ADDENDUM TO THE PAPER: Epipolar Constraints for Vision-Aided Inertial Navigation

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I. Correction to Outlier Protection

Equation 26 has exceptions which could cause valid features to be treated as outliers. The single inequality should be replaced by these two inequalities:

$$\vec{\boldsymbol{e}}_x \circ \left(\vec{\boldsymbol{z}}\,[t_a] - \vec{\boldsymbol{z}}\,[t_b]\right) > 0$$

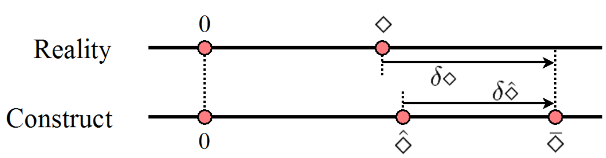
and

$$\left|\vec{\boldsymbol{e}}_x \circ \vec{\boldsymbol{e}}_z\right| < \cos\left(\frac{\pi}{4}\right)$$

These two tests assure that the epipolar constraint is assembled correctly, and that the angle-of-conflict between the IMU data and image data is less than $\frac{\pi}{4}\ radians$.

II. Correction to State Transference

Equations 16–18 contain ambiguous notation and errors. The ornaments (hats and bars) can be clarified by considering the following diagram



Therefore, the notation implies that these statements are always true:

$$\bar{\boldsymbol{x}} = \boldsymbol{x} + \delta\boldsymbol{x} = \hat{\boldsymbol{x}} + \delta\hat{\boldsymbol{x}}$$
$$\hat{\boldsymbol{x}}_T = \bar{\boldsymbol{x}}_T - \delta\hat{\boldsymbol{x}}_T$$
$$\hat{\dot{\boldsymbol{x}}}_T = \bar{\dot{\boldsymbol{x}}}_T - \delta\hat{\dot{\boldsymbol{x}}}_T$$

There are two kinds of updates. The first update is the natural propagation of the state dynamics from Equation 6. After each IMU event, the position and velocity estimates from the new state $\boldsymbol{\psi}\,[t]$ should be transferred according to these equations:

$$\bar{\boldsymbol{x}}_T - \delta\hat{\boldsymbol{x}}_T \Rightarrow \bar{\boldsymbol{x}}_T \ , \ \mathbf{0} \Rightarrow \delta\hat{\boldsymbol{x}}_T$$
$$\bar{\dot{\boldsymbol{x}}}_T - \delta\hat{\dot{\boldsymbol{x}}}_T \Rightarrow \bar{\dot{\boldsymbol{x}}}_T \ , \ \mathbf{0} \Rightarrow \delta\hat{\dot{\boldsymbol{x}}}_T$$

The second update comes from the BLS equations, specifically Equation 13. It should be clear that state estimate is the output of the Kalman filter:

$$\boldsymbol{\psi}^{+} = \begin{bmatrix} \delta\hat{\boldsymbol{x}}_T \\ \delta\hat{\dot{\boldsymbol{x}}}_T \\ \delta\hat{\boldsymbol{b}}_{TurnOn} \\ \delta\hat{\boldsymbol{b}}_{InRun} \\ \delta\hat{\boldsymbol{s}}_{TurnOn} \\ \delta\hat{\boldsymbol{s}}_{InRun} \end{bmatrix}$$

The position and velocity estimates are then transferred through an approximation of spherical coordinates. In other words, the updates occur on the surface of a sphere of radius $\|\Delta\bar{\boldsymbol{x}}_T\|$ as follows:

$$\bar{\boldsymbol{x}}_T\,[t_a] + \vec{\boldsymbol{e}}_{upd}\,\|\Delta\bar{\boldsymbol{x}}_T\| \Rightarrow \bar{\boldsymbol{x}}_T\,[t_b] \ , \ \mathbf{0} \Rightarrow \delta\hat{\boldsymbol{x}}_T$$
$$\bar{\dot{\boldsymbol{x}}}_T - \left(\boldsymbol{I} - \vec{\boldsymbol{e}}_{upd}\vec{\boldsymbol{e}}_{upd}^{\mathrm{T}}\right)\delta\hat{\dot{\boldsymbol{x}}}_T \Rightarrow \bar{\dot{\boldsymbol{x}}}_T \ , \ \mathbf{0} \Rightarrow \delta\hat{\dot{\boldsymbol{x}}}_T$$

with

$$\vec{\boldsymbol{e}}_{upd} = \frac{\Delta\bar{\boldsymbol{x}}_T - \delta\hat{\boldsymbol{x}}_T}{\|\Delta\bar{\boldsymbol{x}}_T - \delta\hat{\boldsymbol{x}}_T\|}$$

III. Public Data Sets

We have applied our method to several real and simulated data sets, and they are available online, along with ground truth data. Please see `http://www.mit.edu/~ddiel/DataSets`, and feel free to contact us with any questions.