6898: Advanced Topics in Software Design MIT Lab for Computer Science March 18, 2002 Daniel Jackson

topics for today

familiar notions (from Scheme)

› let bindings, functions, closures, lists

new notions (from ML)

- inferred types and parametric polymorphism
- side-effects and the unit type
- datatypes (variants)

functions

```
applying an anonymous function
# (fun x -> 2 * x) 3;;
-: int = 6

declaring a function and applying it
# let dbl = fun x -> 2 * x;;
val dbl : int -> int = <fun>
# dbl 3;;
-: int = 6
```

```
functionals, or higher-order functions # let twice = fun f -> (fun x -> (f (f x)));; val twice : ('a -> 'a) -> 'a -> 'a = <fun> # (twice dbl) 3;; -: int = 12
```

let bindings

```
# X;;
# fact 4;;
                                                          # let rec fact i = if i = 0 then 1 else i * fact (i - 1);;
                                                                                                                                                                                                    val x : int = 5
                                                                                                                                                                                                                                                                                                                                                   # let x = 3 and y = 4 in x + y;;
                                                                                                                                                                                                                                                                                                                                                                                 a let expression binds a variable to a value
                              val fact : int -> int = <fun>
                                                                                        recursive let
                                                                                                                                                                                                                                   # let x = 5;;
                                                                                                                                                                                                                                                               read-eval-print-loop uses let instead of define
                                                                                                                                                                                                                                                                                                                       -: int = 7
                                                                                                                                                 -: int = 5
```

-: int = 24

let vs. define

```
# let k = 5;;

val k: int = 5

# let f = fun x -> x + k;;

val f: int -> int = <fun>

# f 3;;

-: int = 8

# let k = 6;;

val k: int = 6

# f 3;;

-: int = 8
```

let is lexical

» no side-effecting top-level define built-in

tuples

```
tuple constructor
# let x = 1, 2;;
val x : int * int = 1, 2
# fst x;;
-: int = 1
# snd x;;
-: int = 2

empty tuple, used instead of 'void'
# ();;
-: unit = ()
# print_string;;
-: string -> unit = <fun>
```

function arguments

```
# (diff 3 4);;
                                                                                                   # diff (3, 4);;
                                                                                                                                val diff: int * int -> int = <fun>
                                                                                                                                                                    # let diff (i,j) = if i < j then j-i else i-j;;
                                                                                                                                                                                                       tupled form: like in an imperative language
This function is applied to too many arguments
                                                                   -: int = 1
```

(diff 3) 4;; # let diff i j = if i < j then j-i else i-j;; val diff: int -> int -> int = <fun> curried form: stages the computation -: int = 1

(diff 3 4);;

-: int = 1

lists

```
# [];;
- : 'a list = []
                                                                                                  # [1;[2]];;
                                                                                                                                                                                                                                                                          # [1;2];;
                                                                                                                         # [[1]];;
-: int list list = [[1]]
                                                                                                                                                                                                            # 1::2::[];;
-: int list = [1; 2]
                                       empty list is polymorphic
                                                                                                                                                                   lists are homogeneous
                                                                                                                                                                                                                                                     -: int list = [1; 2]
                                                                                 This expression has type 'a list but is here used with type int
```

polymorphic functions

```
# cons 1 [];;
                                                                                                                                                                                a polymorphic function over lists
-: int list = [1]
                                                                                        # cons 1 2;;
                                                                                                                      # let cons e l = e :: l;;
val cons : 'a -> 'a list -> 'a list = <fun>
                                                                                                                                                                                                                                                                      # fun x -> x;;
                                                                                                                                                                                                                                              -: 'a -> 'a = <fun>
                                                                                                                                                                                                                                                                                                         the simplest polymorphic function
                                                         This expression has type int but is here used with type 'a list
```

datatypes

```
# Red;;
                               # Tree (Empty, 3, Empty);;
-: numtree = Tree (Empty, 3, Empty)
                                                                                               # Empty;;
                                                                                                                                  type numtree = Empty | Tree of numtree * int * numtree
                                                                                                                                                                                                                                                                                                       # [Red; Green];;
                                                                                                                                                                                                                                                                                                                                                                                                                                          # type color = Red | Green | Blue;;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                          a simple datatype
                                                                                                                                                                 # type numtree = Empty | Tree of numtree * int * numtree;;
                                                                                                                                                                                                     constructors can take arguments
                                                                                                                                                                                                                                                                      - : color list = [Red; Green]
                                                                                                                                                                                                                                                                                                                                                                                                        type color = Red | Green | Blue
                                                                                                                                                                                                                                                                                                                                       - : color = Red
                                                                   : numtree = Empty
```

patterns

```
# sum [1;2;3;4];;
                                                                                                                             # let rec sum l = match l with [] -> 0 l e :: rest -> e + sum rest;;
                                                                                                                                                                           a function on lists
                                                                                                                                                                                                                                                                                                                                              # let tt = Tree (Tree (Empty, 1, Empty), 3, Tree (Empty, 2, Empty));;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     # type numtree = Empty | Tree of numtree * int * numtree;;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   a function on number trees
-: int = 10
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             # let rec treesum t =
                                                                                    val sum: int list -> int = <fun>
                                                                                                                                                                                                                                                                -: int = 6
                                                                                                                                                                                                                                                                                                                                                                                               val treesum : numtree -> int = <fun>
                                                                                                                                                                                                                                                                                                        .# treesum tt;;
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                match t with Empty -> 0
                                                                                                                                                                                                                                                                                                                                                                                                                                      Tree (t1, i, t2) \rightarrow i + treesum t1 + treesum t2;
```

puzzle: poly functional over lists

```
match l with [] -> [] | x :: xs -> (f x) :: (map f xs);; val map : ('a -> 'b) -> 'a list -> 'b list = <fun>
                               # map dbl [1;2];;
                                                                                                                        # let rec map f l =
                                                                                                                                                              solution
                                                                                                                                                                                                                                                       write the function map
-: int list = [2; 4]
                                                                                                                                                                                                                         val map : ('a -> 'b) -> 'a list -> 'b list
```

puzzle: user-defined poly datatypes

```
# type 'a tree = Empty | Tree of ('a tree) * 'a * ('a tree);;
                                                                                               a polymorphic tree
type 'a tree = Empty | Tree of 'a tree * 'a * 'a tree
```

```
val treefold : ('a * 'b * 'a -> 'a) -> 'a -> 'b tree -> 'a = <fun>
                                                                                                                                                                                                             # let rec treefold f b t =
                                                                                                                                                                                                                                                        what is the type of treefold?
                                                                                                 Tree (left, v, right) -> f (treefold f b left, v, treefold f b right);;
                                                                                                                                                    match t with Empty -> b
```

side-effects

```
mutable cells
# let seed = ref 0;;
val seed : int ref = {contents=0}

val seed: int ref = {contents=0}

dereference
# !seed;;
-: int = 0

assignment
# seed := 1;;
-: unit = ()
# !seed;;
-: int = 1
```

puzzle

- write a function *next*> which produces 0, 1, 2, etc

 > takes no arguments

closures and cells

```
# next ();;

-: int = 1

# next ();;

-: int = 2
                                                                         # (next);;
- : unit -> int = <fun>
                                                                                                                                                                           # let next =
                                                                                                                  function () -> seed := !seed+1; !seed);;
val next : unit -> int = <fun>
                                                                                                                                                        (let seed = ref 0 in
```

lazy lists or 'streams'

type 'a stream = Nil | Cons of 'a * (unit -> 'a stream);; define a datatype for streams type 'a stream = Nil | Cons of 'a * (unit -> 'a stream)

let hd s = match s with Cons $(x,f) \rightarrow x;;$ # let cons x s = Cons (x, fun () -> s);; Here is an example of a value that is not matched: Nil val cons : 'a -> 'a stream -> 'a stream = <fun> val hd: 'a stream -> 'a = <fun> Warning: this pattern-matching is not exhaustive

let tl s = match s with Cons $(x, f) \rightarrow f()$; val tl: 'a stream -> 'a stream = <fun> Here is an example of a value that is not matched: Nil Warning: this pattern-matching is not exhaustive

using streams

```
val from : int -> int stream = <fun>
# (from 3);;
- : int stream = Cons (3, <fun>)
                     # hd (tl (from 3));;
-: int = 4
                                                                                                                                           # let rec from k = Cons(k, fun() \rightarrow from(k+1));;
```

puzzle

```
given
type 'a tree = Empty | Tree of 'a * 'a tree list
                                      # type 'a tree = Empty | Tree of 'a * 'a tree list;;
```

write a function that

- performs a depth-first traversal of a tree
- » gives result as a stream

you can assume an infix function @

for appending streams