Lecture 17

- Mechanism Design

↳ • as in "game design", except we won't be designing any video games, so let us be more formal :)

• be open-minded; games could be used to model the market, auctions, elections, policy making; we'll see examples of these later on.

- Warm-up Example: selling an item in an auction

◇ **Setup**: - n bidders, each has private value w_i for item

- if bidder i wins item at price p he derives utility of $w_i - p$; such bidders are called quasi-linear

- set of alternatives $\mathcal{A} = \{1\text{-wins}, 2\text{-wins}, \ldots, n\text{-wins}\}$

$$v_i(j\text{-wins}) = \begin{cases} 0 & , i \neq j \\ w_i & , i = j \end{cases}$$

- suppose auctioneer wants to optimize social welfare

i.e. chose alternative $j\text{-wins} \in \arg\max_\ell \left(\sum_i v_i(\ell\text{-wins})\right)$

that is $j \in \arg\max_\ell (w_\ell)$.

◇ **Candidate Auctions**

- **No payment:** collect bids from bidders $b_1, \ldots, b_n$

give item to bidder $i \in \arg\max_\ell \{b_\ell\}$

trouble: everybody will bid $+\infty$

- **pay your bid (first-price auction)**: collect bids $b_1, .., b_n$
  - give item to bidder $i \in \arg\max_\ell \{b_\ell\}$
  - charge him his bid.

trouble: people will underbid (o.w. even if they get the item they derive zero utility)

e.g. • suppose two bidders $w_1 = \$5$, $w_2 = \$100$
- suppose they need to bid in increments of 1 cent
- then $(b_1, b_2) = (\$5, \$5.01)$ is a Nash Eq.
- i.e. bidders do not truthfully report their value to auctioneer
- in this case OK, since our social objective was met (the highest value bidder got it)
- BUT it's not obvious for the second bidder what the optimal bid is (as this depends on the value/bid of the other bidder), so cycling may occur while the bidders try to learn/guess each others bids, etc.
- also the auctioneer can't verify that her objective was met.

- **Vickrey's 2nd price Auction**:
  - give item to bidder $i \in \arg\max_\ell \{b_\ell\}$ (fix any tie-breaking rule, e.g. lexicographic)
  - charge i the second highest bid $p^* = \max_{j \neq i} b_j$

Proposition: It is a dominant strategy(*) for every bidder i to submit bid $b_i \equiv w_i$.

(*) This means that even without knowledge of the other bidders' values or bids it is in the best interest of every bidder to bid his value

Proof:

Claim: For every $b_1, b_2, \ldots, b_{i-1}, b_{i+1}, \ldots, b_n$ and every w_i, w_i', let u_i be i's utility for the outcome of the auction if she bids w_i and u_i' her utility if she bids w_i'. Then $u_i \geq u_i'$.

proof: case 1: By bidding w_i, i is the winner and second highest bit is b_k for some $k \neq i$. Then i's utility is $w_i - b_k$. Now suppose i considers placing a bid $> w_i$. He will still win and pay same price. So no incentive to do so. Suppose i considers placing a bid $< w_i$. If that new bid is lower than b_k, he loses item and gets $0 \leq w_i - b_k$ utility. If that new bid is higher than b_k, he still wins and pays same price. If new bid is exactly b_k then depending on tie-breaking rule either i loses item, deriving utility $0 \leq w_i - b_k$ OR still wins and pays same price. Overall no incentive not to bid w_i.

case 2: By bidding w_i, is a loser. So his utility is 0. If he changes his bid to $< w_i$ he is still a loser. If he bids $> w_i$, he is either still a loser OR wins but pays price $\geq w_i$, still deriving utility ≤ 0. So no incentive to not bid w_i. ∎

• Mechanism Design without Payments?

- Broader setting (more optimistic):
  - set of alternatives A (or candidates) (or outcomes)
  - set of n bidders I (voters)
  - L set of linear orders on A (i.e. permutations) → anti-symmetric, and transitive
  - each bidder i has a private $<_i \in L$

    $a >_i b$ means that i prefers a to b

Def: A function $F: L^n \to L$ is called a social welfare function.

A function $F: L^n \to A$ is called a <u>social choice function</u> or a mechanism w/out payment.

Def: A social choice function f can be strategically manipulated by bidder i if for some $\prec_1, \dots, \prec_n \in L$ and some $\prec_i' \in L$

$$f(\prec_1, \dots, \prec_i, \dots, \prec_n) \prec_i f(\prec_1, \dots, \prec_i', \dots, \prec_n)$$

i.e. by lying to $\prec_i'$ bidder i gets a better outcome.

Def: f is incentive compatible if it cannot be manipulated.

Def: f is monotone if

$$f(\prec_1 \dots \prec_i \dots \prec_n) = a \neq a' = f(\prec_1, \dots \prec_i', \dots, \prec_n) \Rightarrow \begin{array}{l} a' \prec_i a \\ \text{and } a \prec_i' a'. \end{array}$$

Proposition: f is IC $\Leftrightarrow$ f is monotone

Proof: obvious

e.g. majority vote between two candidates is IC.

Def: Bidder i is a dictator in social choice function f if for all $\prec_1 \dots \prec_n \in L$:

$$f(\prec_1, \dots, \prec_n) = top(\prec_i).$$

f is a dictatorship if some i is a dictator in it

Theorem (Gibbard-Satterthwaite) Let f be an IC social choice function onto A, where $|A| \geq 3$. Then f is a dictatorship.