Sparse Signal Recovery Using Markov Random Fields

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Compressive Sensing

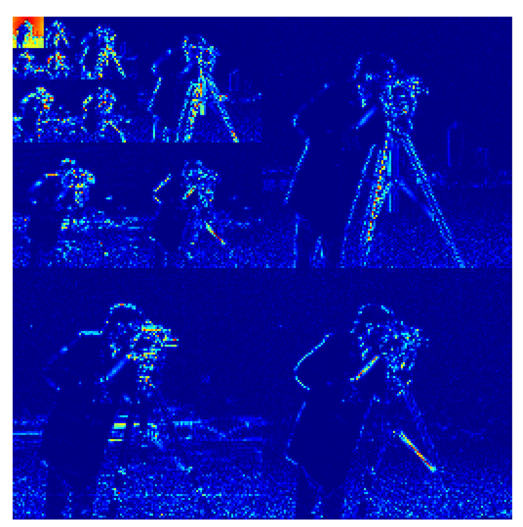
- Natural/manmade signals often have **sparse/compressible** structure
- Traditional signal acquisition: sample first, then compress
- Compressive acquisition: compress and sample simultaneously

Compression and Sparsity

Traditional signal acquisition:

- *Sample* data at Nyquist rate (2x bandwidth)
- *Compress* data (signal dependent, nonlinear)

N pixel image

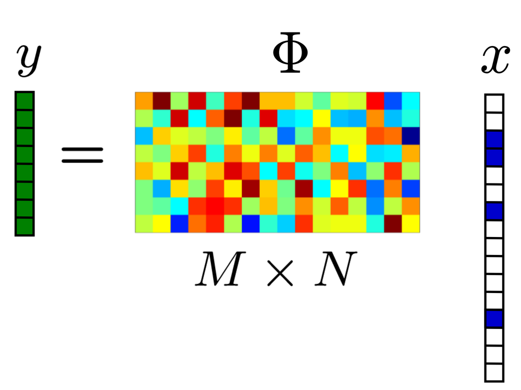


$K \ll N$ large wavelet coefficients

Compressive Sensing (CS)

- Acquire *compressive measurements*

$M \times 1$ measurements



$N \times 1$ signal

K nonzero coefficients

$$\boxed{M = O(K \log(N/K)) \ll N}$$

Signal Recovery

- Recovery algorithm *exploits sparsity*
  - ℓ_1-minimization (slow, uniform guarantees)

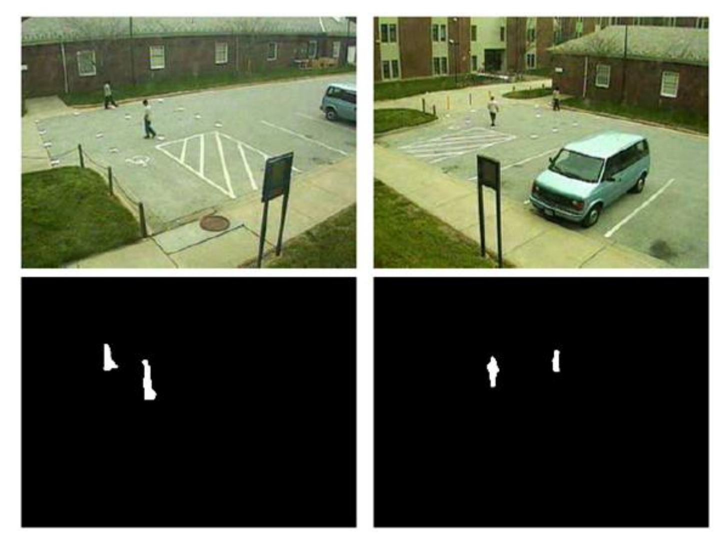
$$\boxed{\hat{x} = \arg\min \|x\|_1 + \lambda \|y - \Phi x\|_2^2} \rightarrow \boxed{\|\hat{x} - x_0\|_2 \le C \|x^* - x_0\|_2}$$

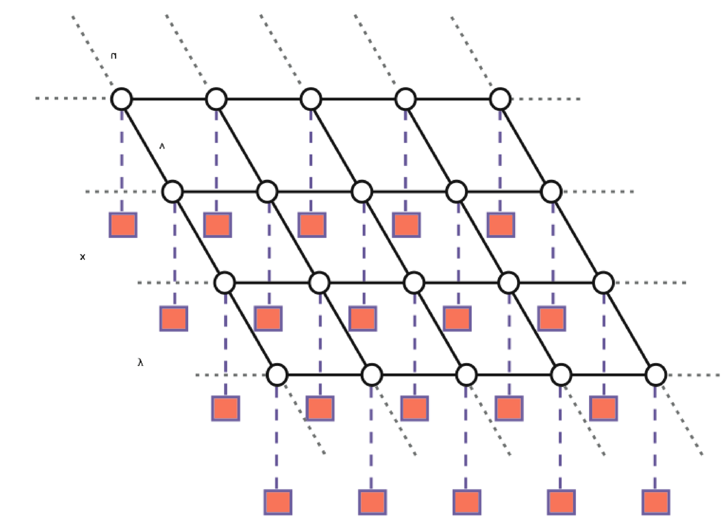
  - orthogonal matching pursuit (faster, weak guarantees)
  - CoSaMP / IHT (faster, uniform guarantees)

Structure-driven Sparse Recovery

- Sparsity assumption does not capture **dependencies** among coefficients
- Model restricts search space; enables faster, more robust recovery
- LaMP : a new algorithm that solves for signals modeled by MRFs

Structured Sparse Representations





Example: background-subtracted images

- *Clustered* nonzeroes
- Modeled by *Markov Random Field* (Ising model)
- Model approximation: *Graph Cuts*
- Graph-cut cost functions derived from signal log-likelihoods

Algorithm : Lattice Matching Pursuit

Given: measurements y, matrix Φ , target sparsity K

Repeat until convergence:

- Form signal proxy: $x \leftarrow \Phi^T(y - \Phi x)$
- Estimate signal support S via *graph cuts*
- Compute least-squares estimate of signal using basis elements indexed by S : $x \leftarrow \Phi_S^{\dagger} y$
- Form best K - term approximation of x

Extensions

- Rigorous theoretical framework derived for *union-of-subspaces* models
- Models studied: connected wavelet trees, jointly-sparse signal ensembles

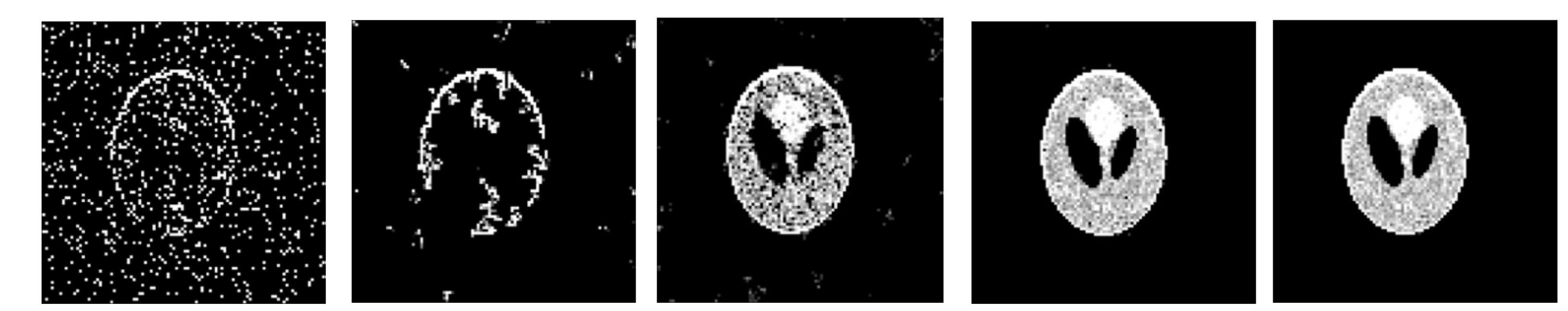
Experimental Results

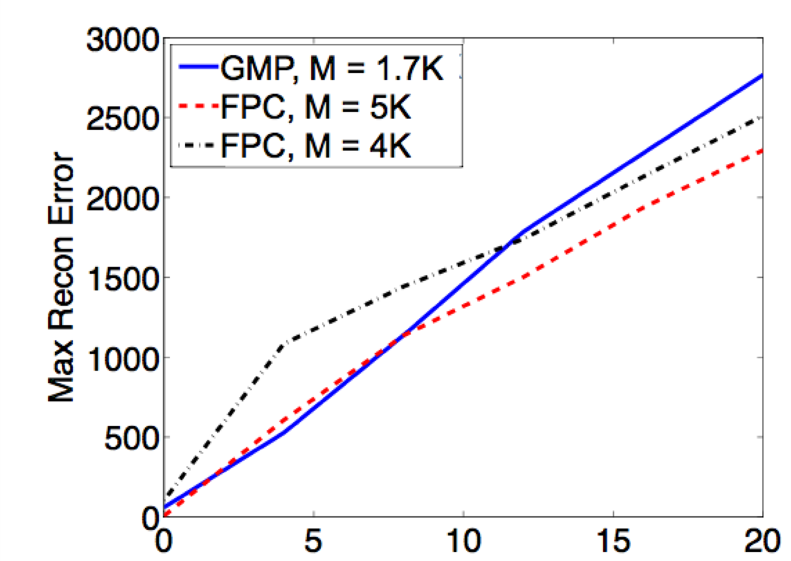
- Requires far fewer measurements than state-of-the art CS methods
- LaMP is fast, robust under measurement noise
- Testing performed on simulated and real data

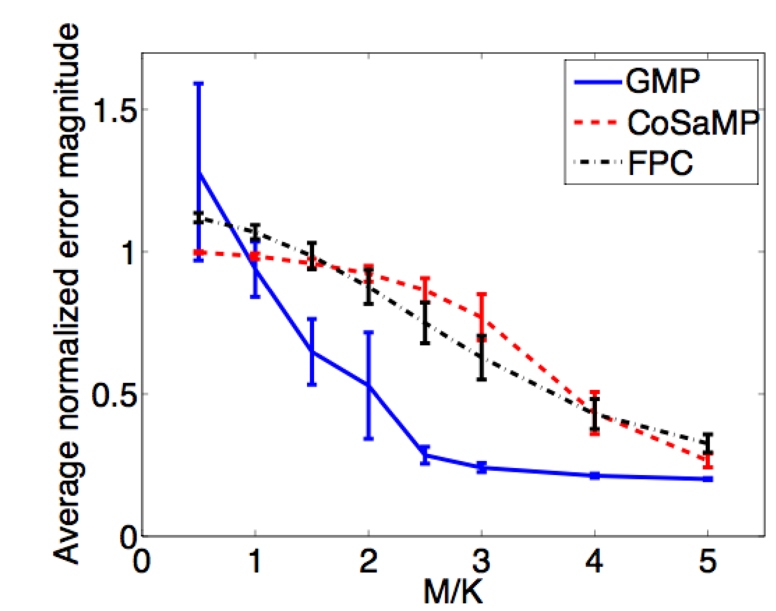
MRF-Driven Sparse Recovery

Synthetic test image: Shepp-Logan Phantom

$N = 10000, K = 1740, M = 2K, \mathrm{SNR} = 10$ dB

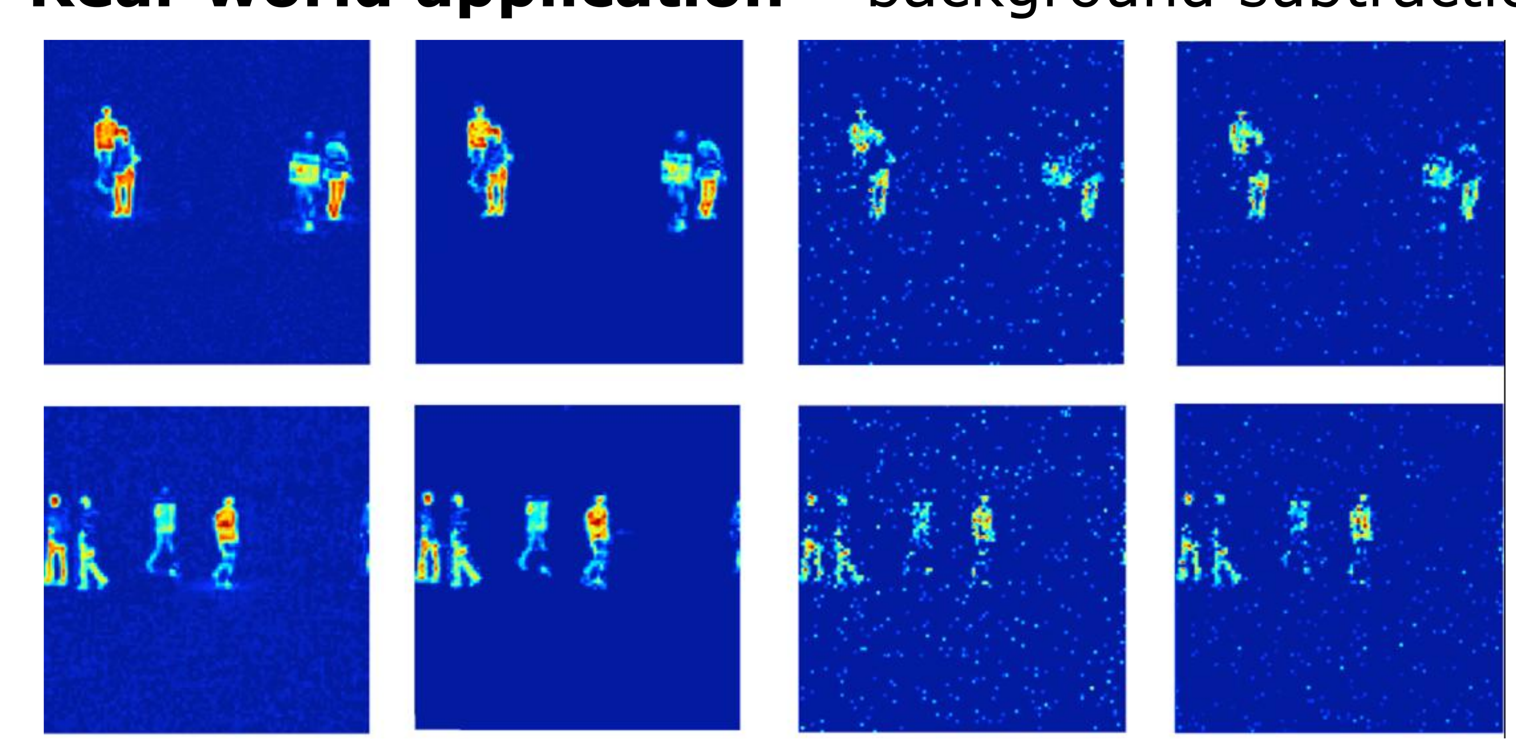






- LaMP works well in the case of compressible signals, noise in samples
- Significant savings in terms of number of measurements

Real-world application – background subtraction



Target **LaMP** **CoSaMP** **ℓ_1-minimization**

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