High-Dimensional Data Fusion via Joint Manifold Learning

Chinmay Hegde

November 13, 2010

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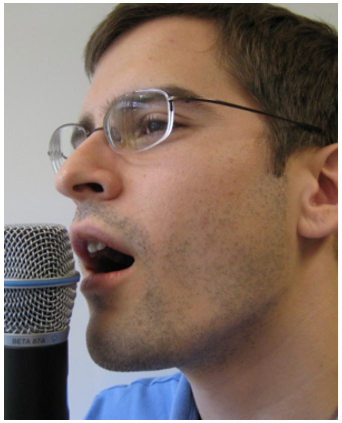

Mark Davenport
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Marco Duarte
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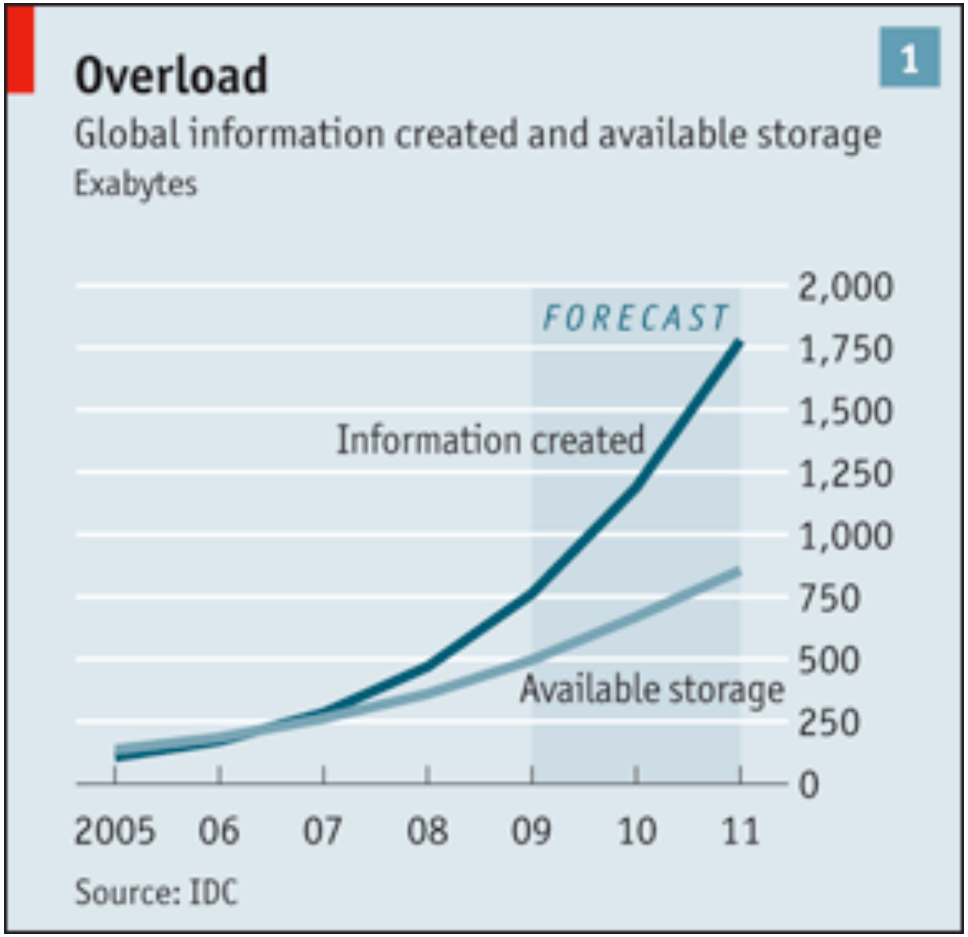
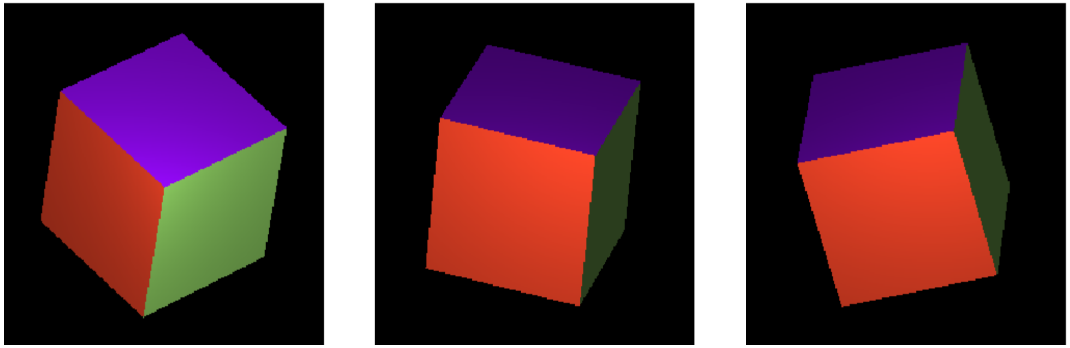
The data deluge
AND HOW TO HANDLE IT: A 14-PAGE SPECIAL REPORT

The Data Deluge



Manifold Models

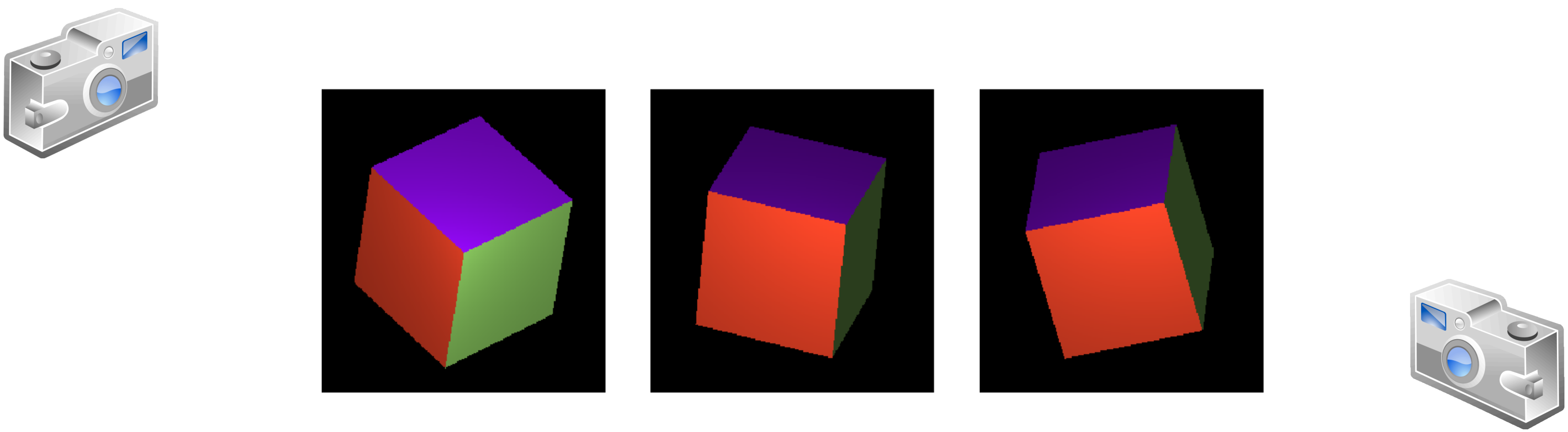
- K-dimensional *parameter vector* captures degrees of freedom in signal $x \in \mathbb{R}^N$



$$SO(3)$$

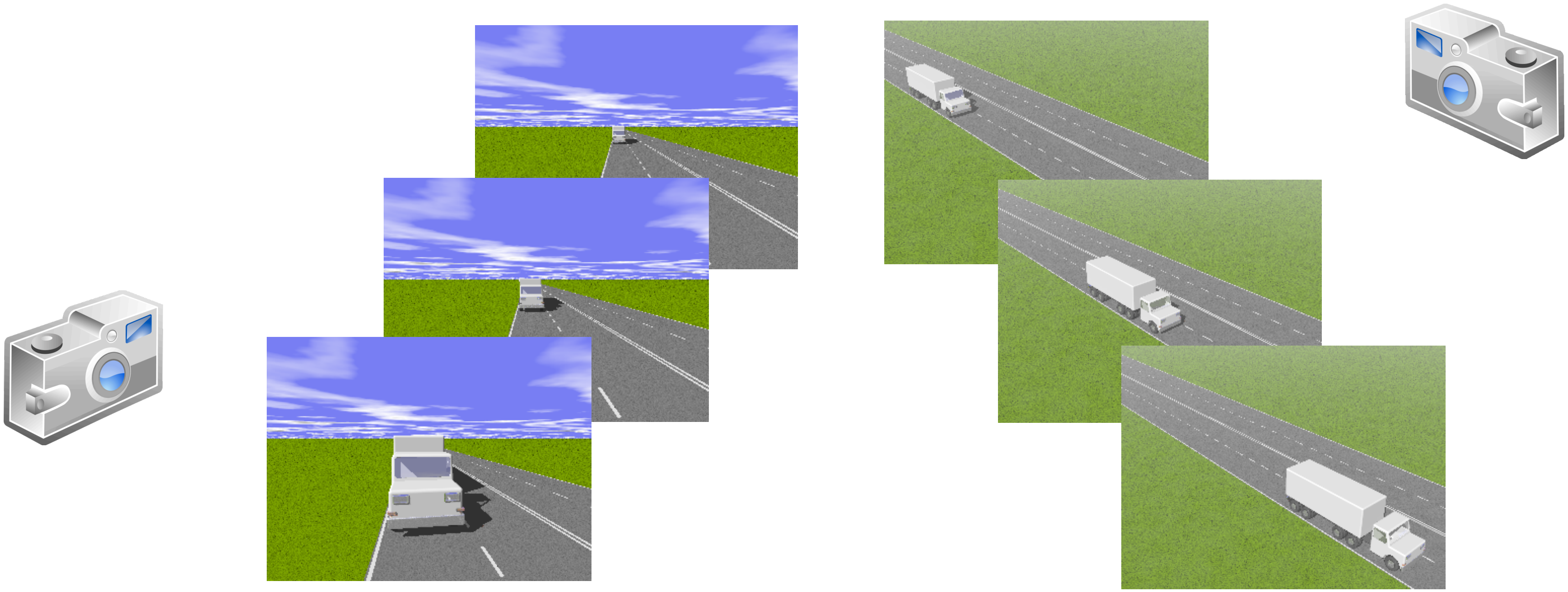
Multi-Manifold Models

- *K*-dimensional *parameter vector* captures degrees of freedom in signal $x \in \mathbb{R}^N$



Multi-sensor Fusion

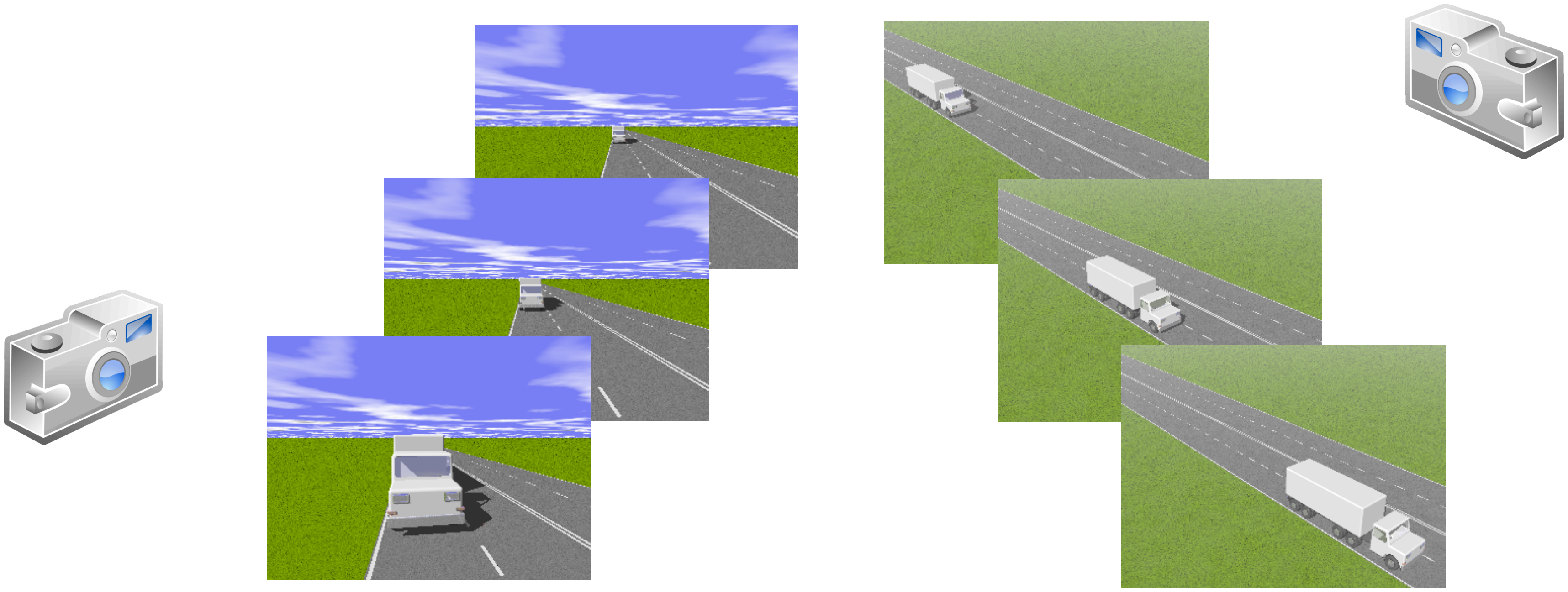
- Example: Network of J cameras jointly observing an articulating object



- Aggregate dimensionality of data = $J\,N$
- Each camera's images lie on K-dim manifold in $\mathbf{R}^N$

Multi-sensor Fusion

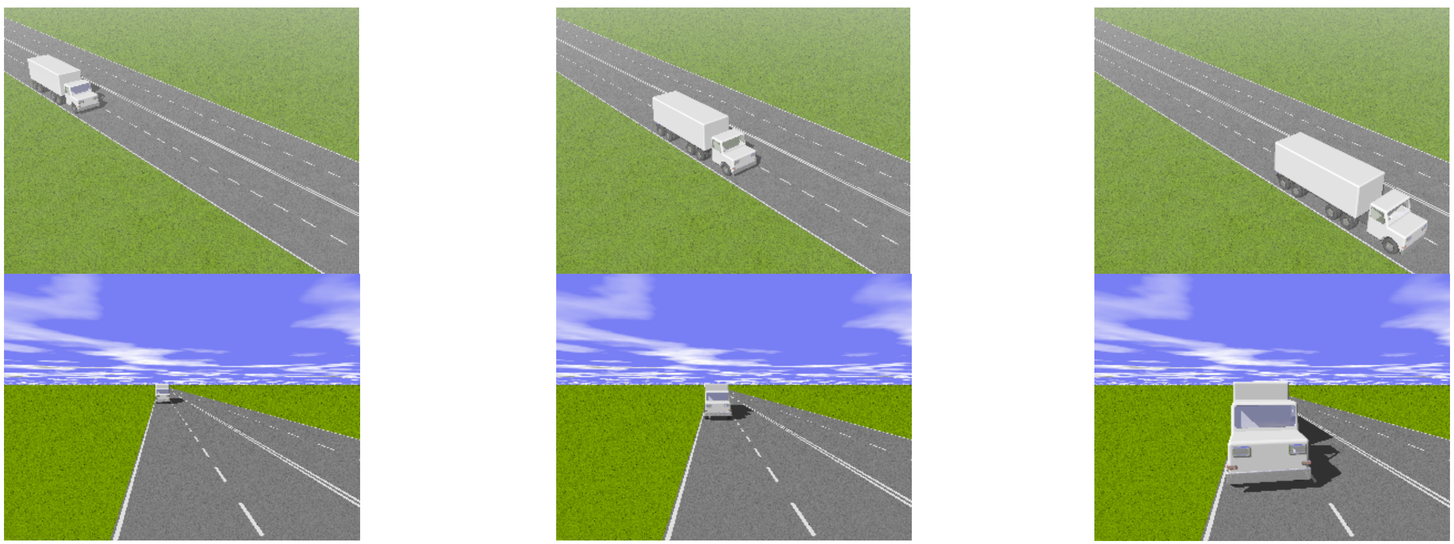
- Example: Network of *J* cameras jointly observing an articulating object



- How to efficiently **fuse imagery** from *J* cameras to perform inference?

Multi-sensor Fusion

- Idea: stack corresponding image vectors taken at the same time



- Stacked images still lie on *K*-dim manifold in $\mathbf{R}^{JN}$

“joint manifold”

Joint Manifolds

- Given manifolds $\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_J \subset \mathbf{R}^N$
  - K-dimensional
  - homeomorphic (we can continuously map between any pair)

- Define **joint manifold** as concatenation of $\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_J$

Joint Manifolds

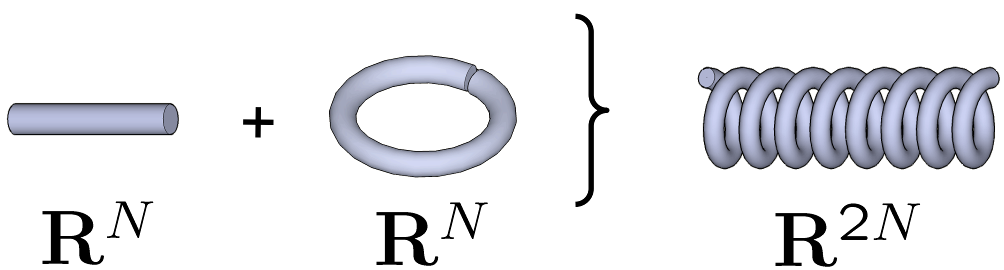
- Given manifolds $\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_J \subset \mathbf{R}^N$
  - K-dimensional
  - homeomorphic (we can continuously map between any pair)

- Define **joint manifold** as concatenation of $\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_J$

- Example: $\mathcal{M}_j = \{f_j(\theta), \theta \in \mathbf{R}^K\}$

$$\mathcal{M}^* = \{f^*(\theta), \theta \in \mathbf{R}^K\} = \{[f_1(\theta); f_2(\theta); \ldots; f_J(\theta)], \theta \in \mathbf{R}^K\}$$

$$\mathcal{M}^* \subset \mathbf{R}^{JN}$$



$$f^*(\theta) = \begin{bmatrix} \theta \\ \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

Joint Manifolds: Theory

- Joint manifold inherits desirable properties from component manifolds
  - compactness
  - smoothness
  - volume:

$$\max_j V_j \leq V^* \leq \sum_{j=1}^{J} V_j$$

  - condition number ($1/\tau$):

$$\frac{1}{\tau^*} \leq \max_j \frac{1}{\tau_j}$$

Joint Manifolds: Theory

- Translate into much better performance of inference algorithms in practice
  - Classification
  - **Manifold learning**

Joint Manifold Learning

- **Theorem (noise robustness)**:

  Suppose $p, q \in \mathcal{M}^*$

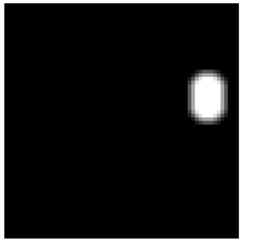
  Observe $r = p + n$ where $\mathrm{var}(n) = \sigma^2$
  $s = q + n$

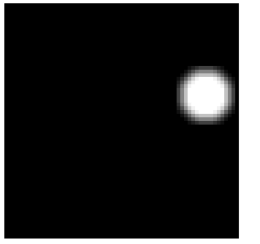
  Then, estimated distance $\|r - s\|$ converges to true distance $\|p - q\|$ with failure probability **exponential** in J^2

Joint Manifold Learning

- **Goal:** Learn embedding of 2D image manifold

  N=45x45=2025 pixels
  J=20 views





Joint Manifold Learning

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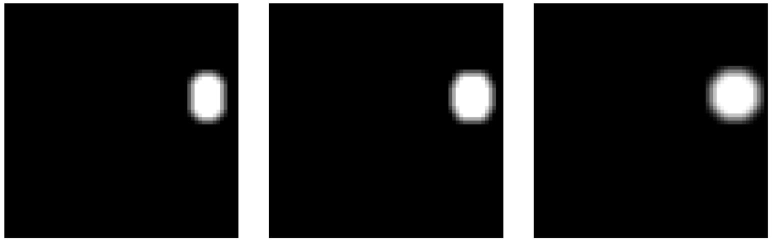
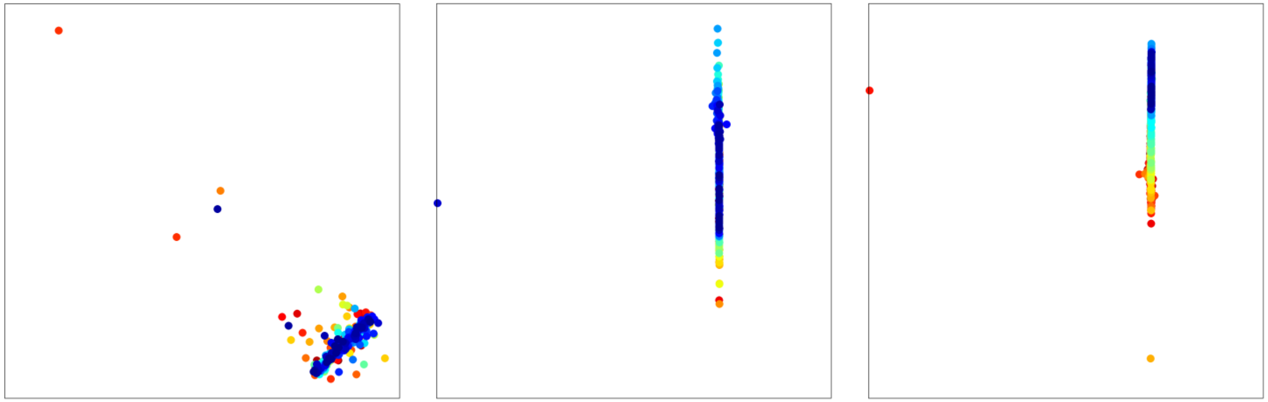
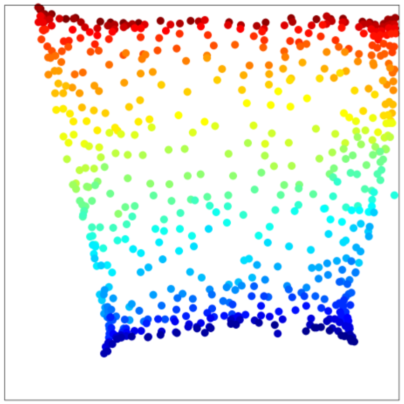


- Embeddings learned **separately**



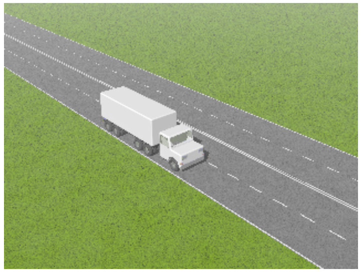
- Embedding learned **jointly**

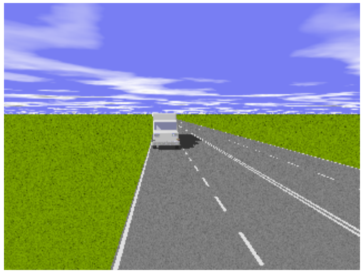


Joint Manifold Learning

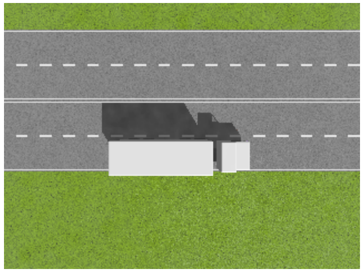
- J=3 CS cameras, each N=240x320 resolution

- 2D joint manifold
  parameters: 2D location
  of the truck on the highway



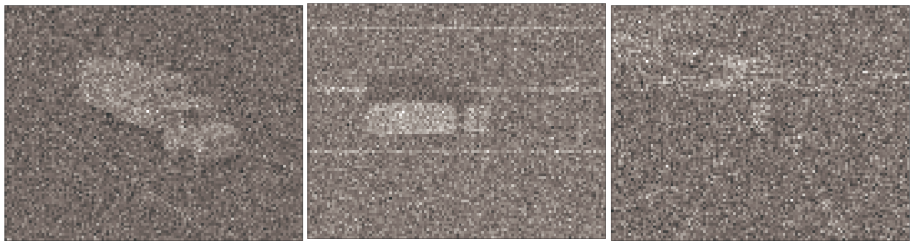


- **Goal:** Learn embedding
  of 2D image manifold

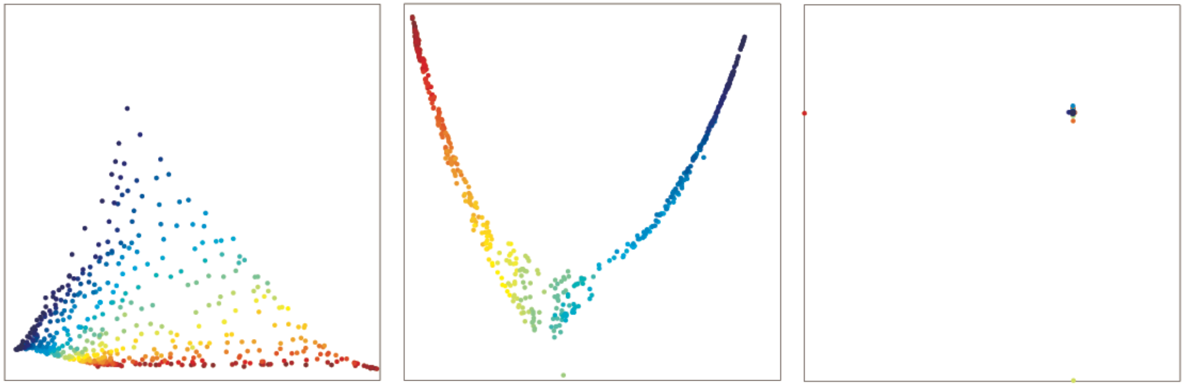


Joint Manifold Learning

- **Goal:** Learn embedding of 2D image manifold (with noise)
  - N=240x320=76800 pixels
  - J=3 views

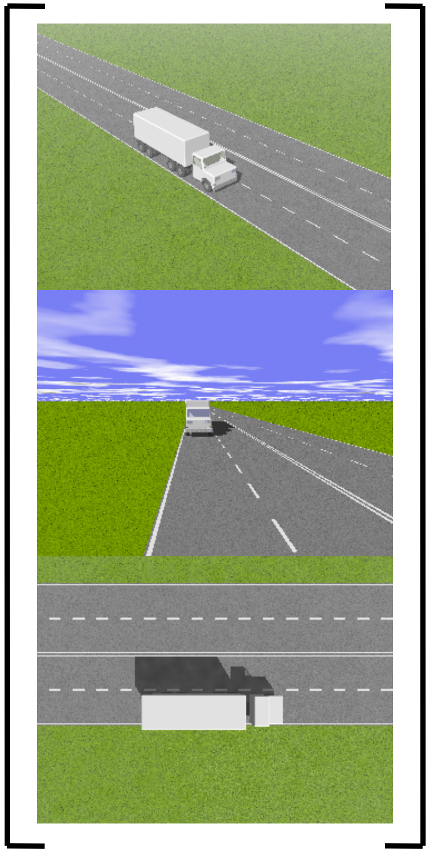


- Embeddings learned **separately**



- Embedding learned **jointly**

Multi-sensor Fusion

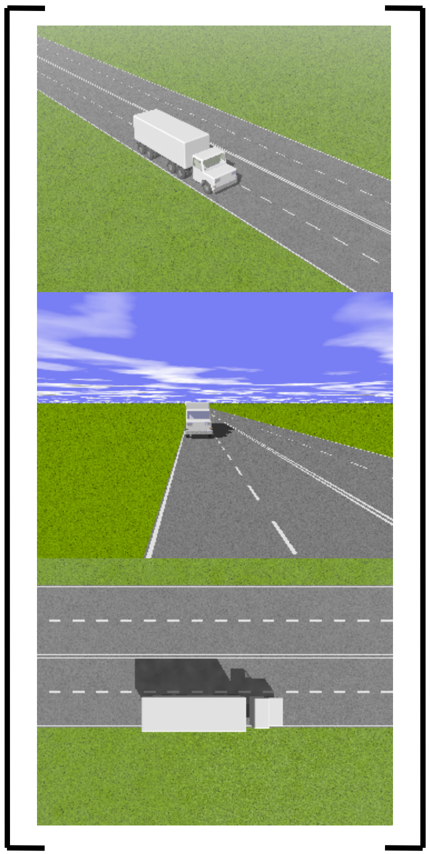


Blessing of dimensionality

[Lawrence]

$$\mathcal{M}_j = \{f_j(\theta), \theta \in \mathbf{R}^K\}$$

Multi-sensor Fusion

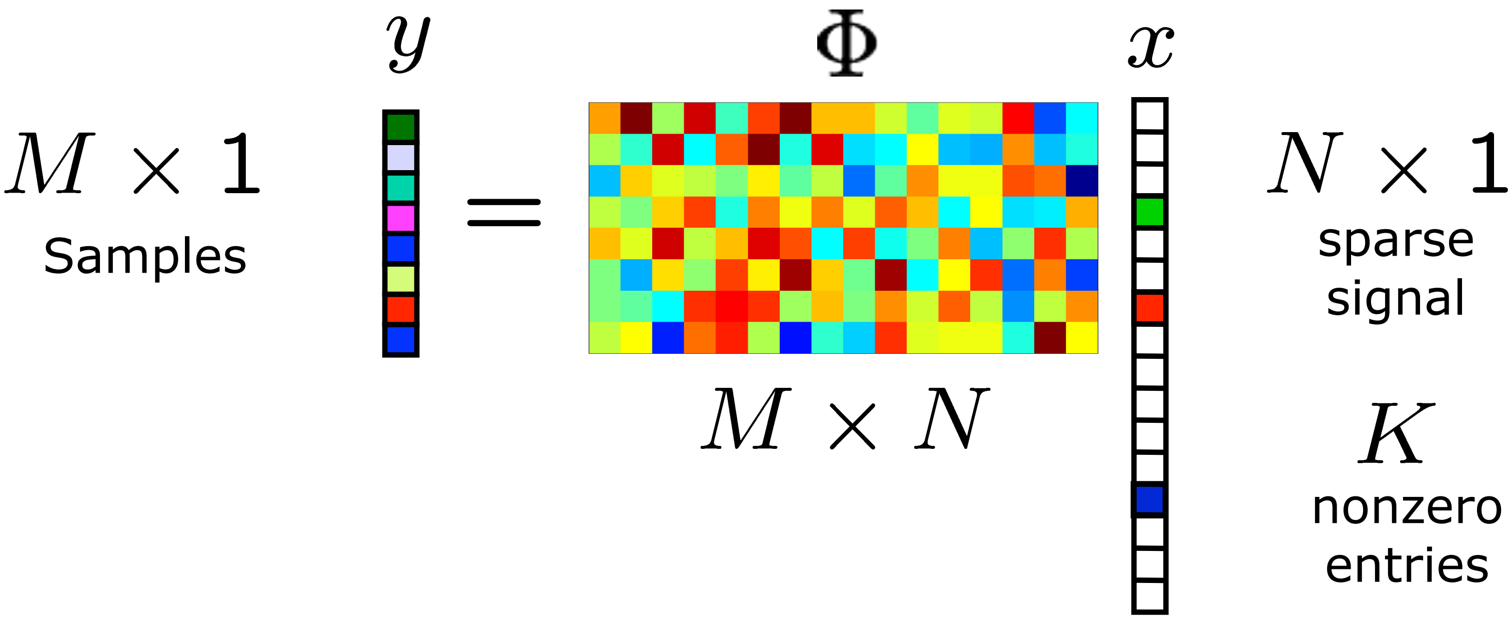


Blessing? of dimensionality

$$\dim(x) = J \times N$$

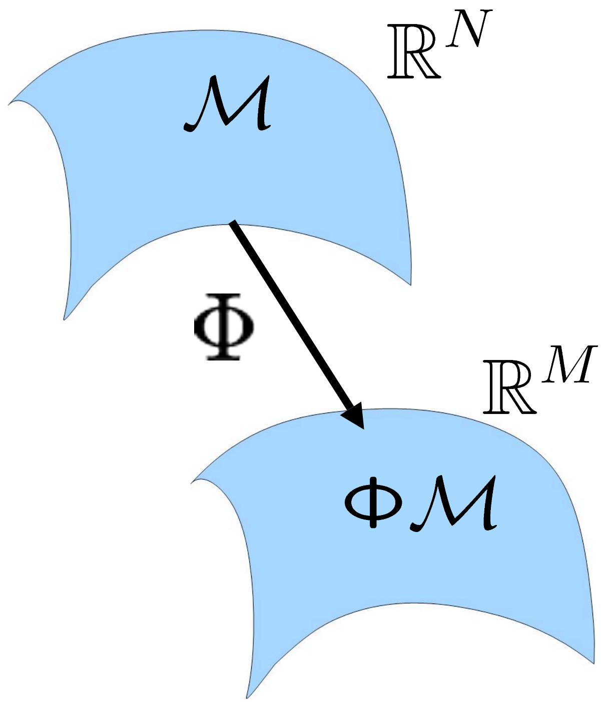
$$\mathcal{M}_j = \{f_j(\theta), \theta \in \mathbf{R}^K\}$$

Compressive Sensing (CS)



Compressive Sensing (CS)

Stable manifold embedding via CS

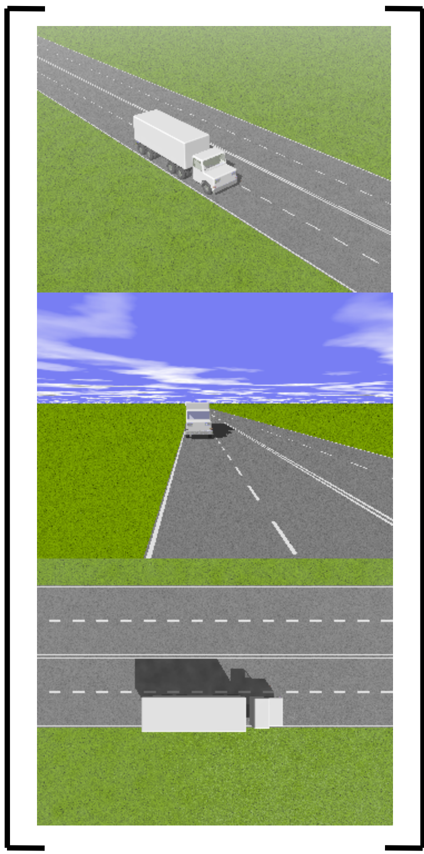


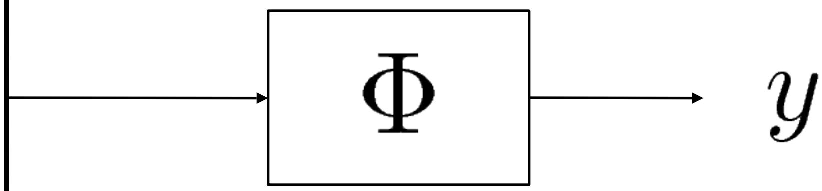
$$M = O\left(\frac{K \log(NV\tau^{-1}\epsilon^{-1}) \log(1/\rho)}{\epsilon^2}\right).$$

[Baraniuk, Wakin 2006]

Multi-sensor Fusion via CS

- Can acquire random CS measurements of stacked images and make inferences





$$\dim(y) = O(K \log(JN))$$

$$\ll$$

$$\dim(y) = O(JK \log(N))$$

$$\ll$$

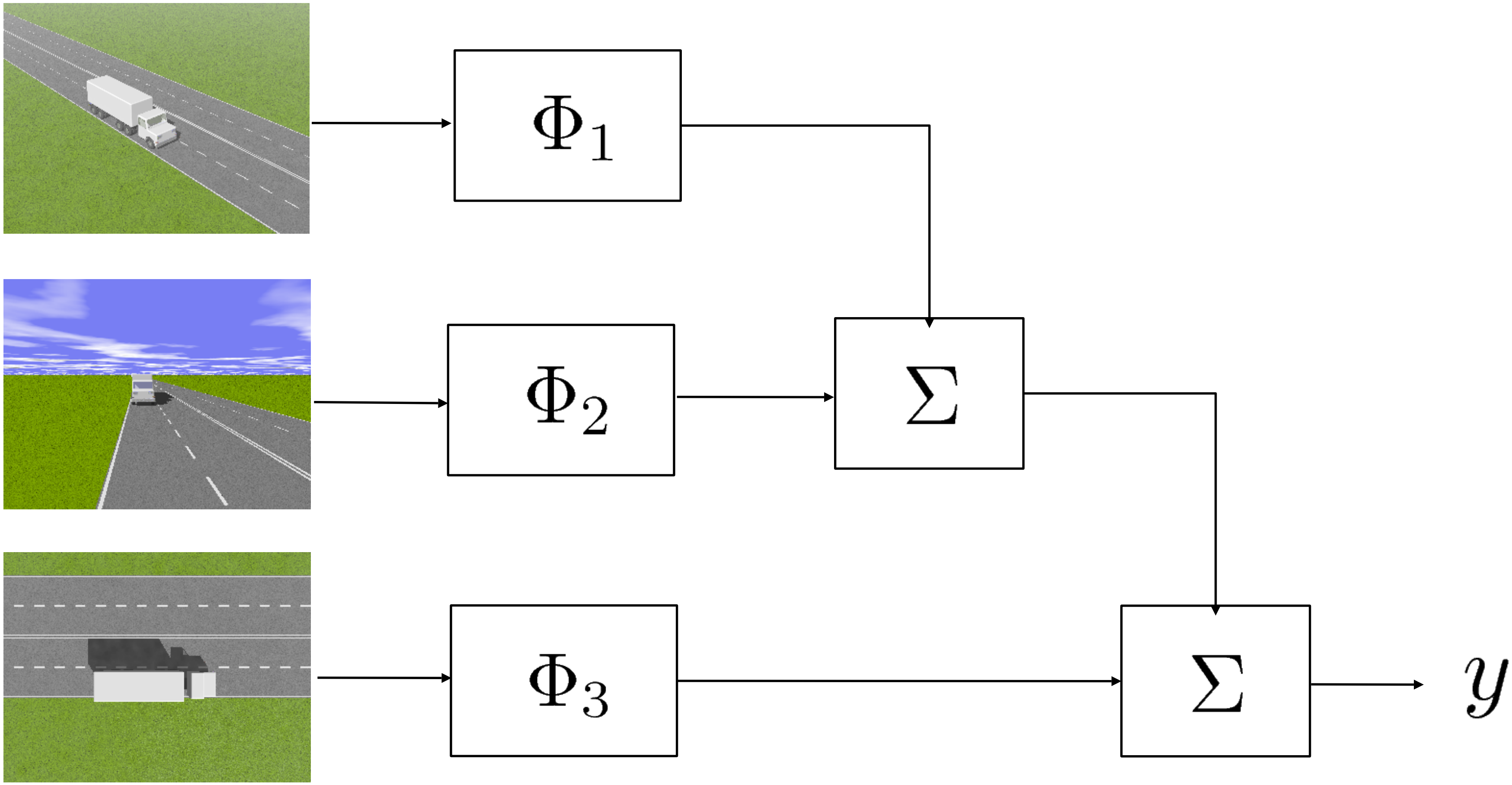
w/ unfused CS

$$\dim(x) = O(JN)$$

w/ unfused and no CS

Multi-sensor Fusion via CS

- Can fuse measurements *efficiently in network*
  - ex: as we transmit to collection/processing point

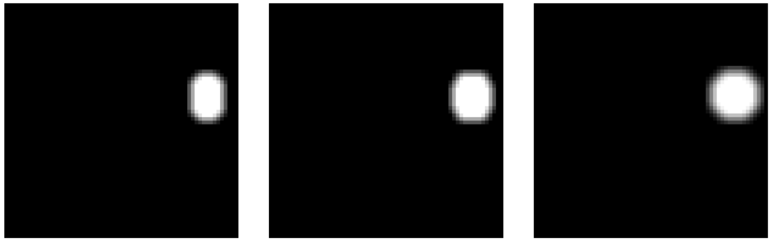


Joint Manifold Learning w/CS

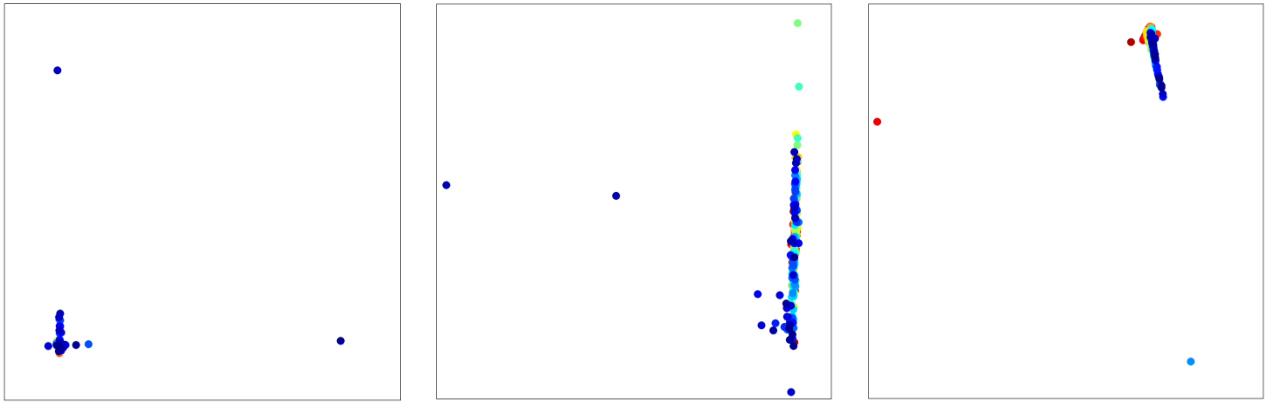
- **Goal:** Learn embedding via **random compressive** measurements

  N=45x45=2025 pixels

  J=20 views

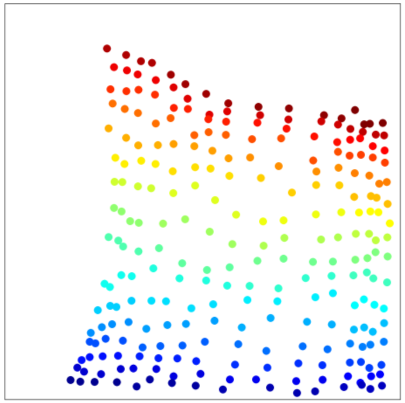


- Embeddings learned **separately**



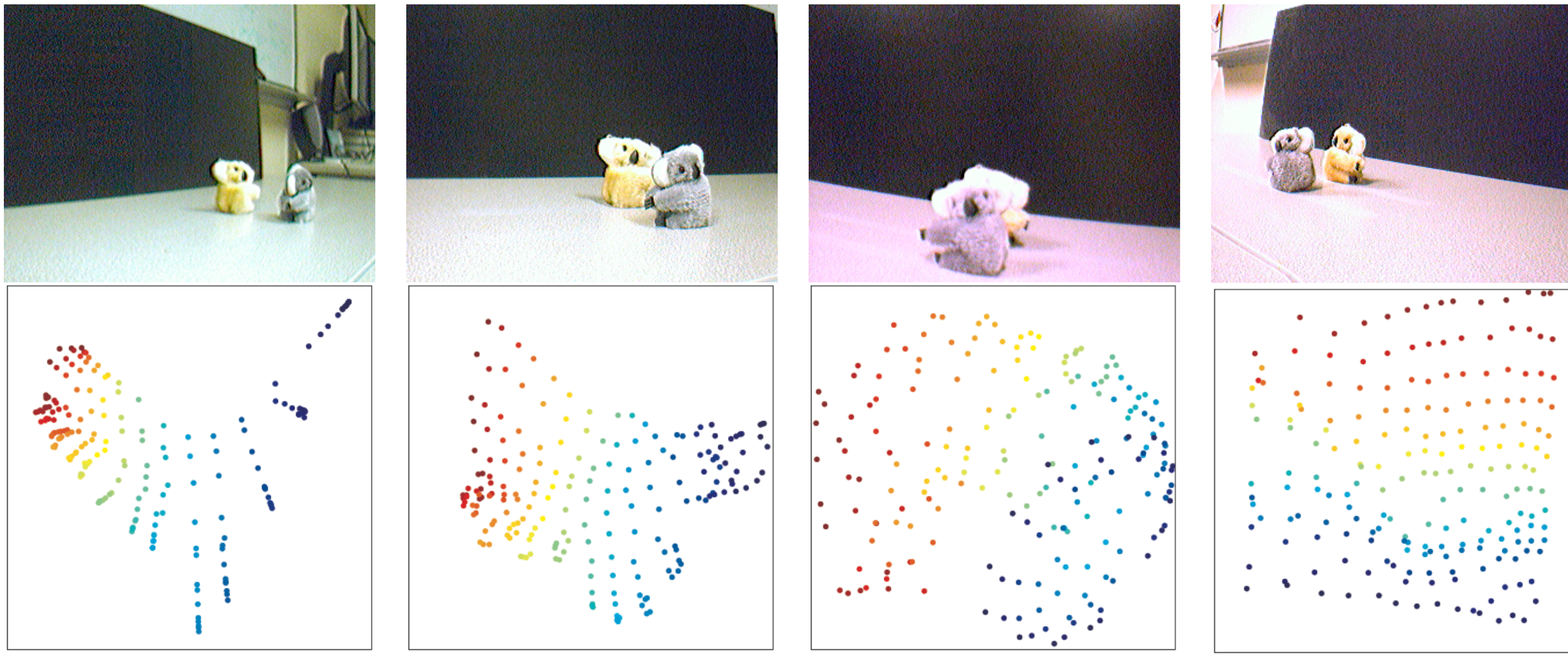
- Embedding learned **jointly**

  M=100 measurements per view

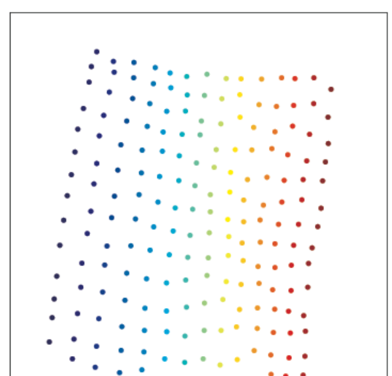


Joint Manifold Learning

- N = 240x320 , J = 4
- Embeddings learned **separately**

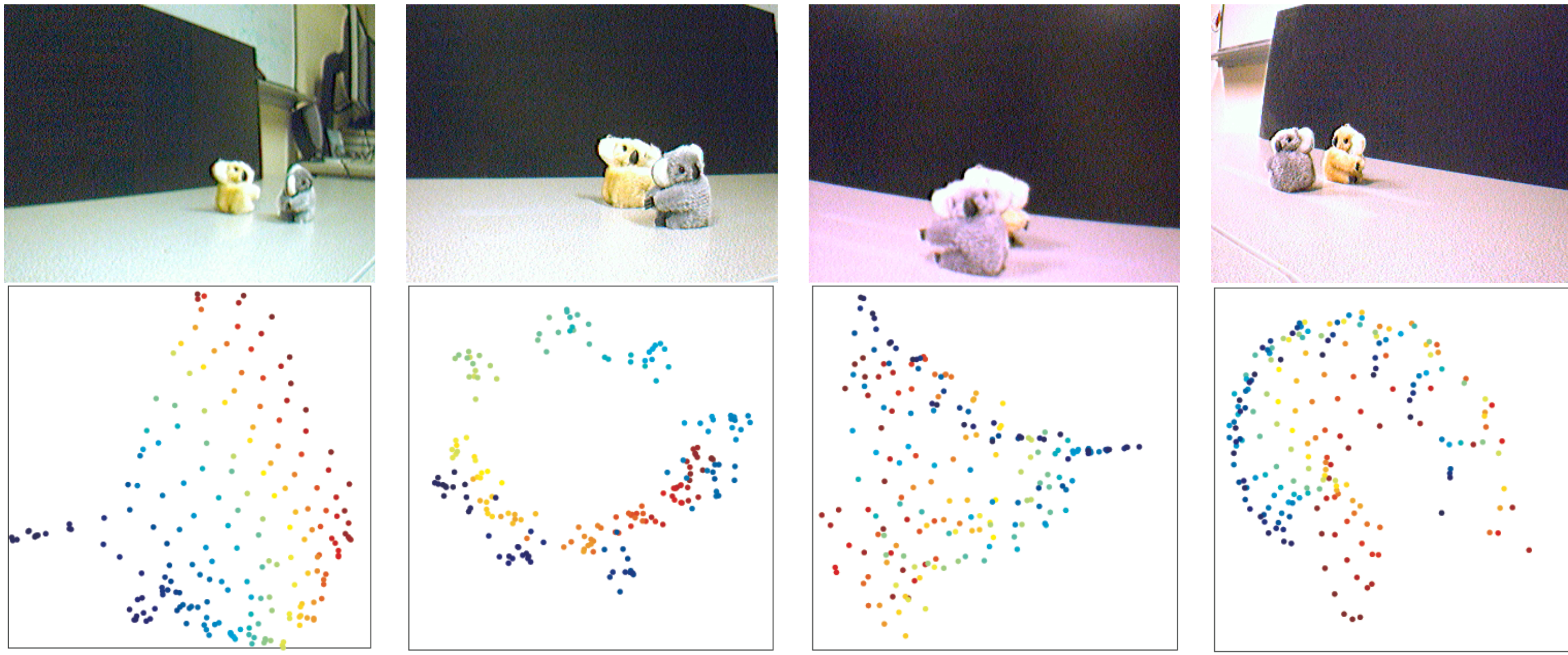


- Embedding learned

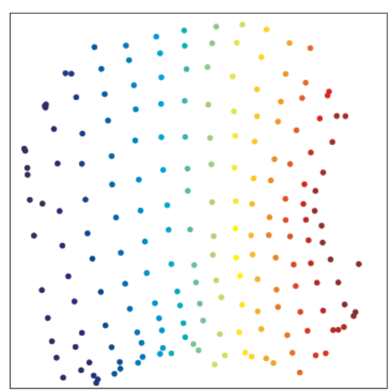


Joint Manifold Learning w/CS

- N = 240x320 , J = 4, M = 2400
- Embeddings learned **separately**

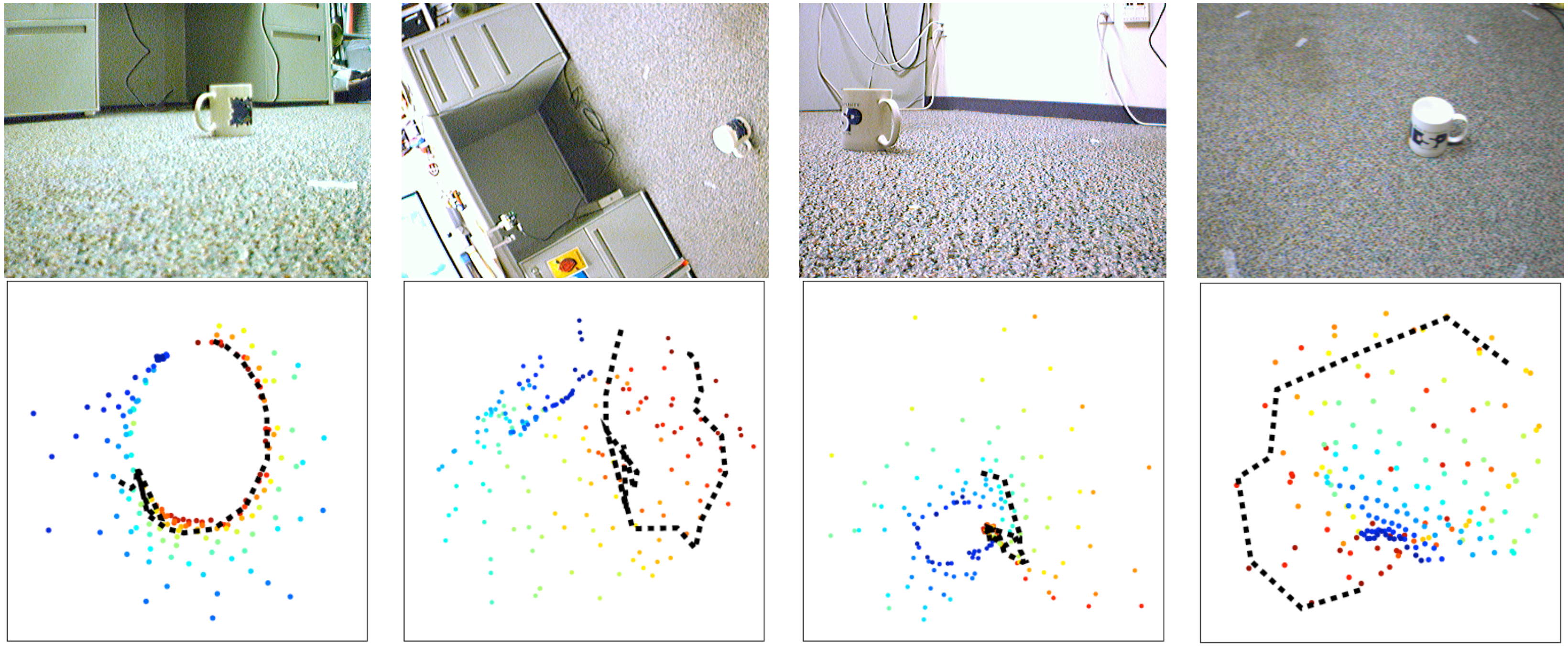


- Embedding learned



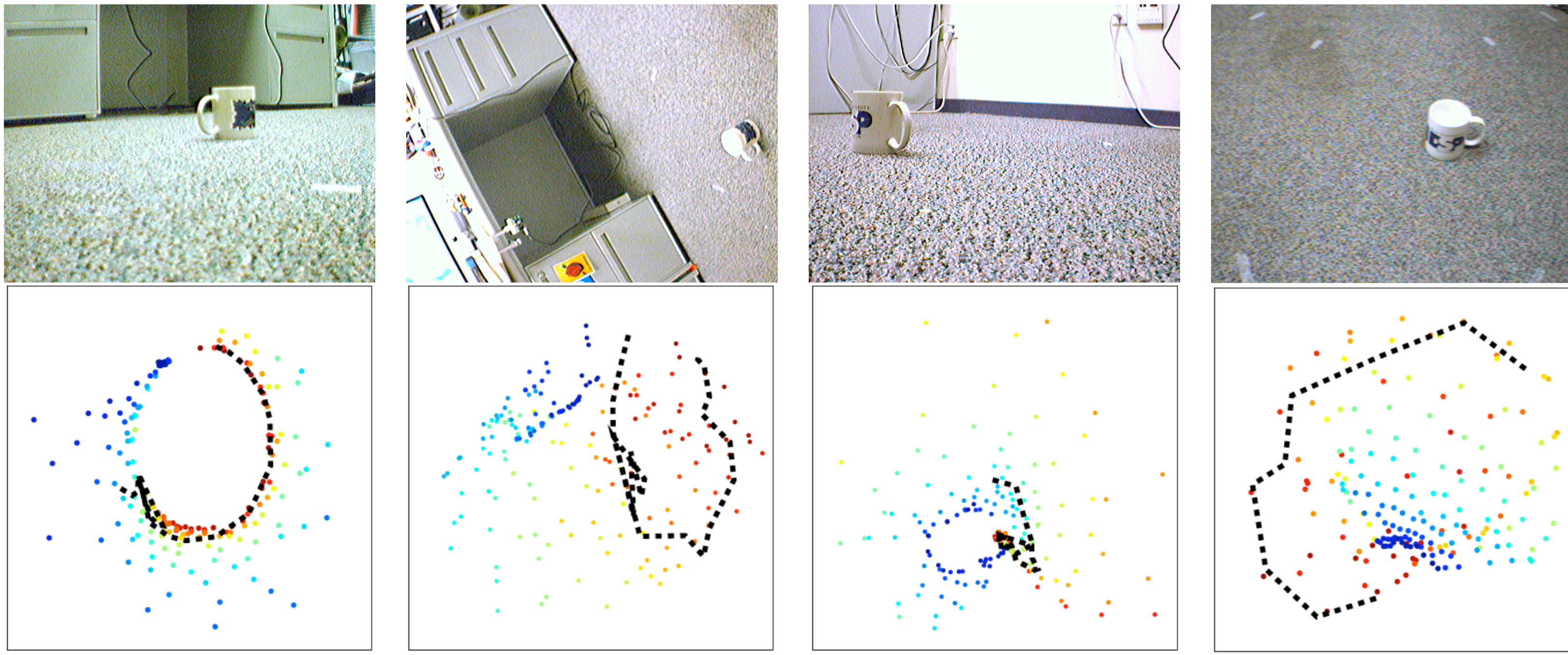
Application: Target Tracking

- N = 240x320 , J = 4

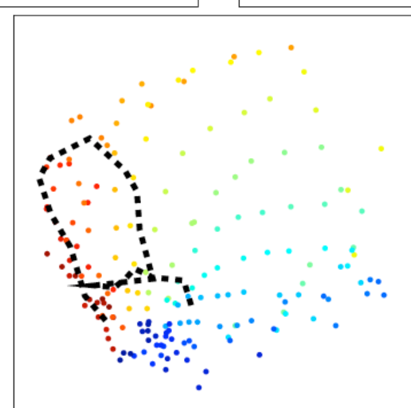


Application: Target Tracking

- N = 240x320 , J = 4
- Trajectory ('R') learned **separately**

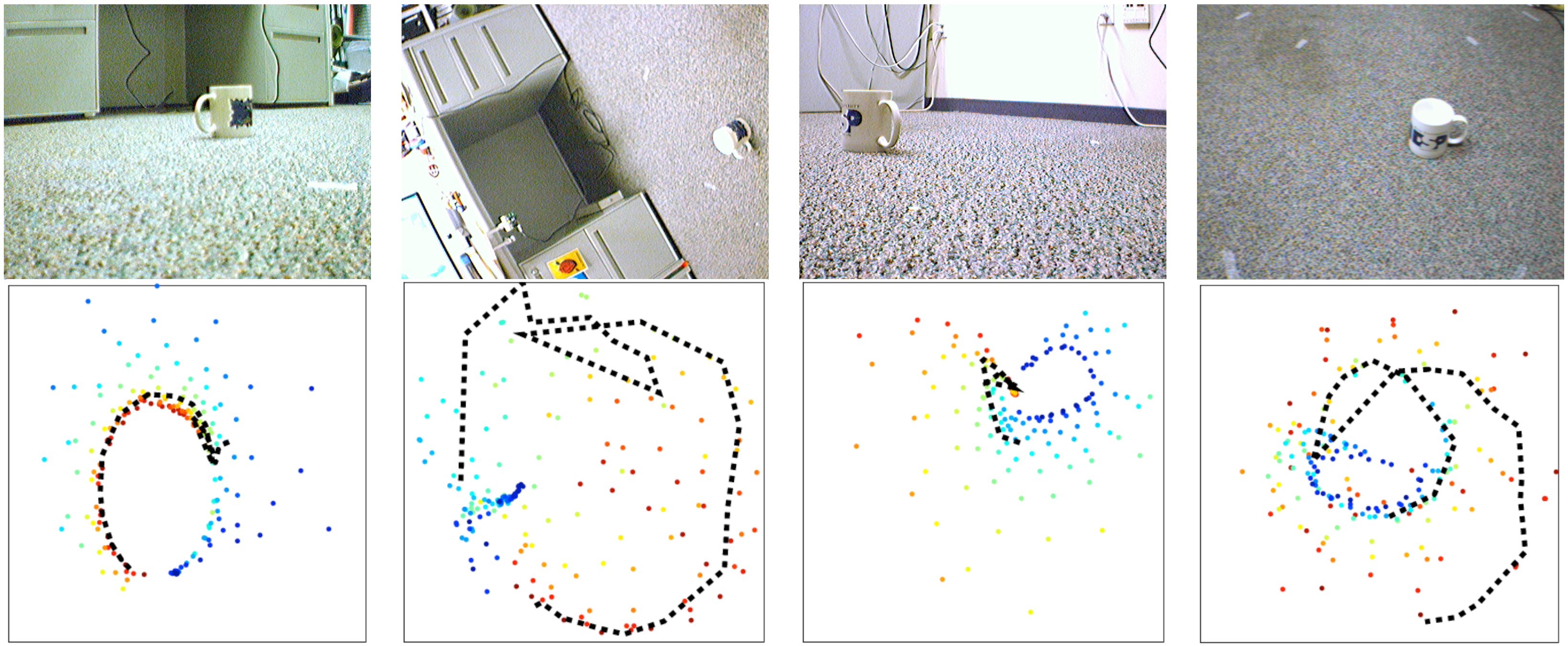


- Trajectory learned **jointly**

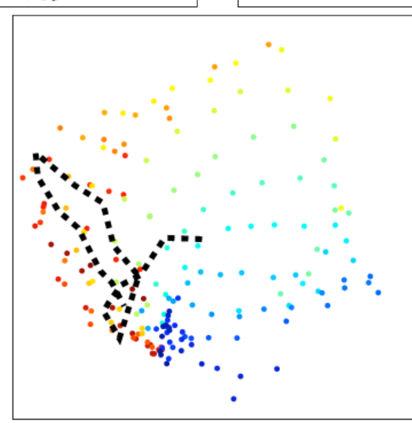


Application: Target Tracking w/CS

- N = 240x320 , J = 4, M = 4800
- Trajectory (‘R’) learned **separately**



- Trajectory learned



Directions

- Better ways to fuse
  - Hierarchical fusion [Rabin]
  - Manifold alignment [Zhi]
  - Optimal feature selection

- Newer problems
  - Clustering [Hunter]
  - homology inference [Wang]
  - anomaly detection
  - regression

Directions

Conclusions

- **Joint manifold**
  - new tool for data fusion
  - attractive geometric properties
  - provable improved guarantees for several inference tasks

- **Multisensor Fusion** with Compressive Sensing (CS)

- **Applications:** classification, manifold learning, target tracking

“Joint Manifolds for Data Fusion”
Davenport, H, Duarte, Baraniuk, Trans IP, Oct 2010

dsp.rice.edu/cs

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Joint Manifolds: Practice

- J=3 CS cameras, each N=320x240 resolution
- M=200 random measurements per camera

- Two classes
  - truck w/ cargo
  - truck w/ no cargo

- Smashed filtering
  - independent
  - majority vote
  - joint manifold