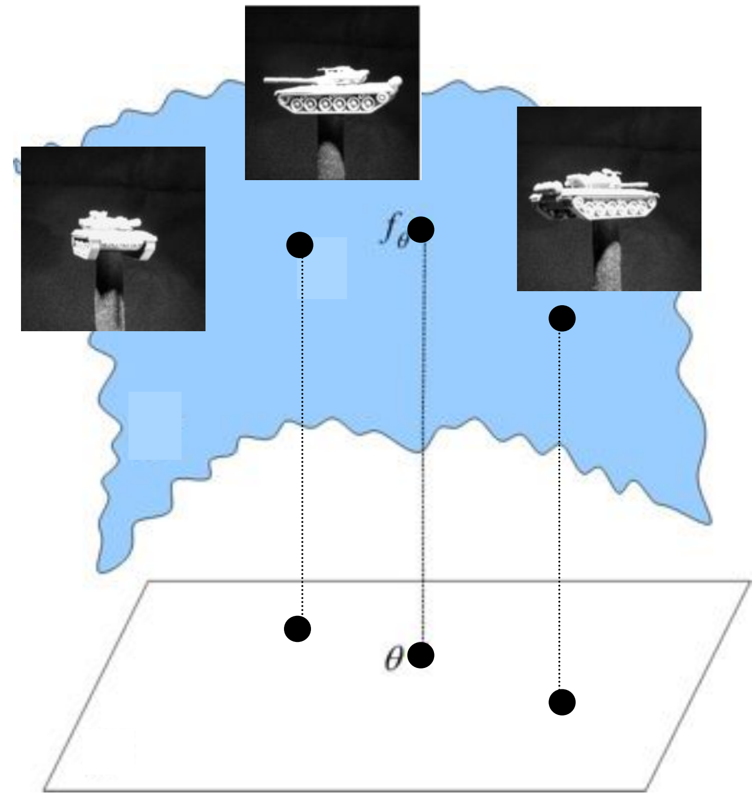
articulation parameter space Θ

Theory/Practice Disconnect

- Practical image manifolds are **not smooth**

- If images have sharp edges, then manifold is everywhere **non-differentiable**

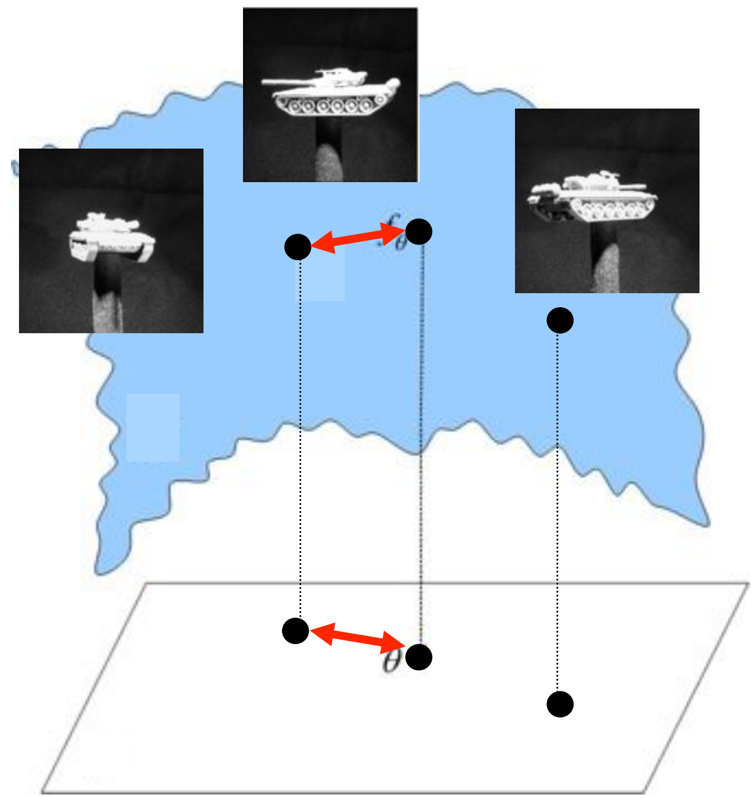
[Donoho, Grimes,2003]



articulation parameter space Θ

Theory/Practice Disconnect – 1

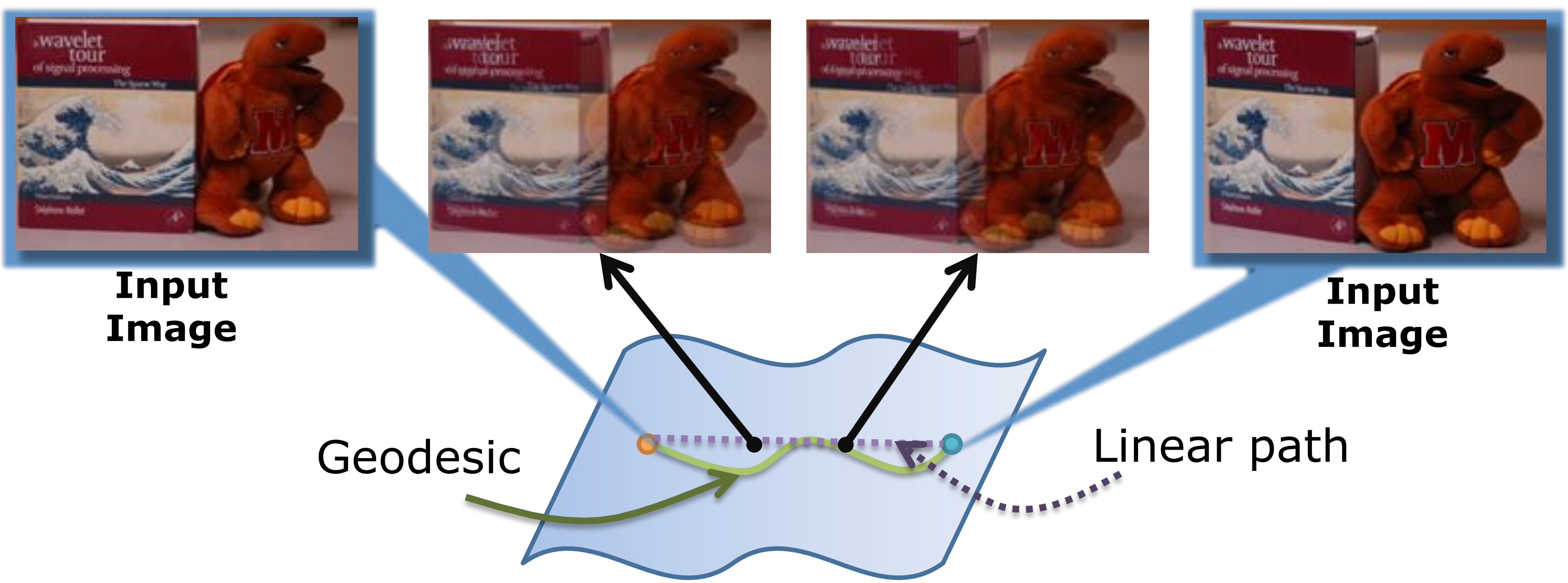
- **Lack of isometry**
- Local image distance on manifold should be proportional to articulation distance in parameter space
- But true only in toy examples
- Result: poor performance in classification, estimation, tracking, learning, …



articulation parameter space Θ

Theory/Practice Disconnect – 2

- **Lack of local linearity**
- Local image neighborhoods assumed to form a **linear tangent subspace** on manifold
- But true only for extremely small neighborhoods
- Result: **cross-fading** when synthesizing images that should lie on manifold



A New Model for Image Manifolds

Key Idea: model the IAM in terms of
Transport operators

$$I_\theta = f \circ I_{\theta_{ref}}$$

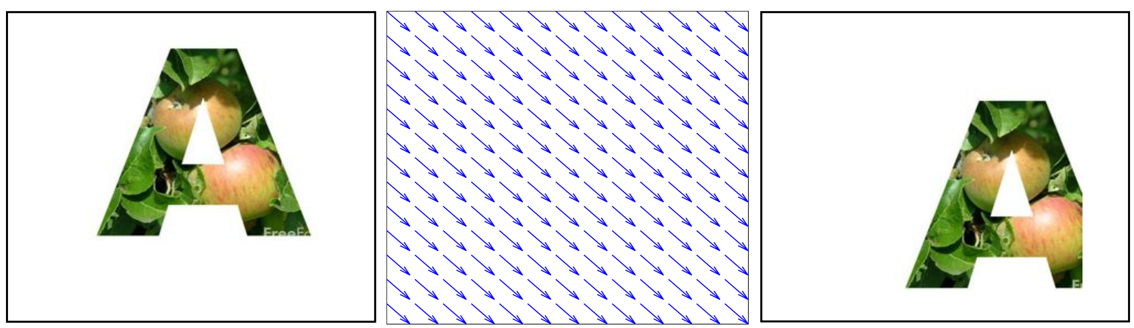
For example:

$$I_\theta(x) = I_{\theta_{ref}}(f(x))$$

Optical Flow

- Given two images I_1 and I_2, we seek a displacement vector field
  $f(x, y) = [u(x, y), v(x, y)]$ such that

$$I_2(x, y) = I_1(x + u(x, y), y + v(x, y))$$

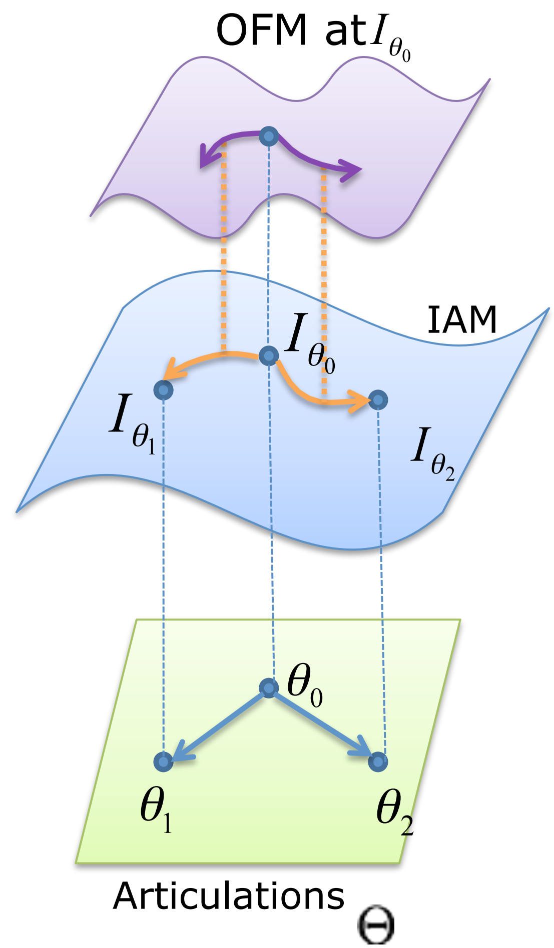


- **Linearized brightness constancy**

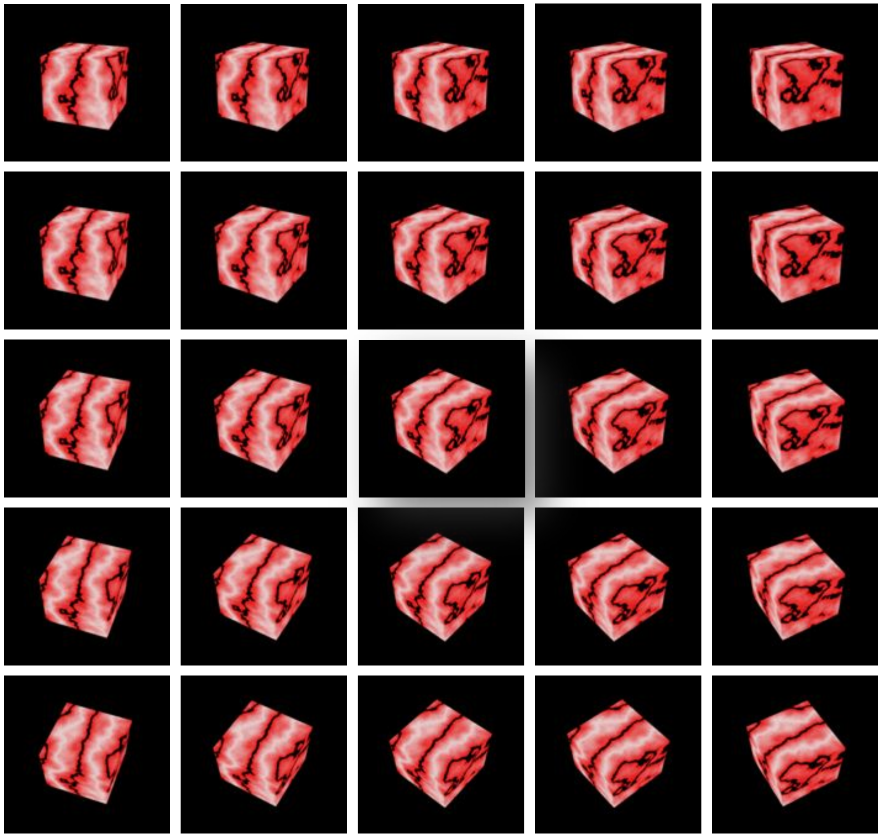
$$I_2(x, y) = I_1(x, y) + (\nabla_X I_1) u(x, y) + (\nabla_Y I_1) v(x, y)$$

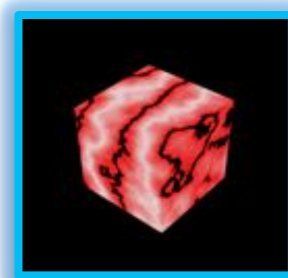
Optical Flow Manifold (OFM)

- Consider a reference image I_{θ_0} and a *K*-dimensional articulation
- Collect optical flows from I_{θ_0} to all images reachable by a *K*-dimensional articulation. Call this the *optical flow manifold* **(OFM)**
- Provides a **transport operator** to propagate along manifold

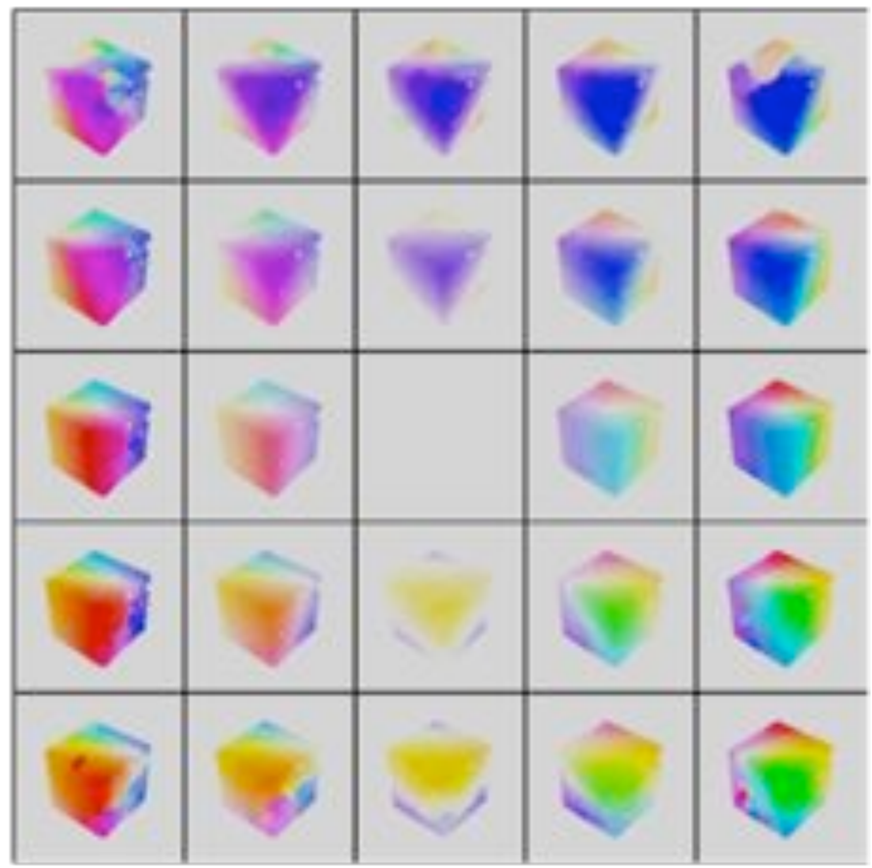


OFM: Example





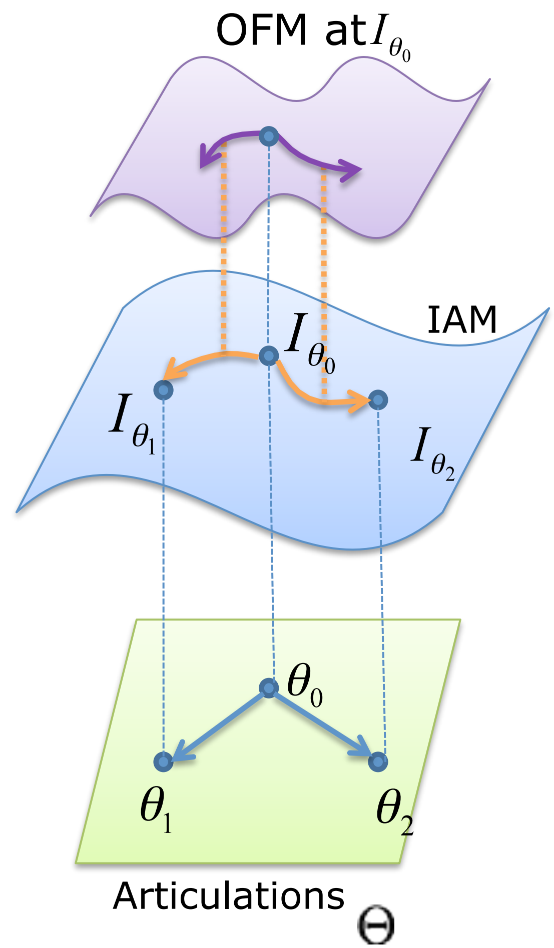
Reference Image



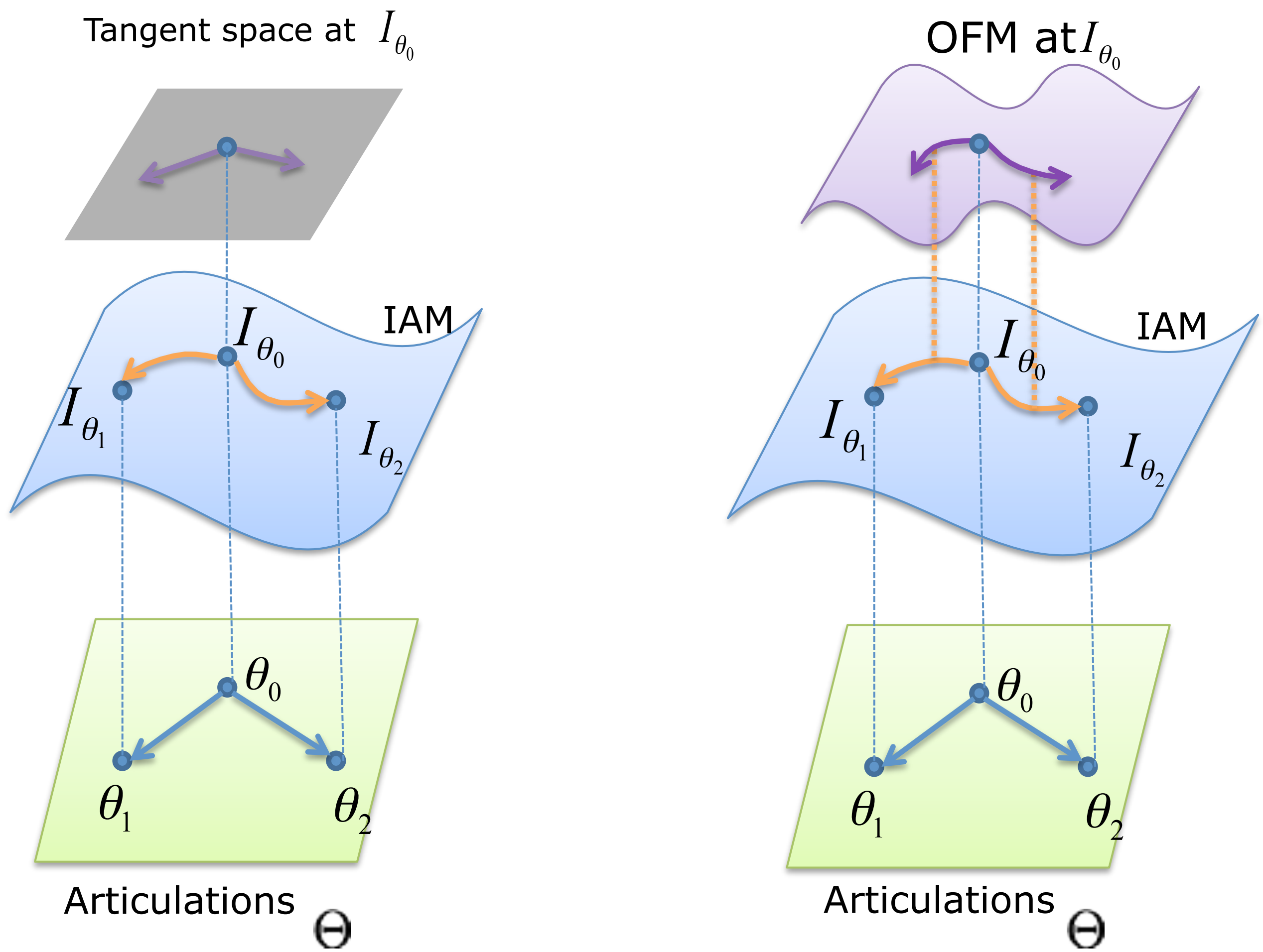
OFM: Properties

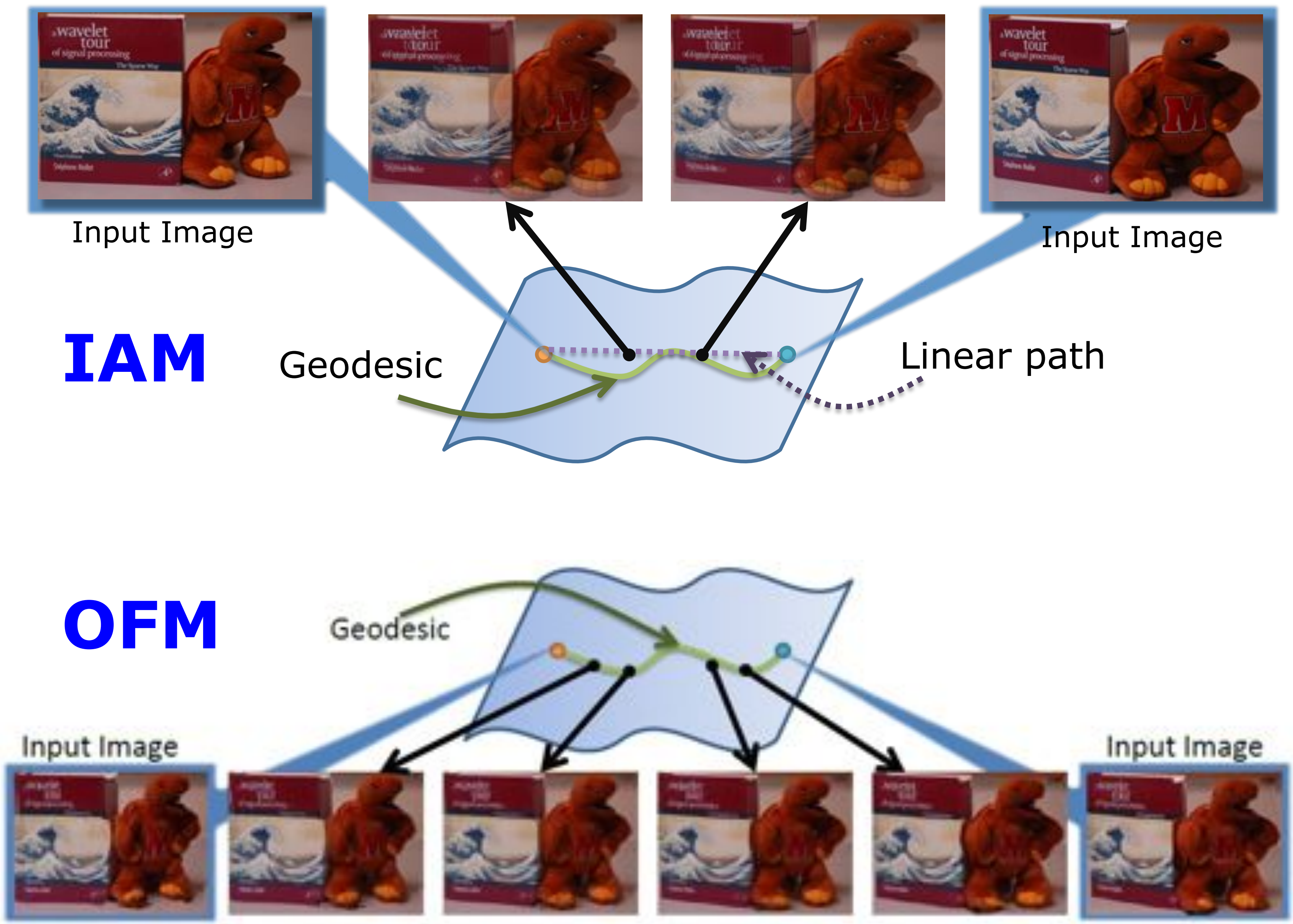
- **Theorem:** Collection of OFs (OFM) is a **smooth *K*-dimensional** submanifold of R^{2N}
  [S,H,N,B,2011]

- **Theorem**: OFM is **isometric** to Euclidean space R^{K} for a large class of IAMs
  [S,H,N,B,2011]

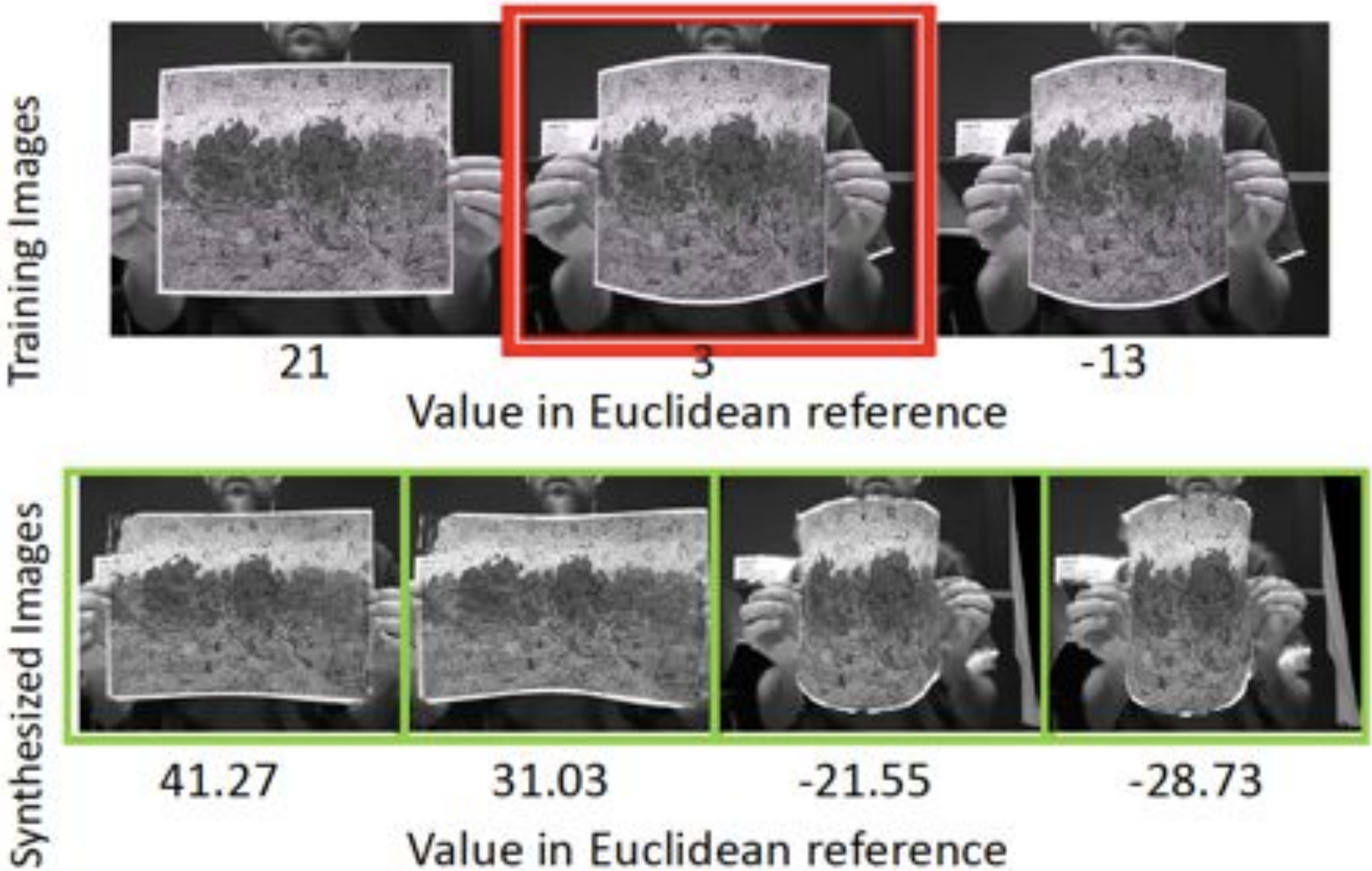


OFM = 'Nonlinear' Tangent Space



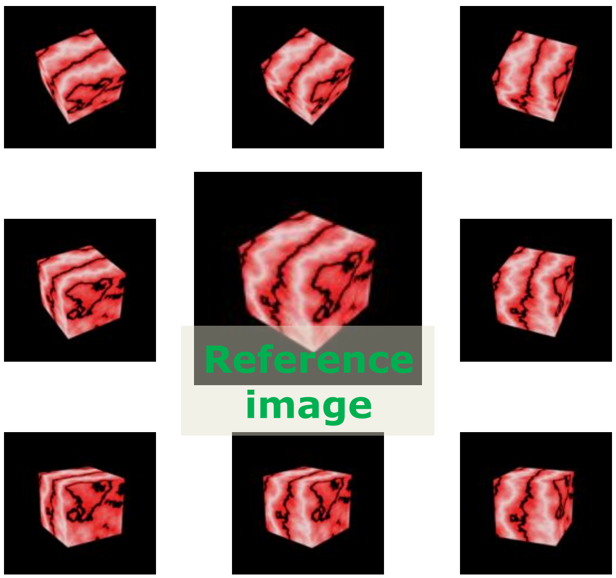

Input Image
Input Image
IAM
Geodesic
Linear path
OFM
Geodesic
Input Image
Input Image

App 1: Image Synthesis

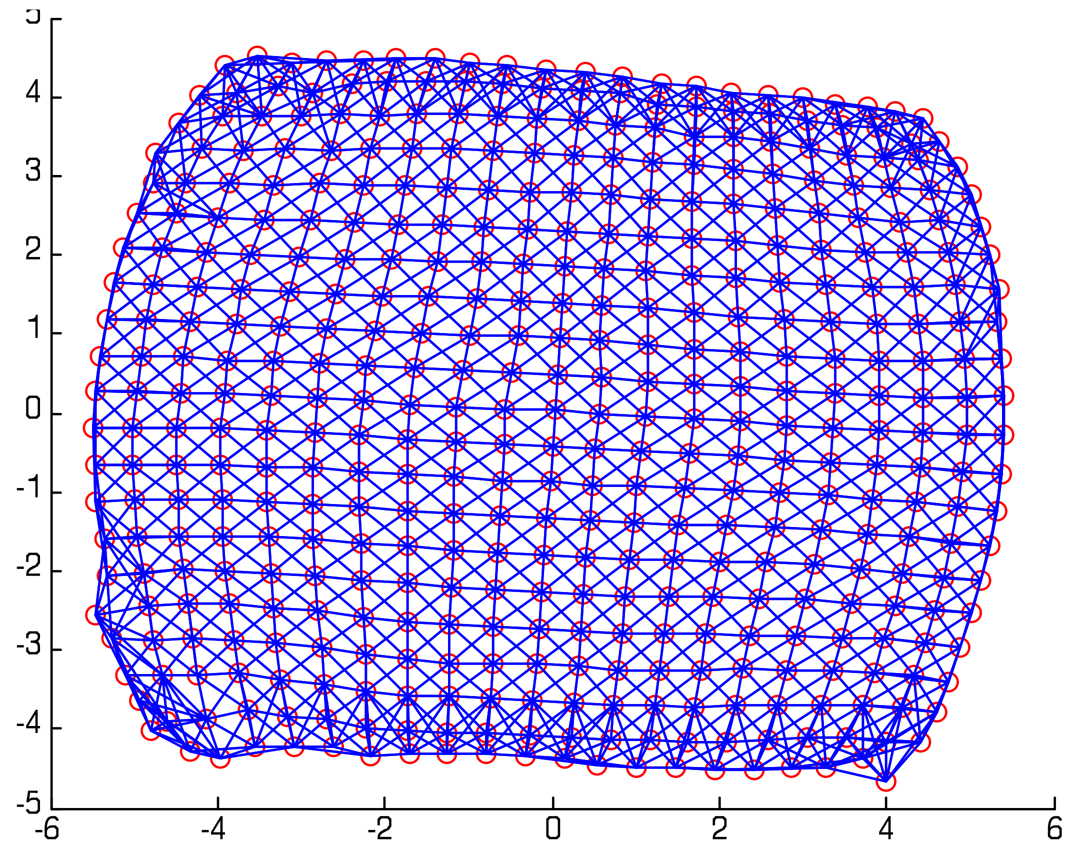


App 2: Manifold Learning

2D rotations



Embedding of **OFM**

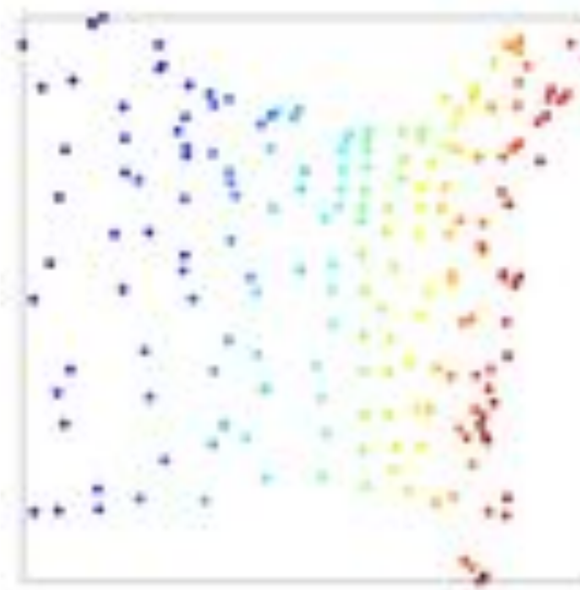


App 2: Manifold Learning

Data
196 images of two bears moving linearly and independently

Task
Find low-dimensional embedding

IAM



OFM

App 3: Karcher Mean Estimation

- Point on the manifold such that the sum of squared *geodesic distances* to every other point is minimized
- Important concept in nonlinear data modeling, compression, shape analysis [Srivastava et al]

10 images from an IAM

ground truth KM

OFM KM

linear KM

Summary

- Manifolds: **concise model** for many image processing problems involving **image collections** and multiple sensors/viewpoints

- But practical image manifolds are non-differentiable
  - manifold-based algorithms have not lived up to their promise

- **Optical flow manifolds** (OFMs)
  - smooth even when IAM is not
  - OFM ~ nonlinear tangent space
  - support accurate image synthesis, learning, charting, …

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Open Questions

- Our treatment is specific to **image manifolds** under brightness constancy

- What are the natural transport operators for **other data manifolds**?

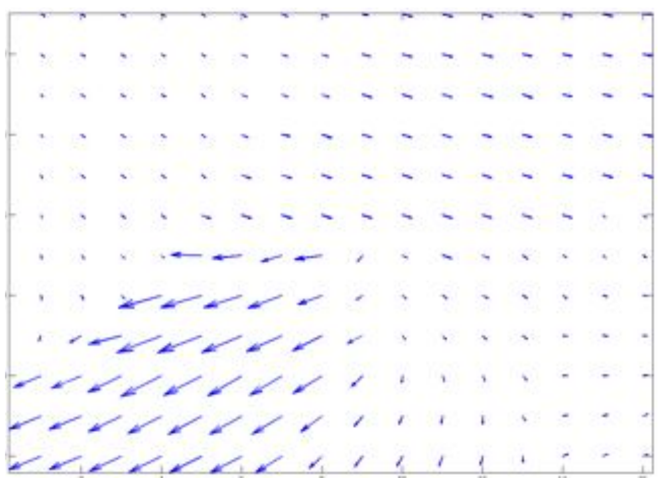
Optical Flow

$$I_2(x,y) = I_1(x + u(x,y), y + v(x,y))$$

$$I_2(x,y) = I_1(x,y) + (\nabla_X I_1)u(x,y) + (\nabla_Y I_1)v(x,y)$$

two-image sequence

optical flow



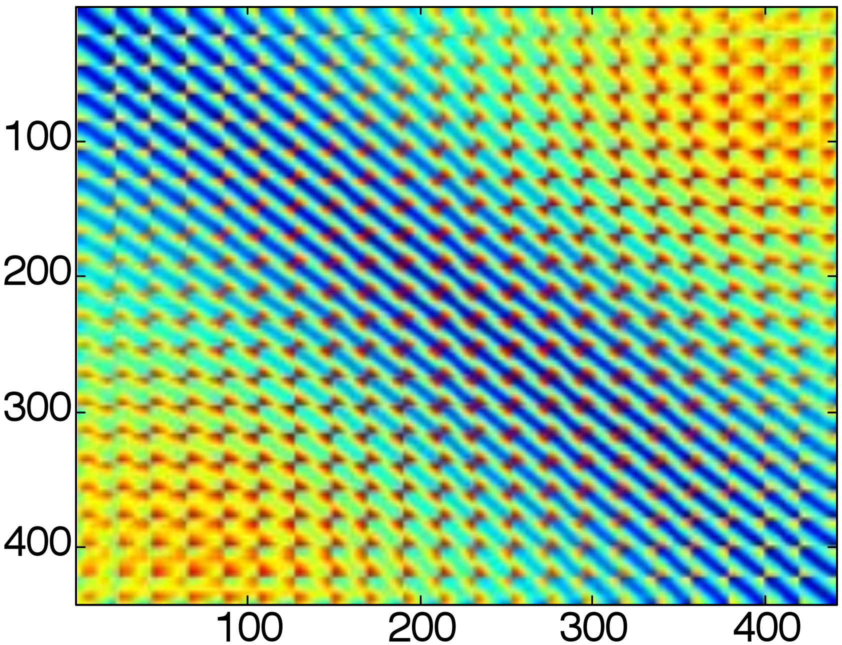
2nd image predicted from 1st via OF

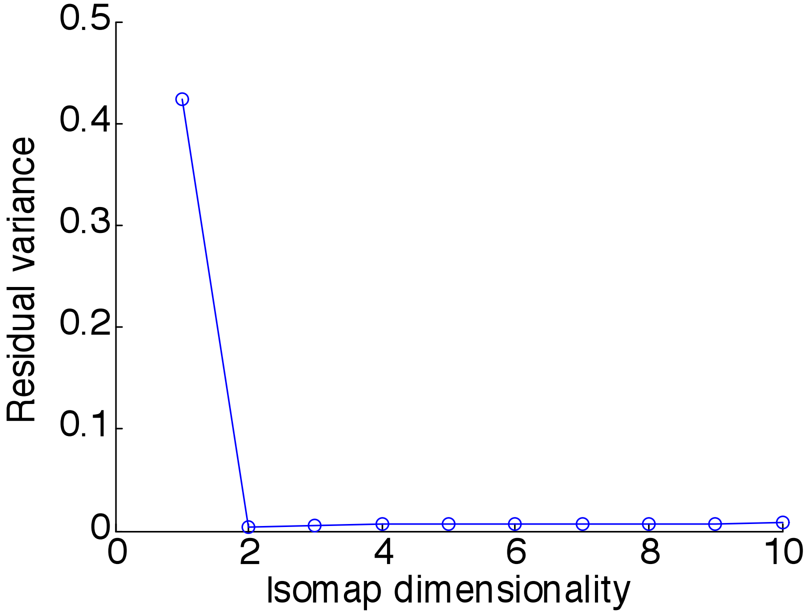
(Figures from Ce Liu's optical flow page and ASIFT results page)

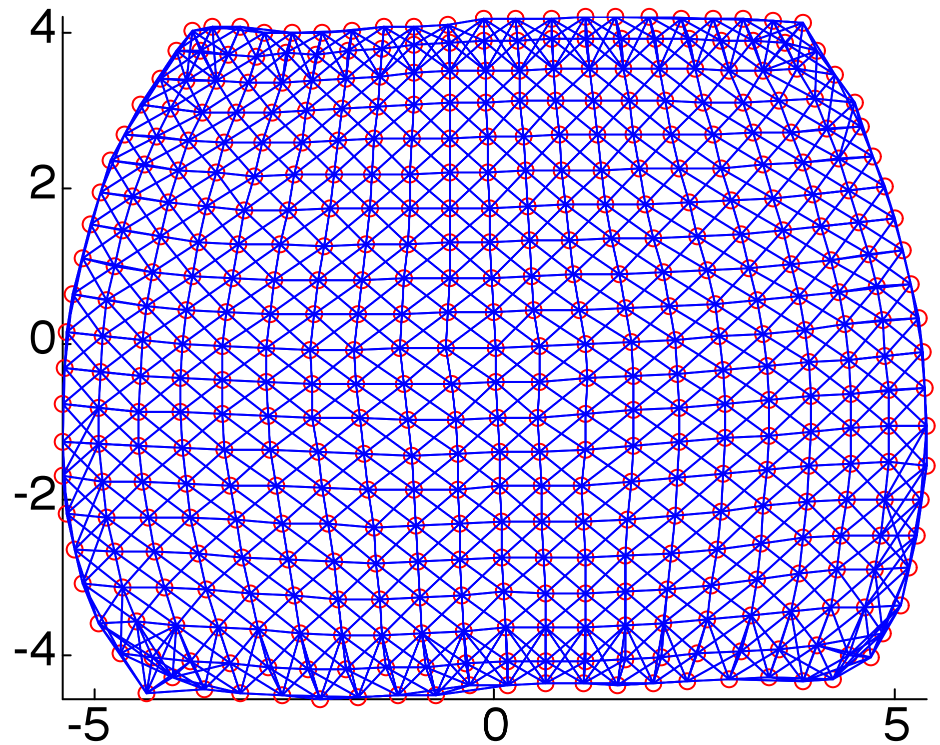
Limitations

- Brightness constancy
  - Optical flow is no longer meaningful
- Occlusion
  - Undefined pixel flow in theory, arbitrary flow estimates in practice
  - Heuristics to deal with it
- Changing backgrounds etc.
  - Transport operator assumption too strict
  - Sparse correspondences ?

Pairwise distances and embedding

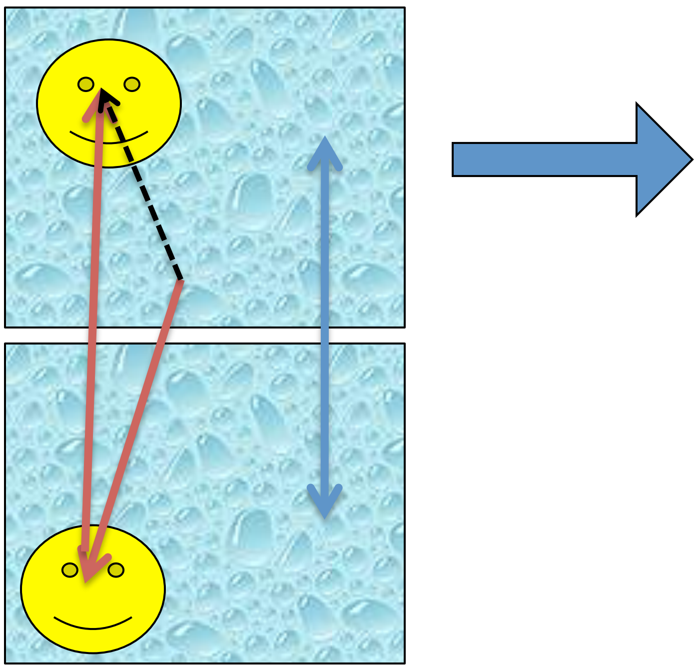


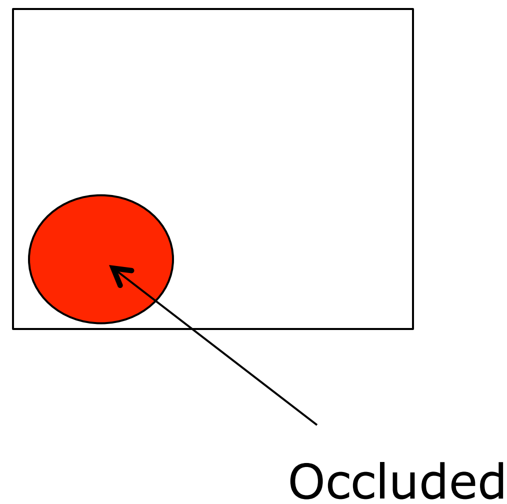




Occlusion

- Detect occlusion using forward-backward flow reasoning





- Remove occluded pixel computations

- **Heuristic** **---** formal occlusion handling is hard

History of Optical Flow

- Dark ages (<1985)
  - special cases solved
  - LBC an under-determined set of linear equations

- Horn and Schunk (1985)
  - Regularization term: smoothness prior on the flow

- Brox et al (2005)
  - shows that linearization of **brightness constancy (BC)** is a bad assumption
  - develops optimization framework to handle BC directly

- Brox et al (2010), Black et al (2010), Liu et al (2010)
  - practical systems with reliable code