

# idioms of logical modelling

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SBMF/ICGT · Natal · Sept 20, 2006



**introduction**

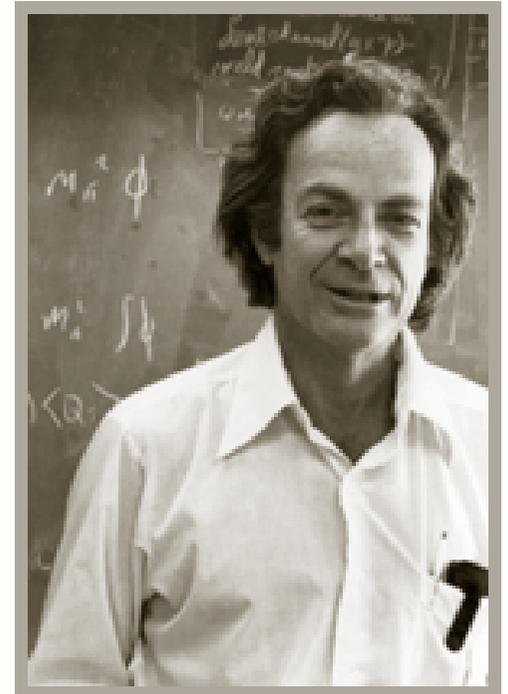
# premises

software development needs

- › simple, expressive and precise notations
- › deep and automatic analyses
- ... especially in early stages

The first principle is that you must not fool yourself, and you are the easiest person to fool.

--Richard P. Feynman



# desiderata

syntax: flexible and easy to use

› eg, declarations & navigations like OMT, Syntropy, etc

semantics: simple and uniform

› eg, relational logic like Z

analysis: fully automatic and interactive

› eg, symbolic model checking like SMV

# transatlantic alloy



Oxford, home of Z



Pittsburgh, home of SMV

# alloy project

*version*

*language*

*analysis*

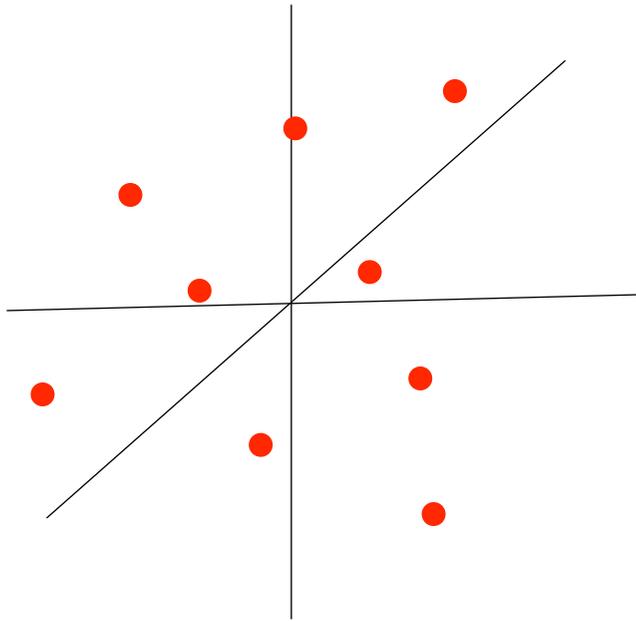
*sample case study*

Nitpick (1995)	relational calculus subset of Z	relation enumeration	IPv6 routing
Alloy 1 (1999)	+ navigation exps quantifiers	WalkSAT, DP	intentional naming
Alloy 2 (2001)	+ relational ops higher arity	Chaff, Berkmin symmetry, sharing	key management, Unison filesync
Alloy 3 (2004)	+ subtyping, overloading	+ atomization (bad)	Mondex electronic purse
Alloy 4 (2007)	+ imperative features	sparse matrices better sharing	

# scope-complete analysis

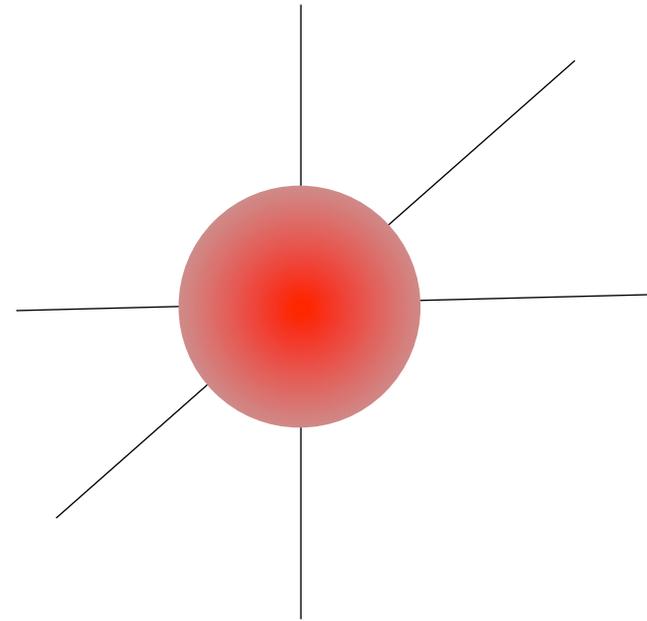
observations about analyzing designs

- › most assertions are wrong
- › most flaws have small counterexamples



testing:

a few cases of arbitrary size



scope-complete:

all cases within small scope

**pure logic modelling**

# traditional approach

built-in notions

- › state, invariant, operation, trace

standard idiom

- › a fixed view of software systems

examples

- › state-invariant-operation (Z, B, VDM, OCL)
- › state-update-formula (SMV, Murphi)
- › state-guarded command-formula (SPIN)
- › heap-stack-if-while (Pathfinder, Bandera)

# pure logic modelling

suppose we had

- › no built-in notions
- › no fixed idiom

what might the language look like?

what idioms could we express?

how naturally could we simulate standard idioms?

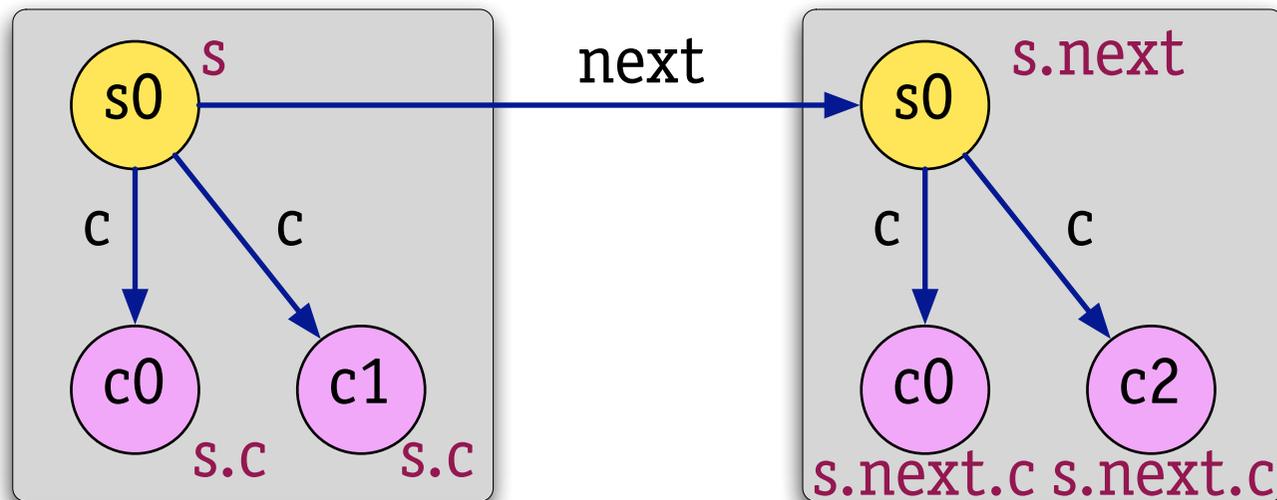
# everything's a relation

Alloy uses relations for

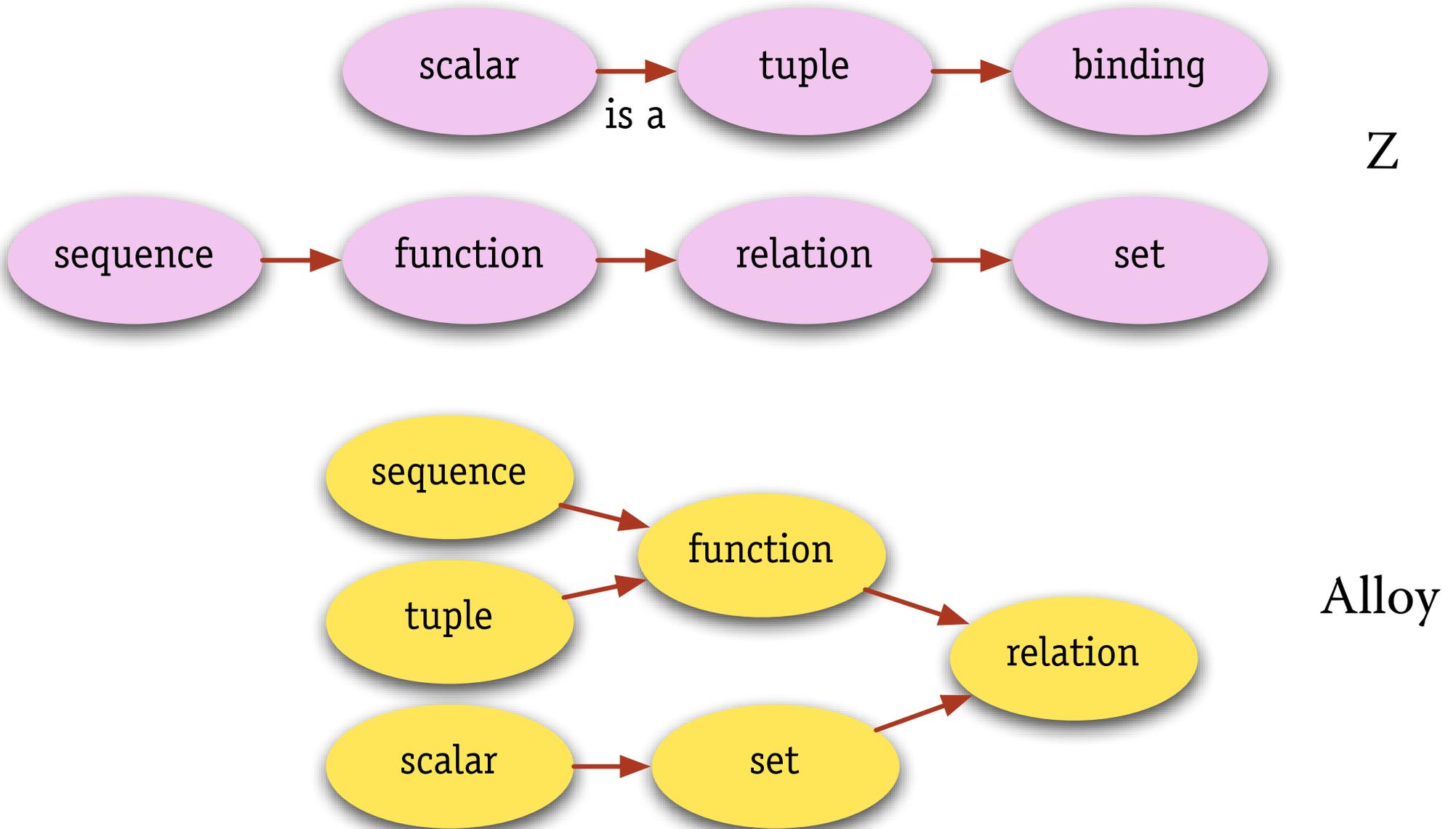
- › all datatypes -- even sets, scalars and tuples
- › structures in space *and* time

key operator is **dot join**

- › for taking components of a structure
- › for indexing into a collection
- › for resolving indirection



# relations from Z to A

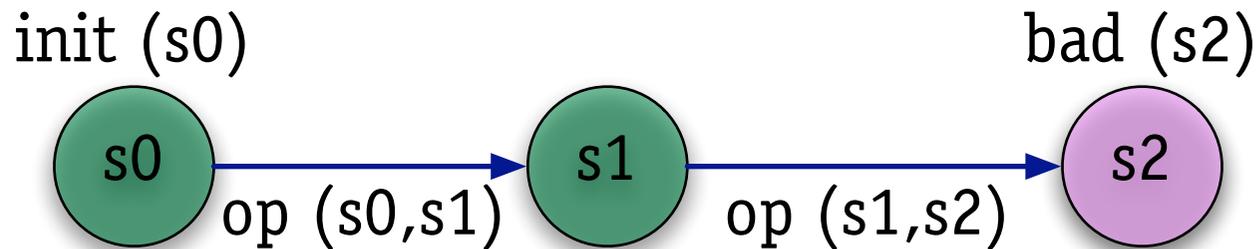


# everything's a constraint

no special syntax or semantics for state machines

use constraints for describing

- › subtypes & classification
- › declarations & multiplicity
- › invariants, operations & traces
- › assertions, including temporal
- › equivalence under refactoring



**an example: hotel locking**

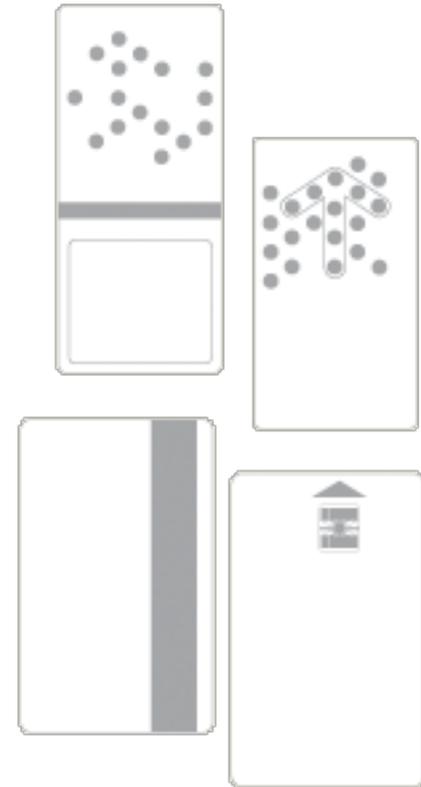
# hotel locking

recodable locks (since 1980)

- › new guest gets a different key
- › lock is 'recoded' to new key
- › last guest can no longer enter

how does it work?

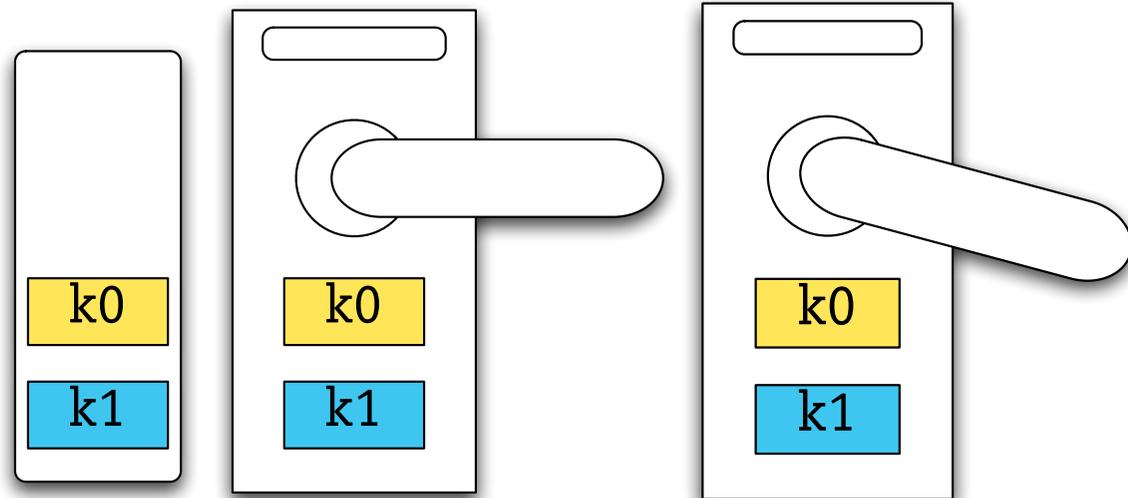
- › locks are standalone, not wired



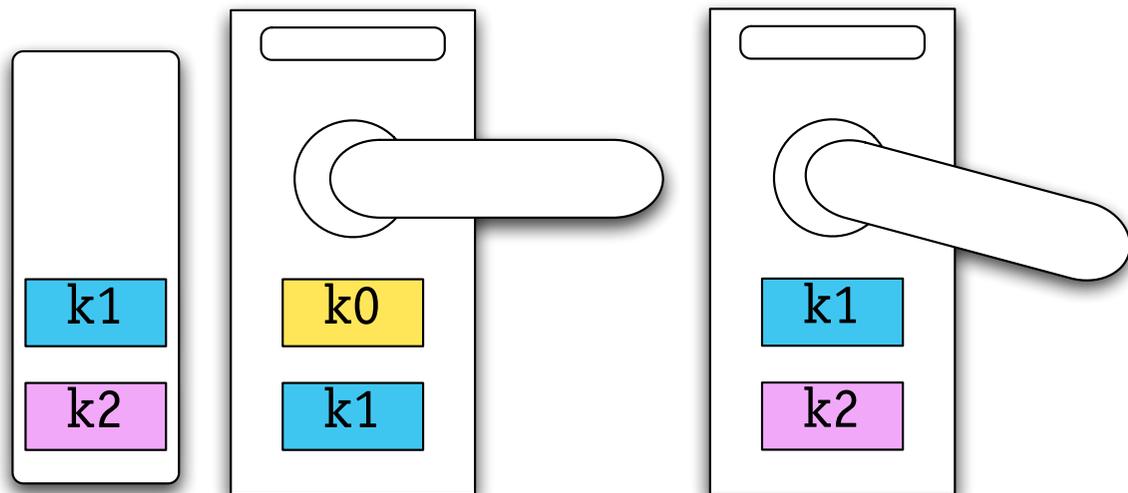
# a recodable locking scheme

from US patent 4511946; many other similar schemes

card & lock have two keys  
if both match, door opens



if first card key matches  
second door key, door opens  
and lock is recoded



# challenge

model this scheme

LOCKS() -- locking mechanism

GUESTS() -- how guests and hotel staff are supposed to behave

formulate a requirement

SAFE() -- only guest who owns a room can enter it

check

LOCKS() **and** GUESTS() **implies** SAFE() ??

**elements of alloy**

# alloy in 3 slides

## signatures

- › provide classification hierarchy for sets
- › composite structure of objects
- › local name space for relations
- › incremental development

## relational logic

- › unusually simple and uniform
- › generalized join

## facts, predicates and assertions

- › simple packaging of constraints

# signatures & fields

**sig** A {}

-- introduces a set of atoms called A

**sig** B **extends** A {}

-- introduces a subset B of A

**sig** C **extends** A {}

-- introduces a subset C of A disjoint from B

**sig** A {f: B}

-- introduces a binary relation from A to B called f

**sig** A {f: B->C}

-- introduces a ternary relation from A to B to C called f

# relational operators

$p + q$      $\{t \mid t \in p \vee t \in q\}$

$p - q$      $\{t \mid t \in p \wedge t \notin q\}$

$p \& q$      $\{t \mid t \in p \wedge t \in q\}$

$p \rightarrow q$      $\{(p_1, \dots, p_n, q_1, \dots, q_m) \mid (p_1, \dots, p_n) \in p \wedge (q_1, \dots, q_m) \in q\}$

$p \cdot q$      $\{(p_1, \dots, p_{n-1}, q_2, \dots, q_m) \mid (p_1, \dots, p_n) \in p \wedge (p_n, q_2, \dots, q_m) \in q\}$

$p \text{ in } q$      $\{(p_1, \dots, p_n) \in p\} \subseteq \{(q_1, \dots, q_n) \in q\}$

$p = q$      $\{(p_1, \dots, p_n) \in p\} = \{(q_1, \dots, q_n) \in q\}$

eg, given **sig**  $A \{f: B \rightarrow C\}$

some expressions and their types:

$a.f: B \rightarrow C$

$f.c: A \rightarrow B$

$b.(a.f): \text{set } C$

# constraints & commands

**fact** {F}

-- establishes formula F, as an assumption

**pred** P () {Fp}

-- declares predicate P; invocation equivalent to inlining Fp

**assert** A () {Fa}

-- declares assertion A, claiming that formula Fa is valid

**run** P

-- instructs analyzer to find instance satisfying facts and Fp

**check** A

-- instructs analyzer to find instance satisfying facts and **not** Fa

**a parade of idioms**

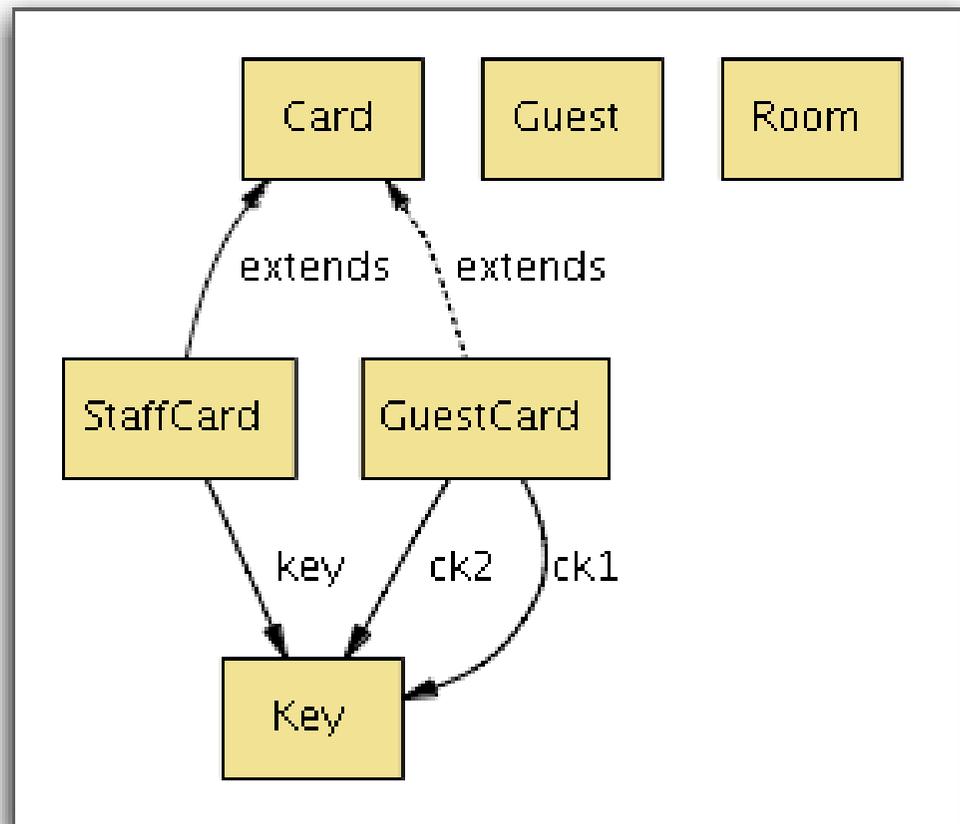
# object model, OCL style

**sig** Room, Guest, Key {}

**sig** Card {}

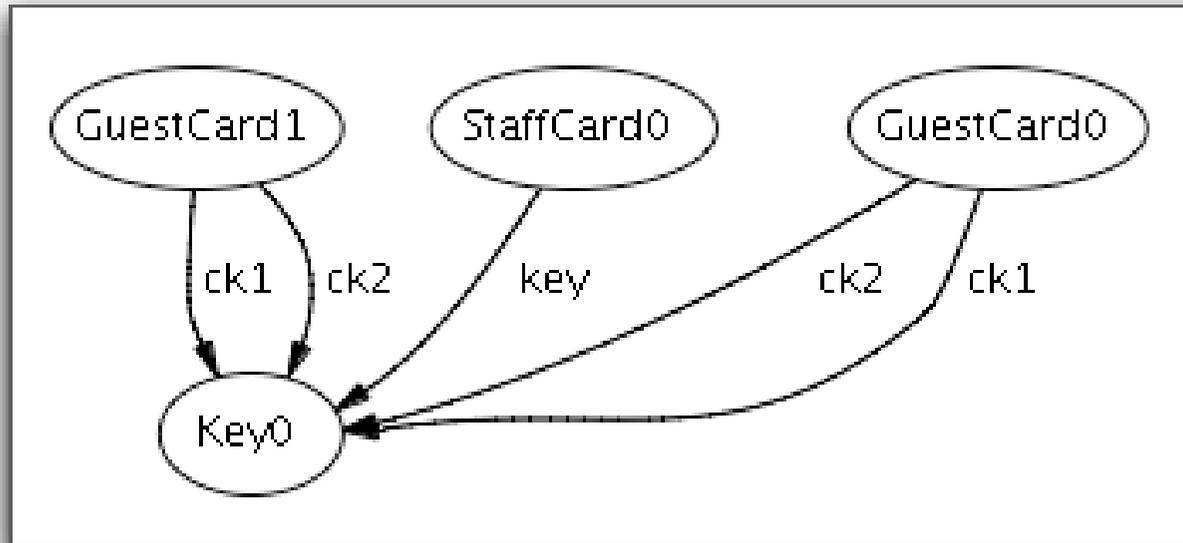
**sig** StaffCard **extends** Card {key: Key}

**sig** GuestCard **extends** Card {ck1, ck2: Key}



# generating an instance

```
pred show () {}  
run show
```



# state/operation, Z style

```
sig Room, Guest {}
```

```
sig State {  
  owns: Room -> Guest  
}
```

no special interpretation for ' mark

```
pred checkin (s, s': State, r: Room, g: Guest) {  
  s'.owns = s.owns + r -> g  
}
```

```
run checkin
```

# checking an invariant

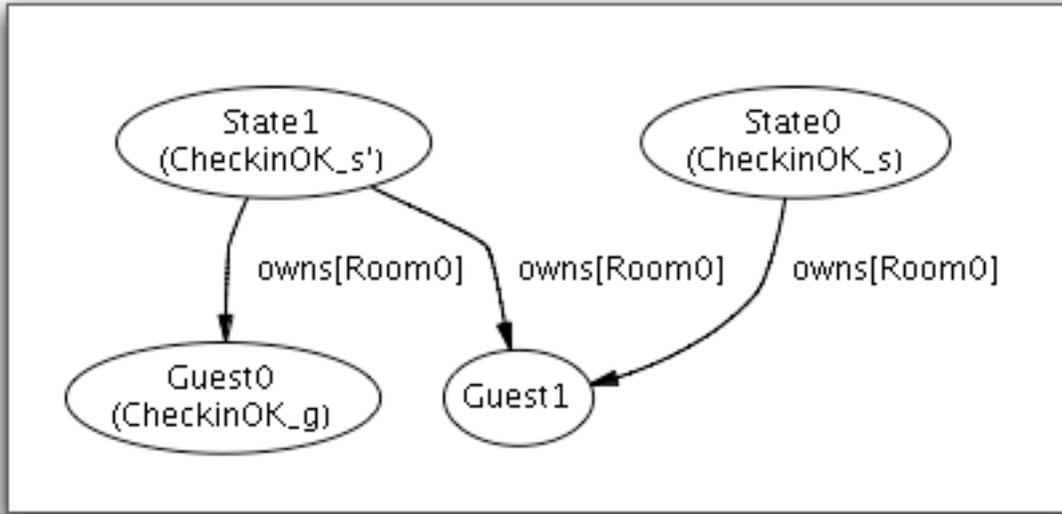
```
pred NoDoubleBooking (s: State) {  
  s.owns : Room -> lone Guest  
}
```

```
assert CheckinOK {  
  all s, s': State, r: Room, g: Guest |  
    NoDoubleBooking (s) and checkin (s, s', r, g)  
    implies NoDoubleBooking (s')  
}
```

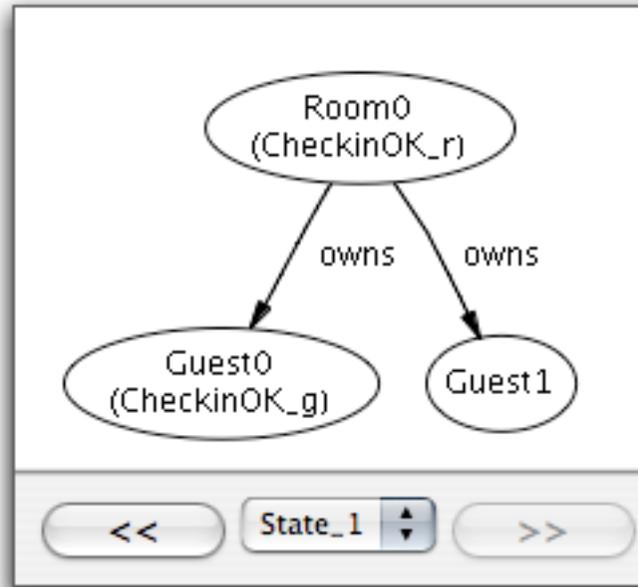
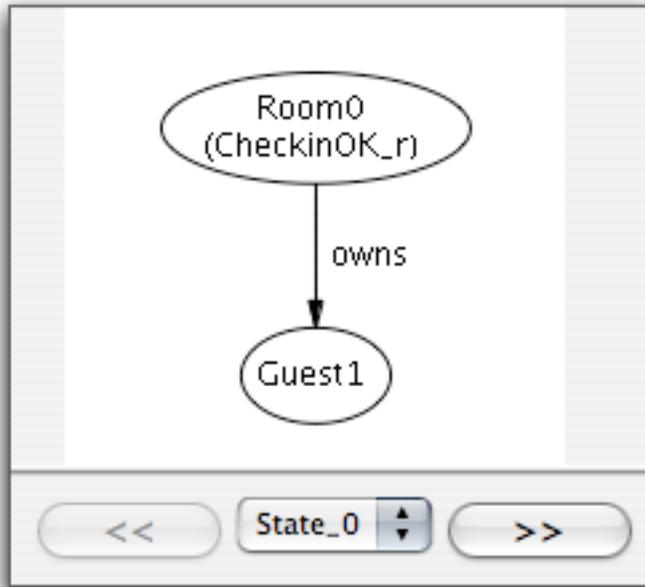
no metalanguage for theorems

```
check CheckinOK for 3 but 2 State
```

# counterexample!



default visualization



visualization  
after 'projecting'  
*State*

can project *any* type

**sig** State {owns: Room -> Guest}

# implicit precondition, Z style

```
pred checkin (s, s': State, r: Room, g: Guest) {  
  no r.(s.owns)  
  s'.owns = s.owns + r -> g  
}
```

**no**  $e$  means  $e$  is empty relation

*analyzer says:*

No counterexample found: CheckinOK is valid within the specified scope. (00:02)

no counterexample  $\Rightarrow$  valid? no!

# growing the state, Z style

```
sig State {  
  owns: Room -> Guest  
}
```

```
sig State1 extends State {  
  issued: set Card  
}
```

```
pred checkin1 (s, s': State1, r: Room, g: Guest, c: Card) {  
  checkin (s, s', r, g)  
  c not in s.issued  
  s'.issued = s.issued + c  
}
```

unlike Z schema: **semantic, not syntactic**  
can add defined component like this:

```
sig State2 extends State1 {empty: set Room}  
  empty = {r: Room | no r.owns}  
}  
fact {State2 = State1}
```

# reachability, BMC style

**module** traces

**open** util/ordering [State]

**sig** Room, Guest {}

**sig** State {owns: Room -> Guest}

**pred** init (s: State) {**no** s.owns}

**pred** checkin (s, s': State, r: Room, g: Guest) {s'.owns = s.owns + r -> g}

**fact** traces {

init (first())

**all** s: State - last () |

**some** r: Room, g: Guest | checkin (s, next(s), r, g)

}

**assert** NoDoubleBooking {

**all** s: State | s.owns : Room -> **lone** Guest

}

**check** NoDoubleBooking

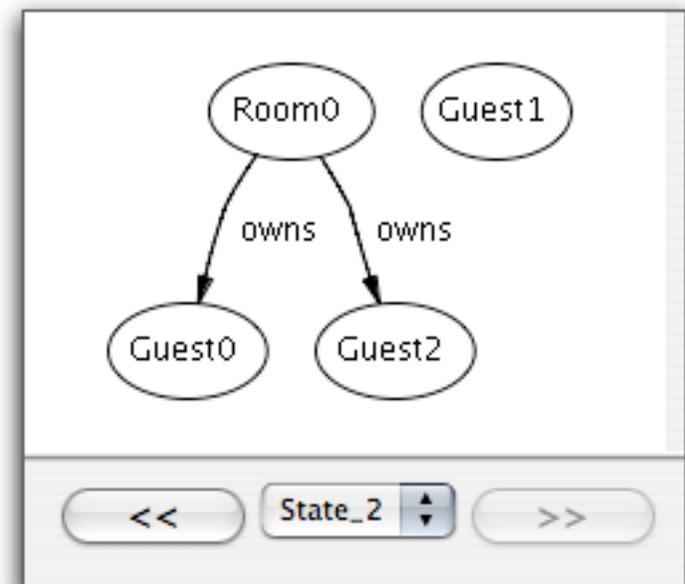
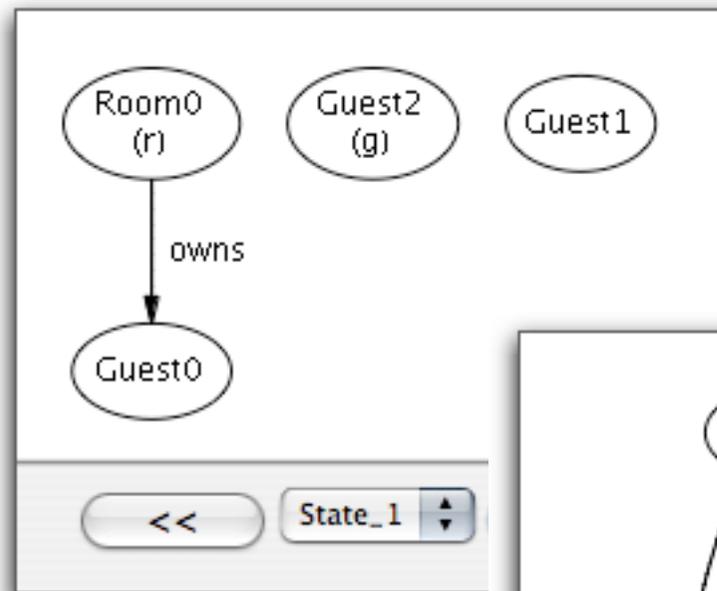
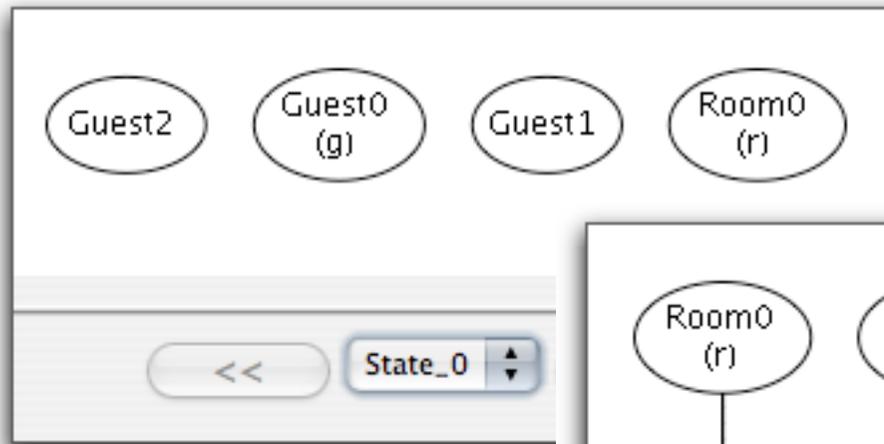
order states with library module

constrain order to satisfy ops

an assertion over *reachable* states

# counterexample trace

operation names don't appear:  
they're not objects, just names  
for constraints



# objects with local state

```
sig Key, Time {}  
sig Card {k1, k2: Key}
```

signatures define local namespaces;  
overloading resolved automatically

```
sig Room {  
  k1, k2: Key one -> Time  
}
```

mutable component has Time column

```
pred enter (r: Room, c: Card, t, t': Time) {  
  c.k1 = r.k2.t  
  k1.t' = k1.t ++ r -> c.k1  
  k2.t' = k2.t ++ r -> c.k2  
}
```

f.t is field f at time t

# events as objects

```
sig Key, Time {}  
sig Card {k1, k2: Key}  
sig Room {k1, k2: Key one -> Time}  
sig Guest {cards: Card -> Time}
```

```
abstract sig HotelEvent {  
  pre, post: Time,  
  guest: Guest  
}
```

```
abstract sig RoomCardEvent extends HotelEvent { room: Room, card: Card }
```

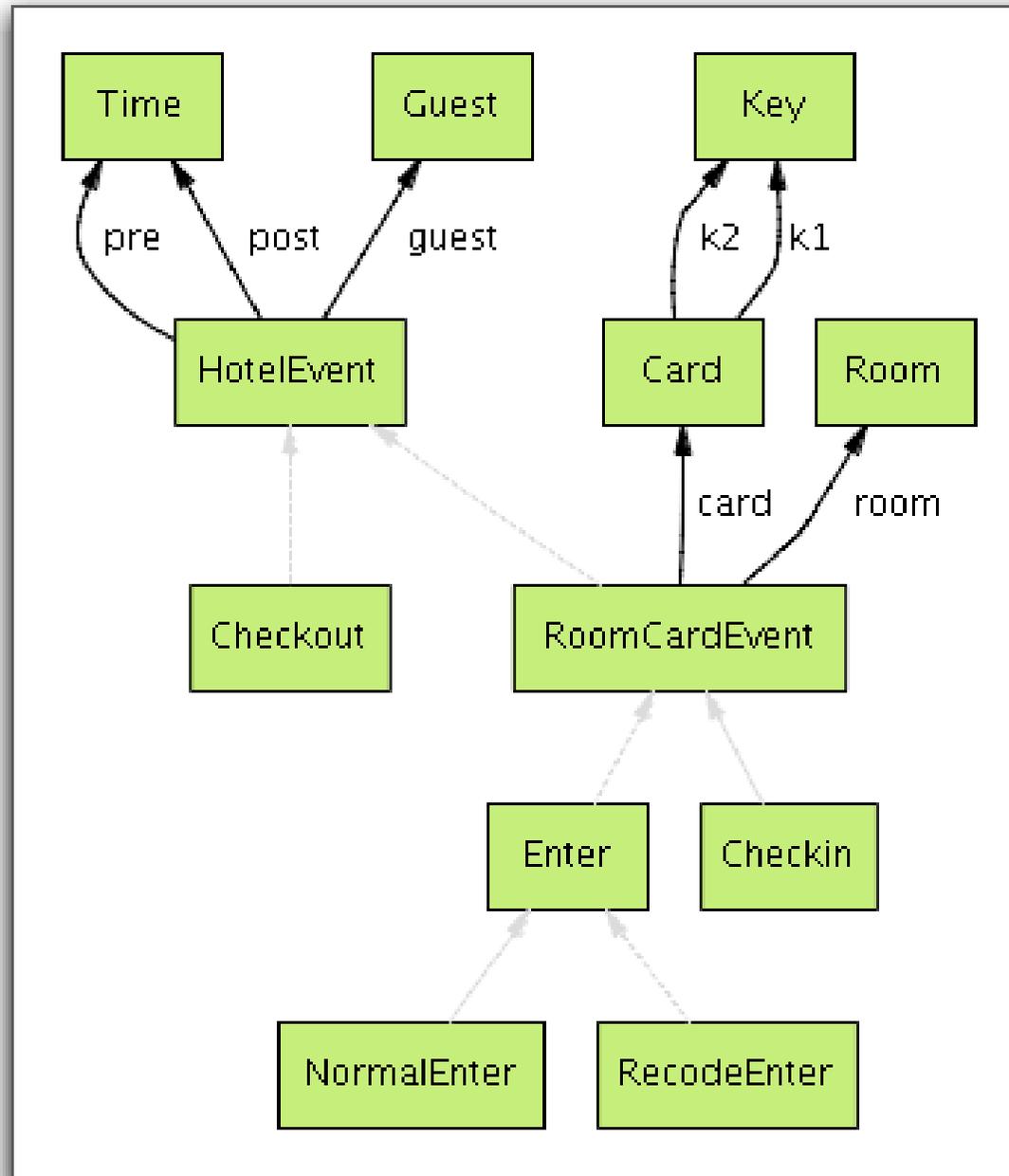
```
abstract sig Enter extends RoomCardEvent { } { card in guest.cards.pre }
```

```
sig NormalEnter extends Enter { } { card.k1 = room.k1.pre }
```

```
sig RecodeEnter extends Enter { } {  
  card.k1 = room.k2.pre  
  k1.post = k1.pre ++ room -> card.k1  
  k2.post = k2.pre ++ room -> card.k2  
}
```

like Z's schema components  
and Java's instance variables,  
fields of signatures are *free variables*  
in extending signature

# object model for events



# Reiter's frame conditions

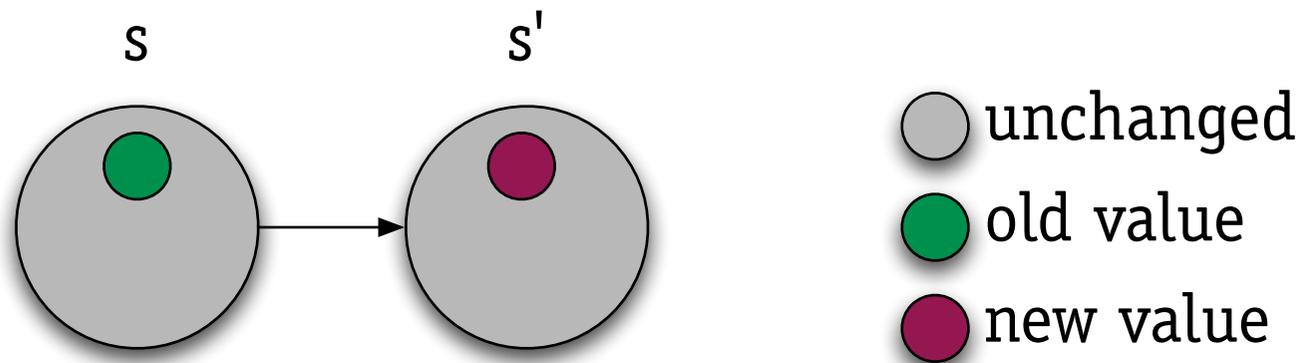
in declarative models

> unmentioned  $\neq$  unchanged

Ray Reiter's scheme

> add 'explanation closure axioms'

if field **f** changed, then event **e** happened



See: Alex Borgida, John Mylopoulos and Raymond Reiter.

On the Frame Problem in Procedure Specifications.

IEEE Transactions on Software Engineering, 21:10 (October 1995), pp. 785-798.

# frame conditions, Reiter style

**fact** Traces {

**all** t: Time - last () | let t' = next (t) |

**some** e: HotelEvent {

e.pre = t **and** e.post = t'

k1.t = k1.t' **and** k2.t = k2.t' **or** e **in** RecodeEnter

issued.t = issued.t' **and** cards.t = cards.t' **or** e **in** Checkin

owns.t = owns.t' **or** e **in** Checkin + Checkout

}

}

if k1 or k2 changed, then  
RecodeEnter must have happened

# a safety assertion

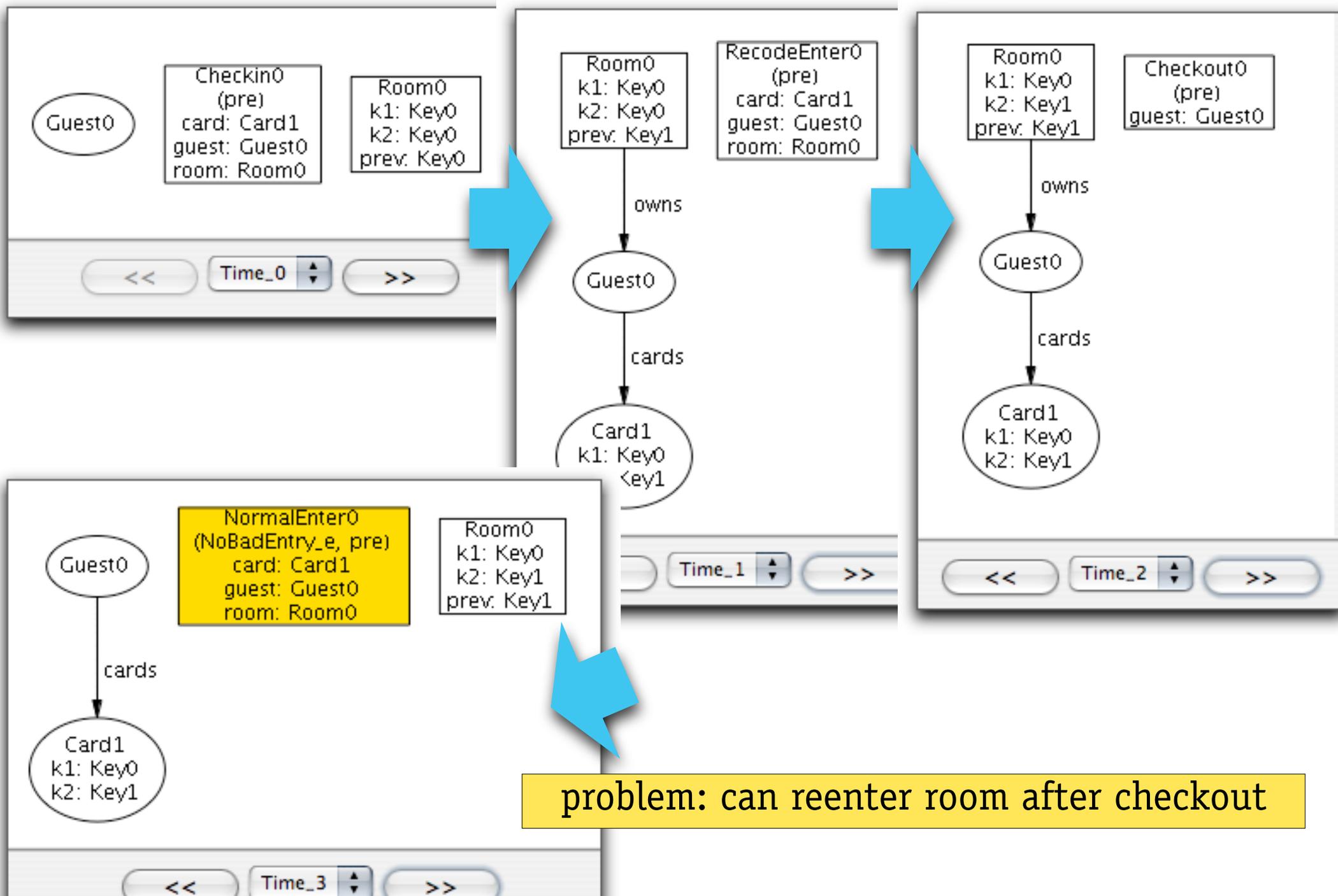
safety condition

- › if an enter event occurs,  
then the guest who enters is an occupant

```
assert NoBadEntry {  
  all e: Enter | e.guest in e.room.owns.(e.pre)  
}
```

this assertion is about events, and is not expressible in purely state-based formalisms

# counterexample!



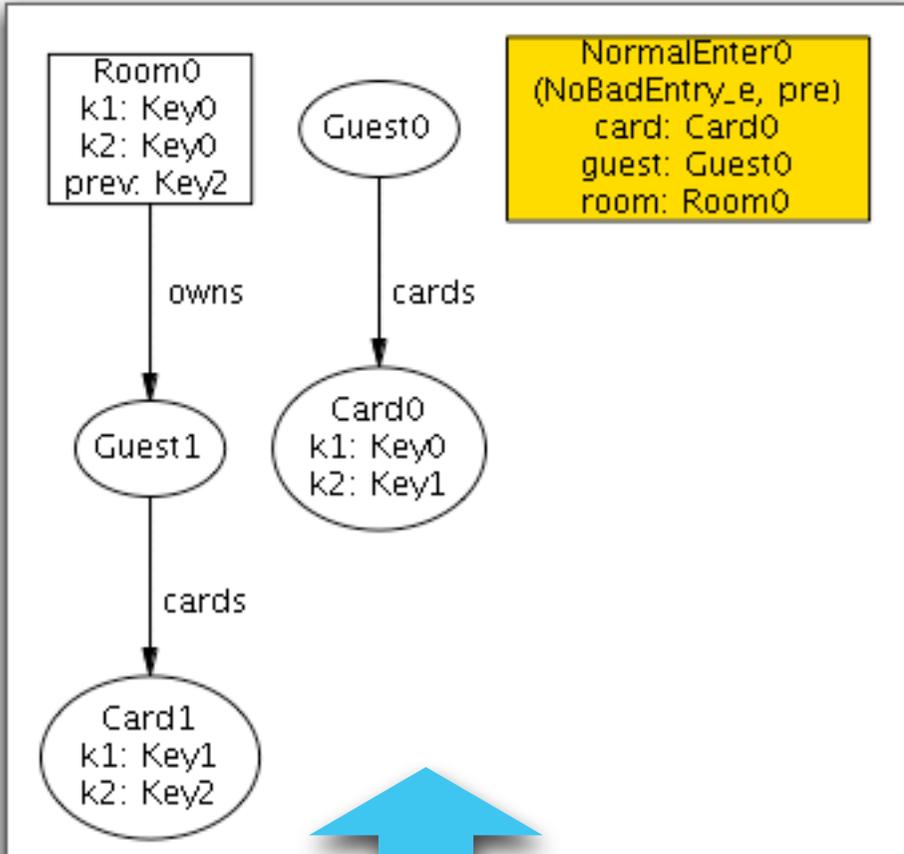
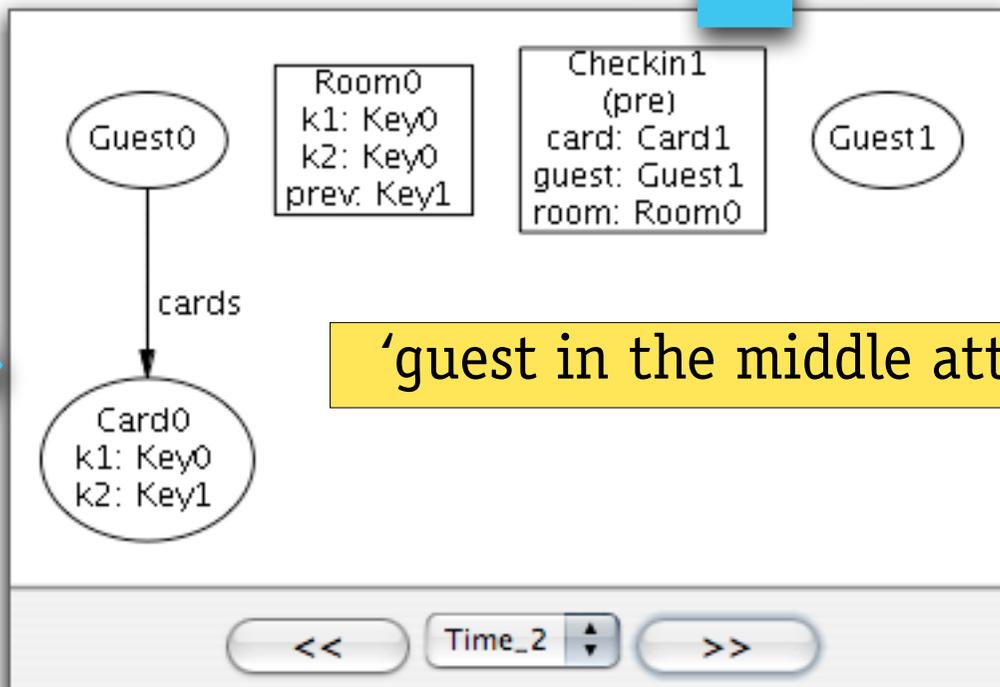
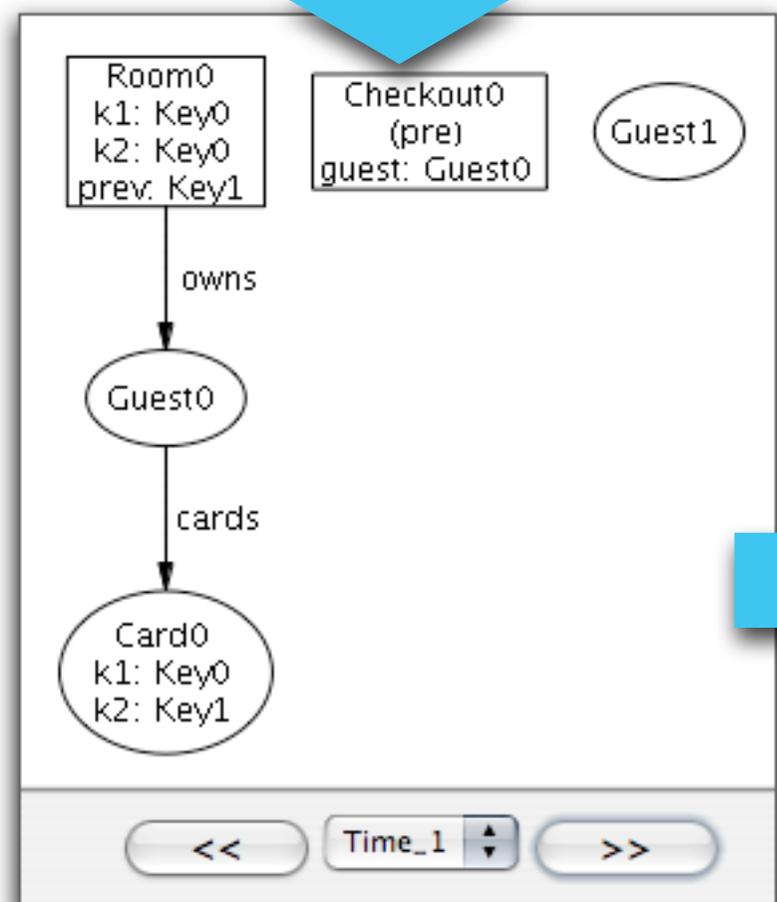
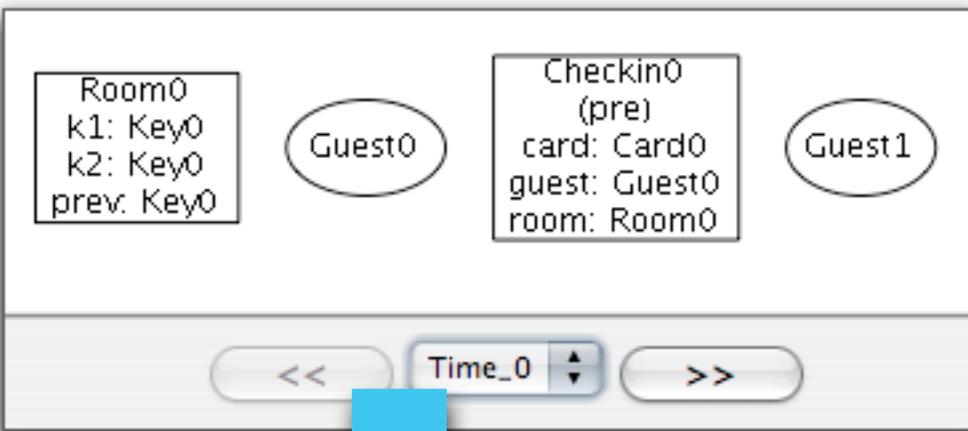
# a relaxed safety assertion

safety condition

- › if an enter event occurs, and the room is occupied, then the guest who enters is an occupant

```
assert NoBadEntry {  
  all e: Enter | let owners = e.r.owns.(e.pre) |  
    some owners => e.g in owners  
}
```

# counterexample!



'guest in the middle attack'

# constraining the environment

after checking in, guest immediately enters room:

```
fact NoIntervening {  
  all c: CheckinEvent |  
    some e: EnterEvent {  
      e.pre = c.post  
      e.room = c.room  
      e.guest = c.guest  
    }  
}
```

**conclusions**

# how to be safe in a hotel

don't let the bellboy open your door!

› must open it yourself to satisfy **NoIntervening**

# pluralistic modelling

Alloy supports a wide range of idioms and styles

good for teaching

- › what you see is what you get
- › simple underlying logic
- › all analysis is model finding

good for research

- › can experiment easily with new idioms

good for practice

- › can tailor idiom to the problem
- › example: Jazayeri's model of Apple's Bonjour mentioned 'two states ago'

# hotel locking case study

contributions in my book from

- › Martin Gogolla (OCL)
- › Jim Woodcock (Z)
- › Peter Gorm Larsen and John Fitzgerald (VDM)
- › Michael Butler (B)

chapter available at <http://softwareabstractions.org/>

recently also by Tobias Nipkow in Isabelle

- › proves safety for weaker (localized) condition
- › shows equivalence of trace- and state-based models

# acknowledgments

*current students  
& collaborators  
who've worked on Alloy*

Greg Dennis  
Derek Rayside  
Robert Seater  
Mana Taghdiri  
Emina Torlak  
Jonathan Edwards  
Vincent Yeung

*former students  
who've worked on Alloy*

Sarfraz Khurshid  
Mandana Vaziri  
Ilya Shlyakhter  
Manu Sridharan  
Sam Daitch  
Andrew Yip  
Ning Song  
Edmond Lau  
Jesse Pavel  
Ian Schechter  
Li-kuo Lin  
Joseph Cohen  
Uriel Schafer  
Arturo Arizpe

# for more info

[alloy.mit.edu](http://alloy.mit.edu)

- › downloads, papers, tutorial

[alloy@mit.edu](mailto:alloy@mit.edu)

- › questions about Alloy
- › send us a challenge

[dnj@mit.edu](mailto:dnj@mit.edu)

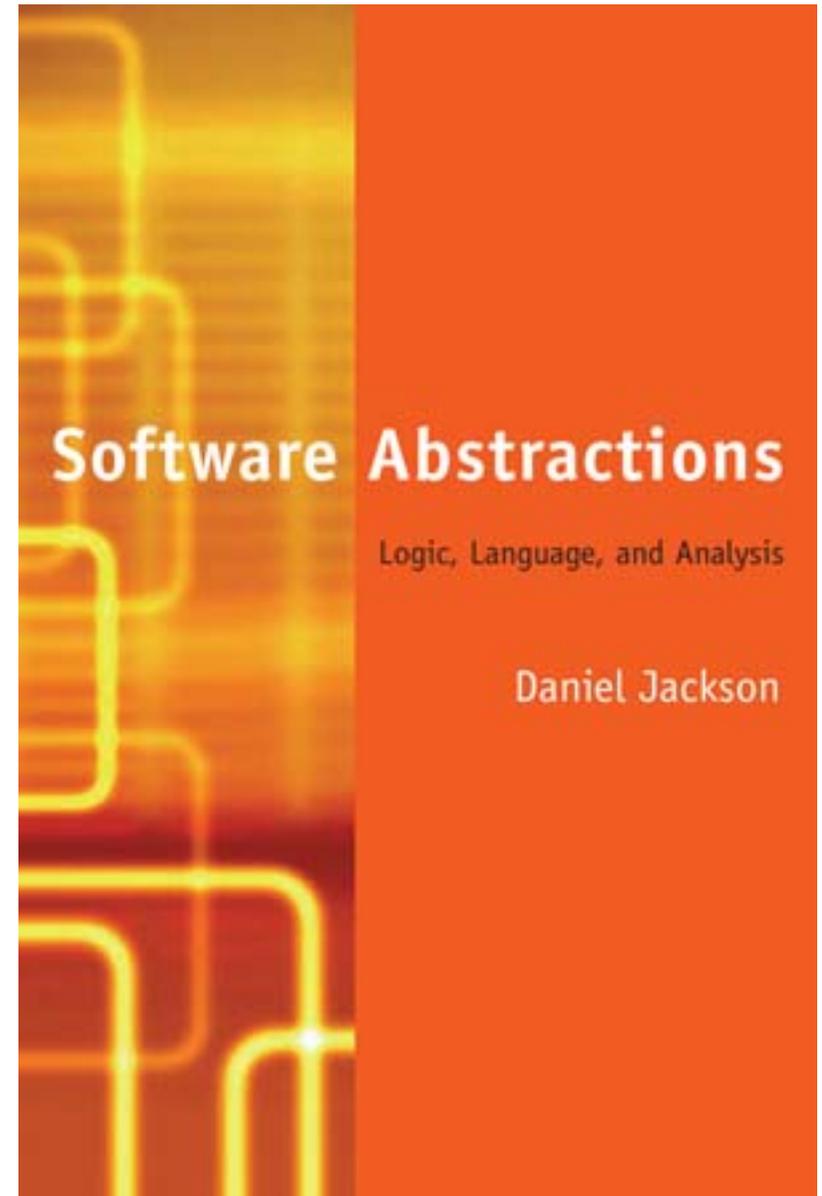
- › happy to hear from you!

[mit.edu/people/emina/kodkod.html](http://mit.edu/people/emina/kodkod.html)

- › Alloy as an API

*Software Abstractions*

- › MIT Press, March 2006
- › discount available to ICGT/SBMF



**spare slides: evaluation of Alloy**

# alloy case studies at MIT

many small case studies

- › intentional naming [Balakrishnan+]
- › Chord peer-to-peer lookup [Kaashoek+]
- › Unison file sync [Pierce+]
- › distributed key management
- › beam scheduling for proton therapy
- › Mondex electronic purse

typically

- › 100-1000 lines of Alloy
- › analysis in 10 secs - 1 hour
- › 3-20 person-days of work

# some alloy applications

## in industry

- › animating requirements (Venkatesh, Tata)
- › military simulation (Hashii, Northrop Grumman)
- › role-based access control (Zao, BBN)
- › generating network configurations (Narain, Telcordia)

## in research

- › exploring design of switching systems (Zave, AT&T)
- › checking semantic web ontologies (Jin Song Dong)
- › enterprise modelling (Wegmann, EPFL)
- › checking refinements (Bolton, Oxford)
- › security features (Pincus, MSR)

# alloy in education

**courses using Alloy** at Michigan State (Laura Dillon), Imperial College (Michael Huth), National University of Singapore (Jin Song Dong), University of Iowa (Cesare Tinelli), Queen's University (Juergen Dingel), University of Waterloo (Joanne Atlee), Worcester Polytechnic (Kathi Fisler), University of Wisconsin (Somesh Jha), University of California at Irvine (David Rosenblum), Kansas State University (John Hatcliff and Matt Dwyer), University of Southern California (Nenad Medvidovic), Georgia Tech (Colin Potts), Politecnico di Milano (Carlo Ghezzi), Rochester Institute of Technology (Michael Lutz), University of Auckland (John Hamer, Jing Sun), Stevens Institute (David Naumann), USC (David Wilczynski)

# good things

conceptual simplicity and minimalism

- › very little to learn
- › WYSIWYG: no special semantics (eg, for state machines)
- › expressive declarations

high-level notation

- › constraints -- can build up incrementally
- › relations flexible and powerful
- › much more succinct than most model checking notations

automatic analysis

- › no lemmas, tactics, etc
- › counterexamples are never spurious
- › visualization a big help
- › can do many kinds of analysis: refinement, BMC, etc

# bad things

relations aren't a panacea

- › sequences are awkward
- › treatment of integers limited

limitations of logic

- › recursive functions hard to express
- › sometimes, want iteration and mutation

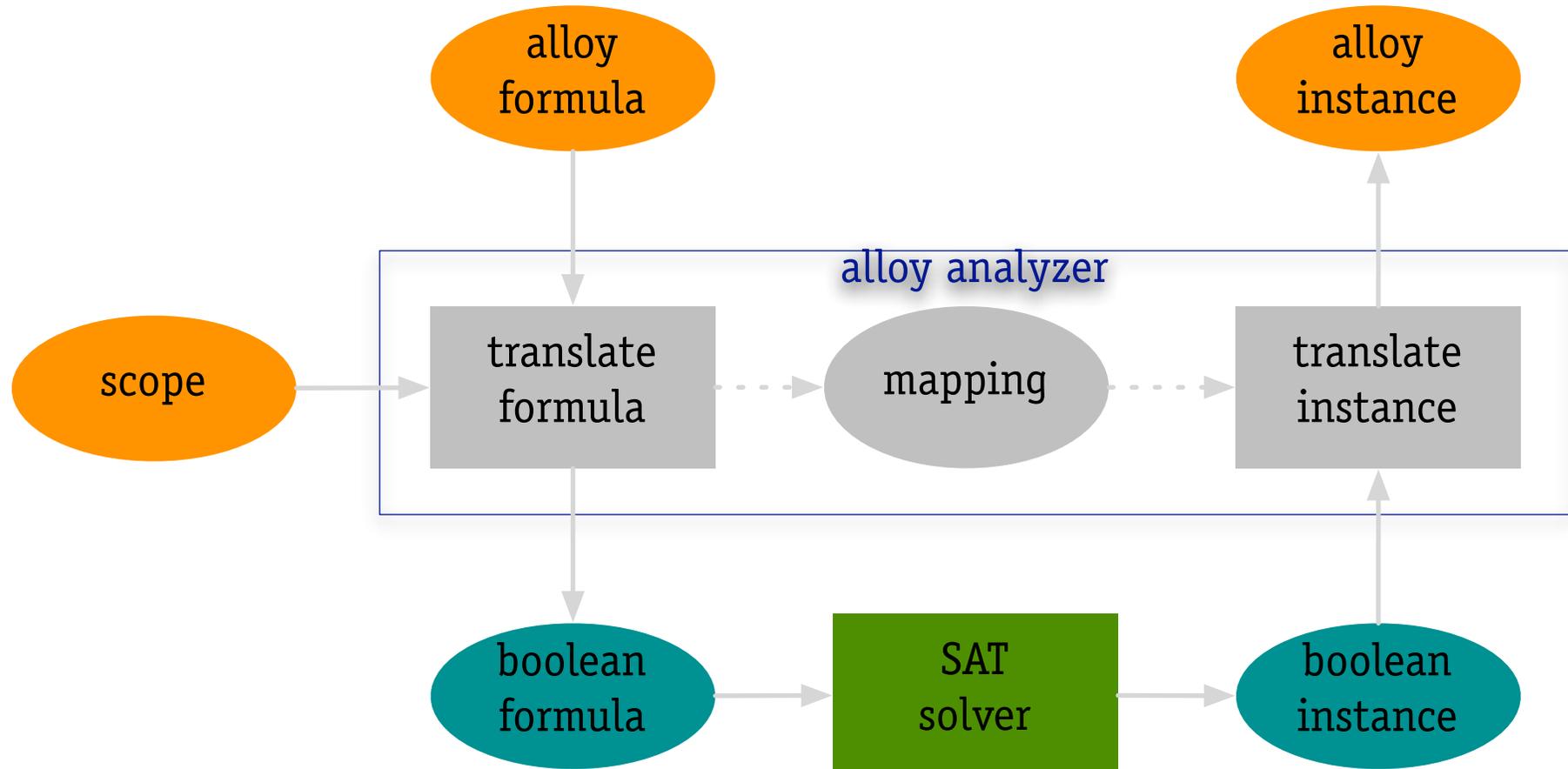
limitations of language

- › module system doesn't offer real encapsulation

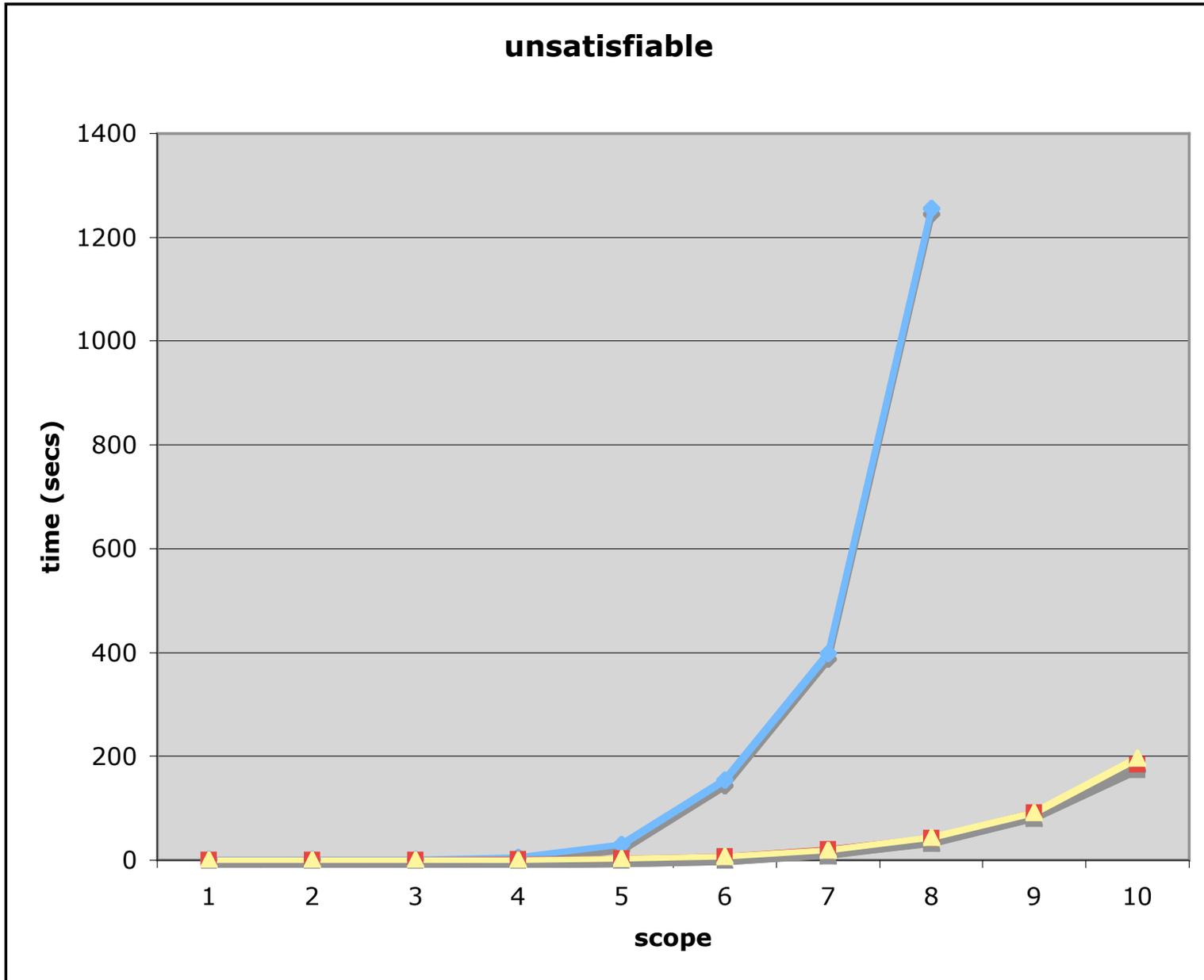
limitations of tool

- › tuned to generating instances (hard) rather than checking instances (easy)

# alloy analyzer architecture



# performance (unsat)



# performance (sat)

