Structured Sparse Representations

Example: background-subtracted images

- Clustered nonzeros
- Modeled by Markov Random Field (Ising model)
- Model approximation: Graph Cuts
- Graph-cut cost functions derived from signal log-likelihoods

Algorithm: Lattice Matching Pursuit

Given: measurements $y$, matrix $\Phi$, target sparsity $K$.
Repeat until convergence:
- Form signal proxy: $x \leftarrow \Phi^T(y - \Phi x)$
- Estimate signal support $S$ via graph cuts
- Compute least-squares estimate of signal using basis elements indexed by $S$: $x \leftarrow \Phi_S y$
- Form best $K$-term approximation of $x$

Extensions

- Rigorous theoretical framework derived for union-of-subspaces models
- Models studied: connected wavelet trees, jointly-sparse signal ensembles

Experimental Results

- Requires far fewer measurements than state-of-the art CS methods
- LaMP is fast, robust under measurement noise
- Testing performed on simulated and real data

MRF-Driven Sparse Recovery

Synthetic test image: Shepp-Logan Phantom
$N = 10000, K = 1740, M = 2K, SNR = 10$ dB

Target
LaMP
CoSaMP
$\ell_1$-minimization

Compression and Sparsity

Traditional signal acquisition:
- Sample data at Nyquist rate (2x bandwidth)
- Compress data (signal dependent, nonlinear)

Compressive Sensing (CS)

- Acquire compressive measurements

$M \times 1$ measurements

$y = \Phi x$

$N \times 1$ signal
$K$ nonzero coefficients

Signal Recovery

- Recovery algorithm exploits sparsity
  - $\ell_1$-minimization (slow, uniform guarantees)
  - Orthogonal matching pursuit (faster, weak guarantees)
  - CoSaMP / IHT (faster, uniform guarantees)

$\hat{x} = \arg \min ||x||_1 + \lambda \|y - \Phi x\|^2$

$||\hat{x} - x_0||^2 \leq C ||x^* - x_0||^2$

Compressive Sensing

- Natural/manmade signals often have sparse/compressible structure
- Traditional signal acquisition: sample first, then compress
- Compressive acquisition: compress and sample simultaneously

Structured Sparse Recovery

- Sparsity assumption does not capture dependencies among coefficients
- Model restricts search space; enables faster, more robust recovery
- LaMP : a new algorithm that solves for signals modeled by MRFs
- Requires far fewer measurements than state-of-the art CS methods
- LaMP is fast, robust under measurement noise
- Testing performed on simulated and real data

Structured Sparse Representations

Synthetic test image: Shepp-Logan Phantom
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Real-world application – background subtraction

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