

Inference and Representation:

Case study in Computational Cognitive Science

Brenden Lake

“Learning classifiers” in cognitive science

concept learning
(cognitive science
&
psychology)

=

classification
(data science
&
machine learning)

**labeled data for
“dogs”**



**generalization task:
dog or cat?**



**labeled data for
“cats”**



human-level concept learning

the speed of learning

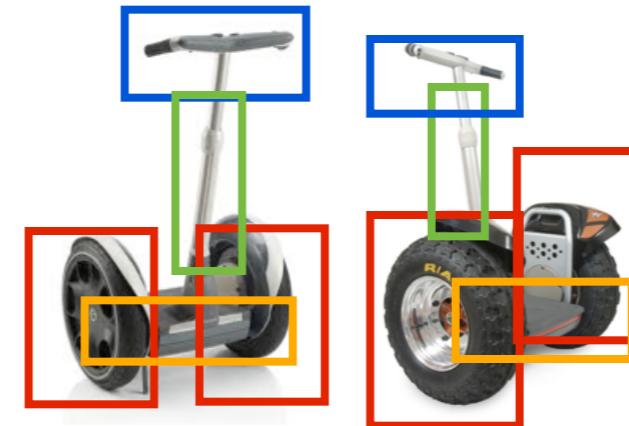


“one-shot
learning”



the richness of representation

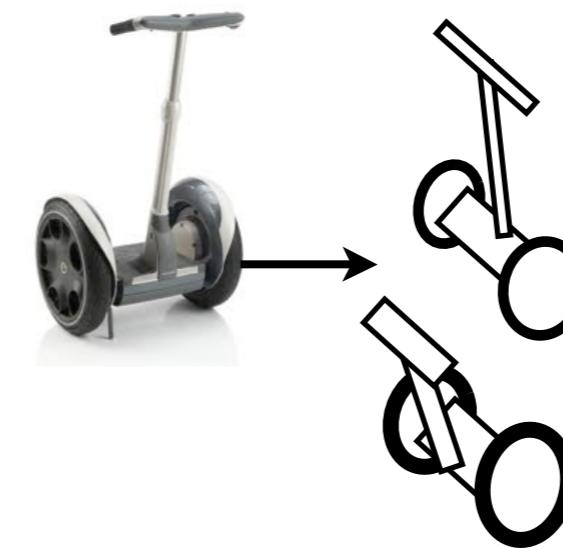
parsing



generating
new concepts



generating
new examples



portable immersion circulator



bucket-wheel excavator



spring-loaded camming device



drawing knife



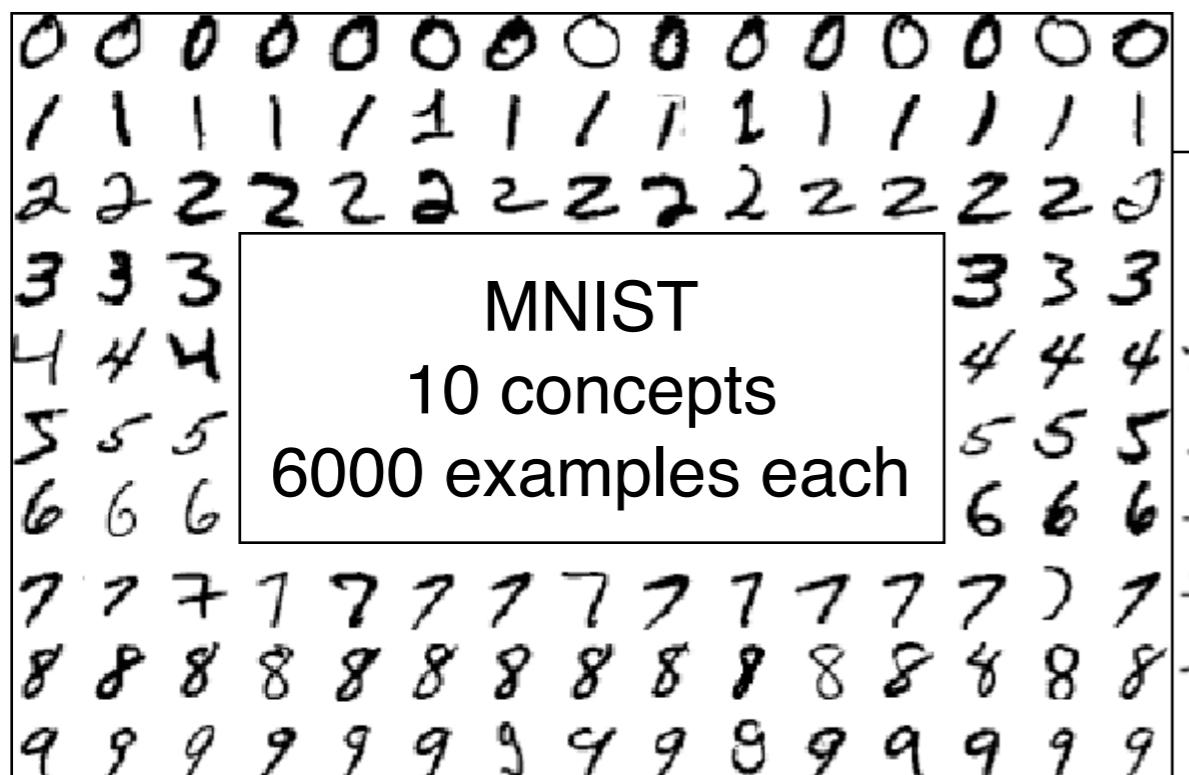
A testbed for studying human-level concept learning

We would like to investigate a domain with...

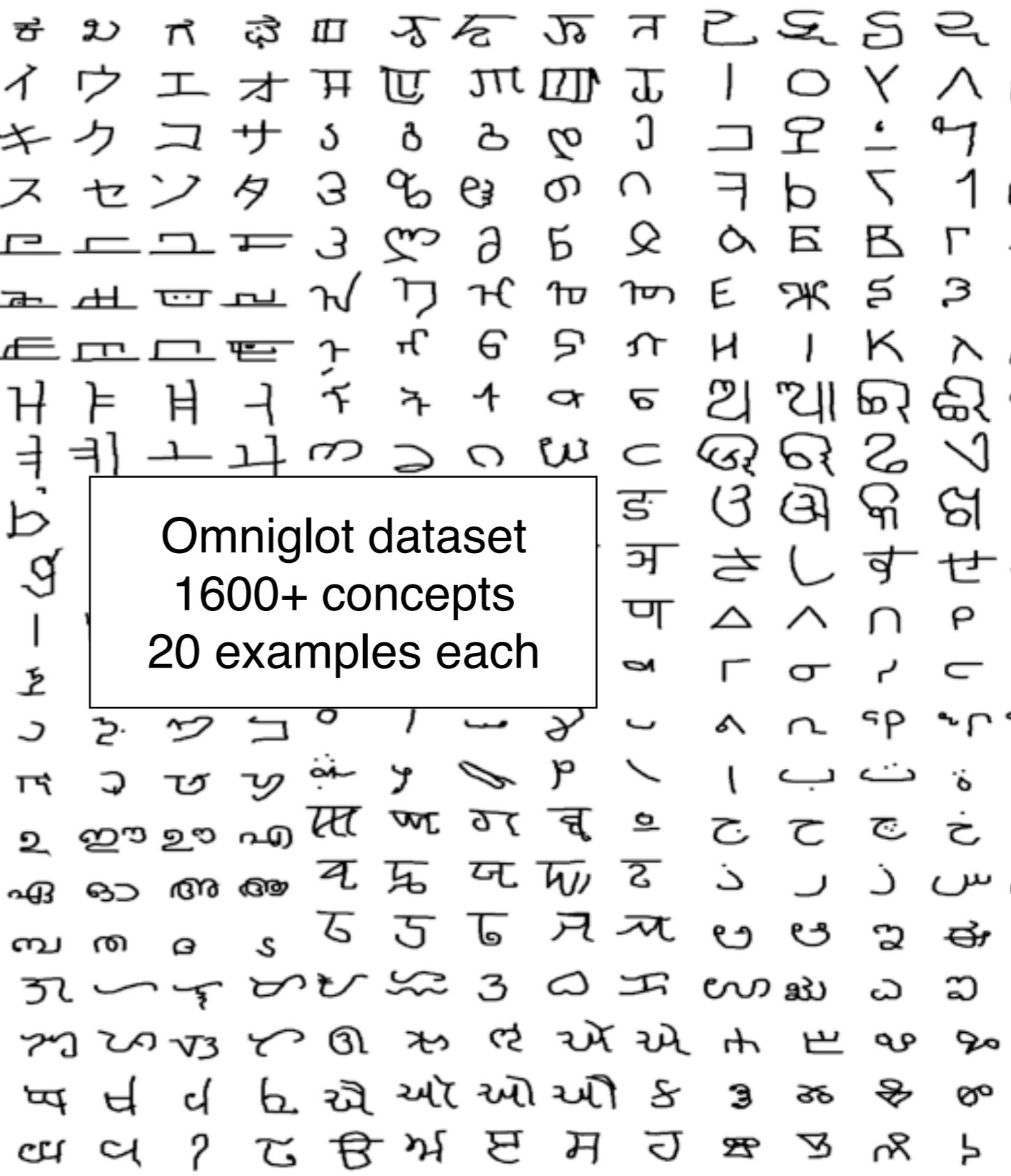
- 1) A relatively even slate for comparing humans and machines.
- 2) Natural, high-dimensional concepts.
- 3) A reasonable chance of building computational models that can see most of the structure that people see.
- 4) Insights that generalize across domains.

Standard machine learning

<https://github.com/brendenlake>



Our testbed



Sanskrit

क	ख	ग	ঘ	ক
চ	ছ	জ	ঝ	ঝ
ট	ড	ত	ঢ	ঢ

Tagalog

ກ	ຂ	ဒ	ପ	ଫ
ଚ	ଛ	ସ	ମୁ	ମୁ
ତ୍ର	ଦ୍ର	ତ୍ରୀ	ବ୍ରାହ୍ମ	ବ୍ରାହ୍ମ

Balinese

ମା	ମା	ମା	ମା	ମା
ମି	ମି	ମି	ମି	ମି
ମା	ମା	ମା	ମା	ମା

Hebrew

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ל	צ	ח	ט	י
כ	ל	ג	ל	ב

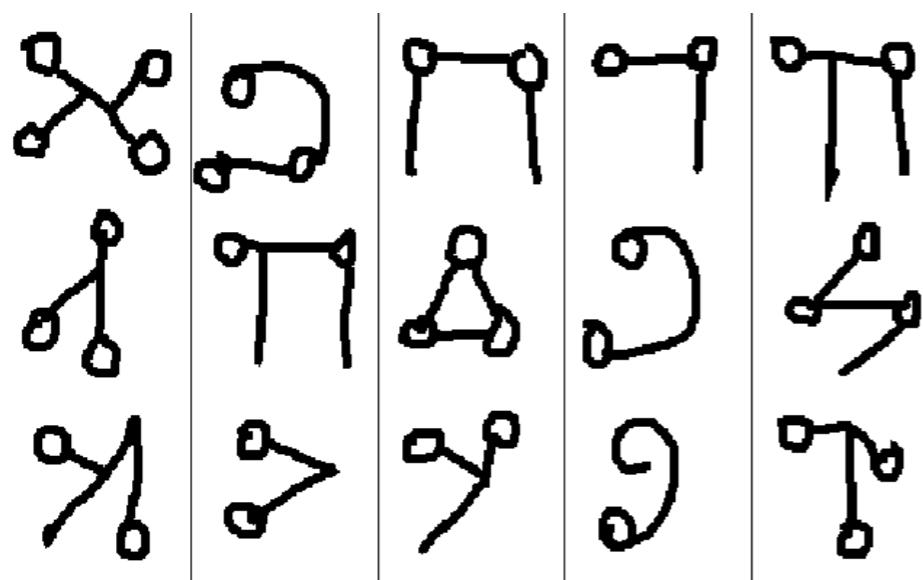
Latin

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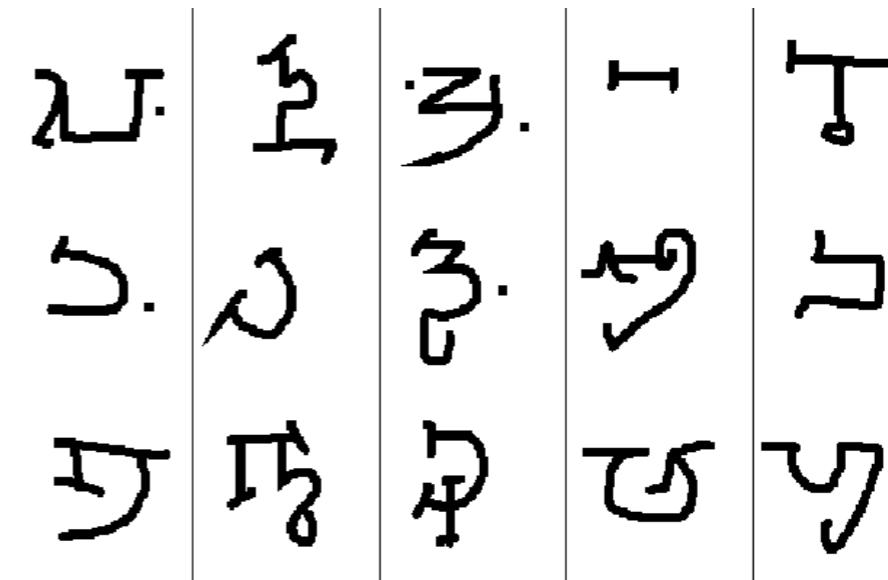
Braille

•	•	•	•	•
• •	•	•	•	•
•	•	•	•	•
•	•	•	•	•

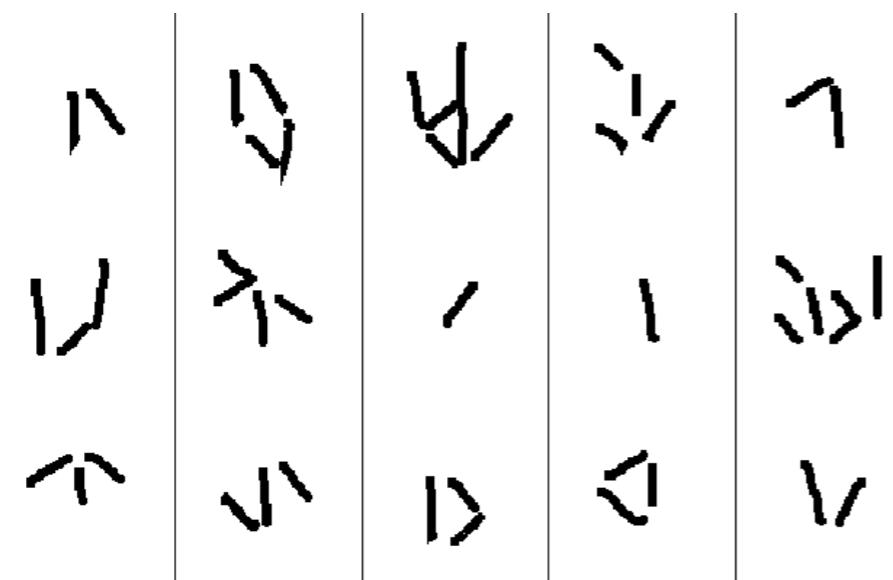
Angelic



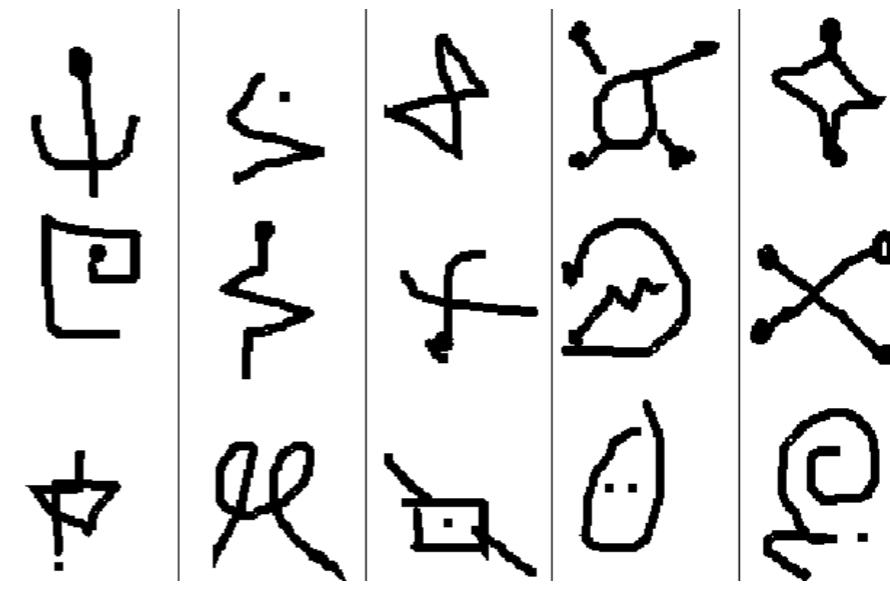
Alphabet of the Magi



ULOG



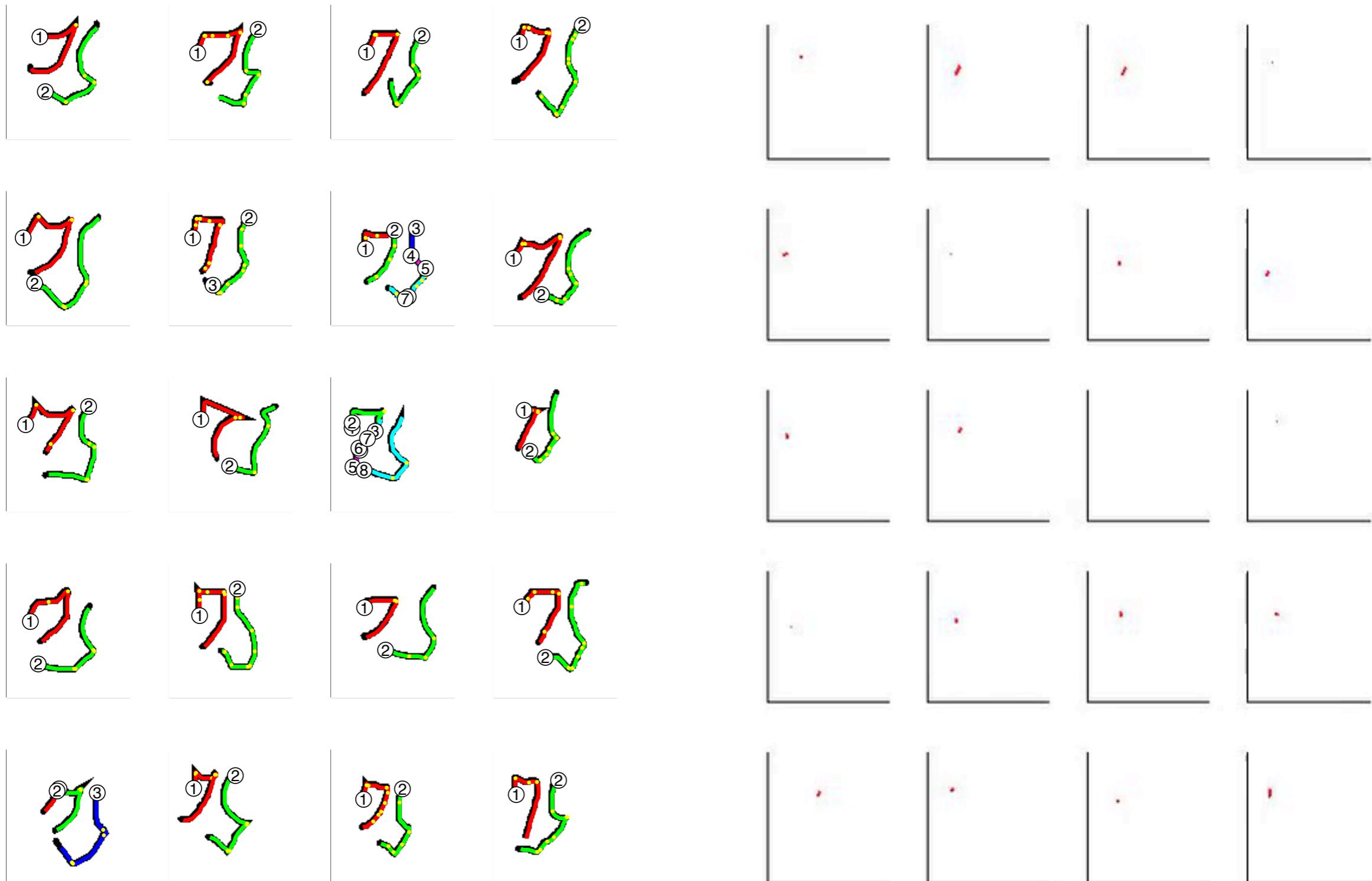
Futurama



Original Image



20 People's Strokes



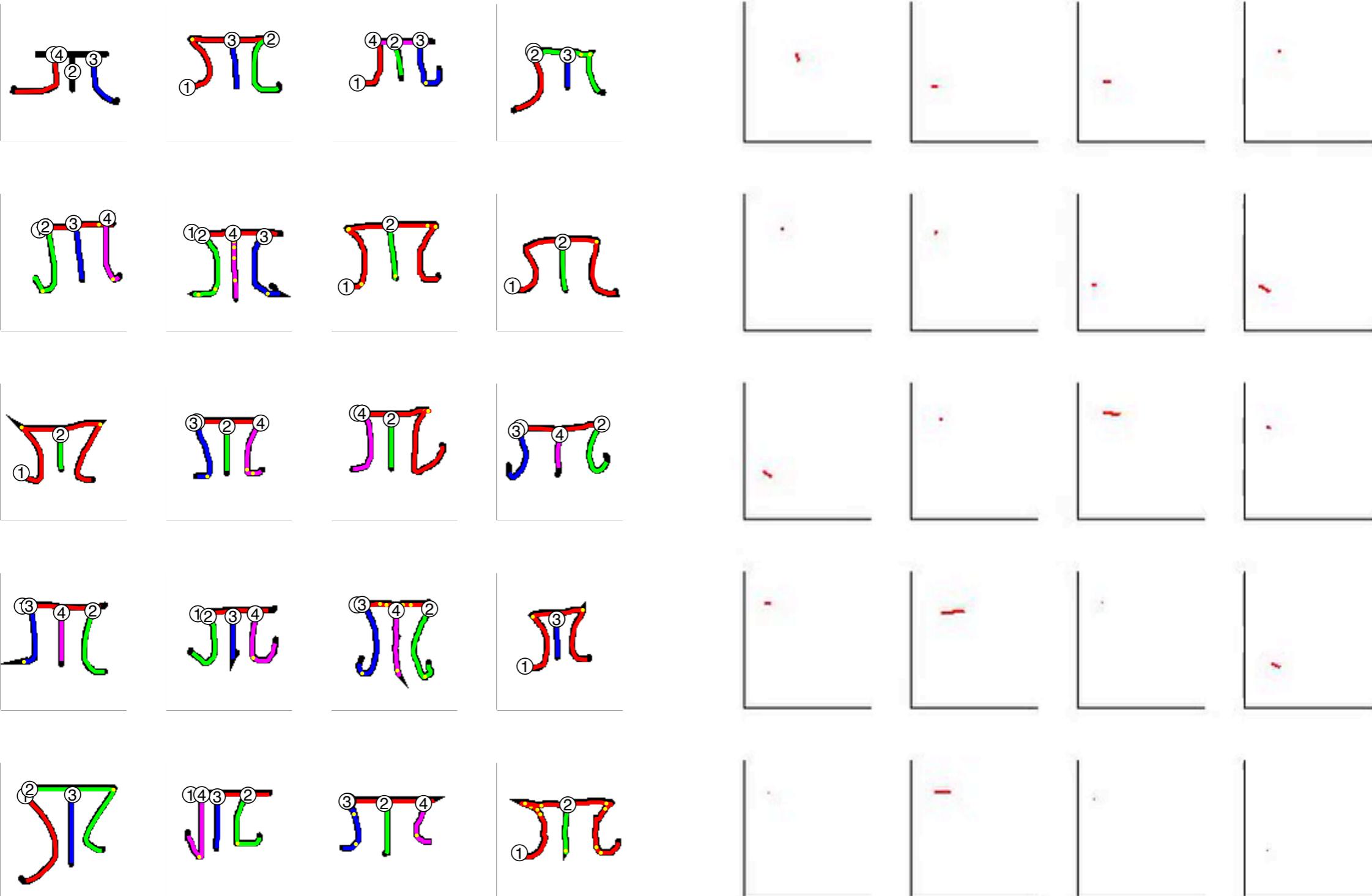
Stroke order:

— 1st — 2nd — 3rd — 4th — 5th — 6th (or higher)

Original Image



20 People's Strokes



Original Image



20 People's Strokes



human-level concept learning

the speed of learning

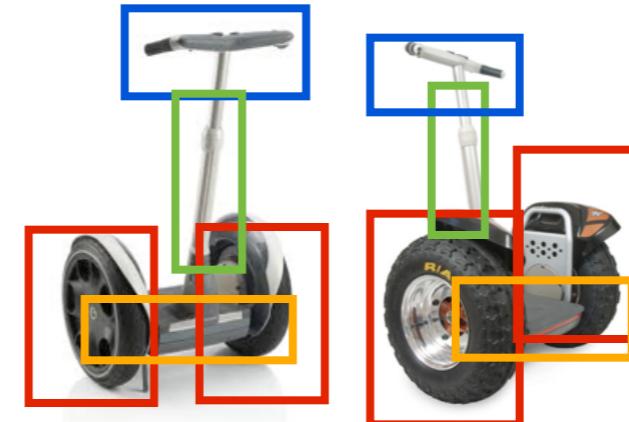


“one-shot
learning”



the richness of representation

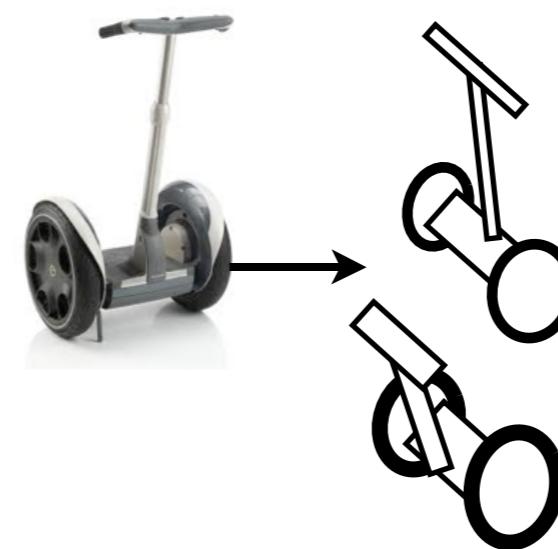
parsing



generating
new concepts



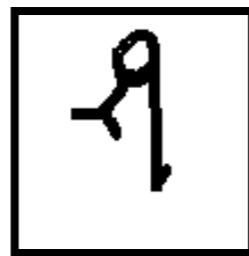
generating
new examples



human-level concept learning

the speed of learning

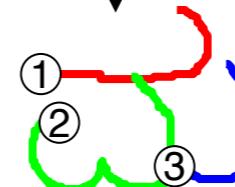
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the richness of representation

parsing

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generating
new examples

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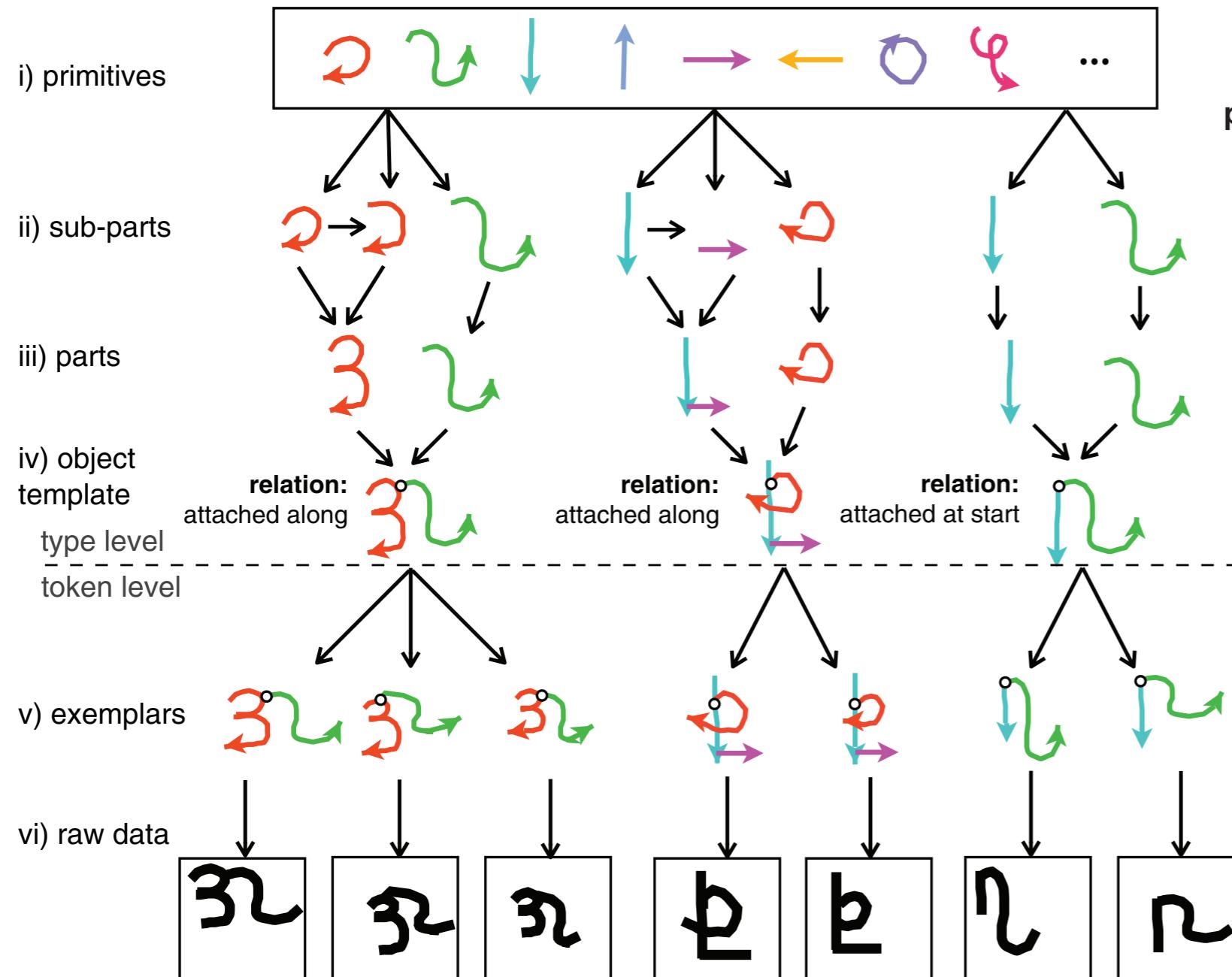
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generating
new concepts

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Bayesian Program Learning



```

procedure GENERATETYPE
   $\kappa \leftarrow P(\kappa)$   $\triangleright$  Sample number of parts
  for  $i = 1 \dots \kappa$  do
     $n_i \leftarrow P(n_i|\kappa)$   $\triangleright$  Sample number of sub-parts
    for  $j = 1 \dots n_i$  do
       $s_{ij} \leftarrow P(s_{ij}|s_{i(j-1)})$   $\triangleright$  Sample sub-part sequence
    end for
     $R_i \leftarrow P(R_i|S_1, \dots, S_{i-1})$   $\triangleright$  Sample relation
  end for
   $\psi \leftarrow \{\kappa, R, S\}$ 
  return @GENERATETOKEN( $\psi$ )  $\triangleright$  Return program

```

```

procedure GENERATETOKEN( $\psi$ )
  for  $i = 1 \dots \kappa$  do
     $S_i^{(m)} \leftarrow P(S_i^{(m)}|S_i)$   $\triangleright$  Add motor variance
     $L_i^{(m)} \leftarrow P(L_i^{(m)}|R_i, T_1^{(m)}, \dots, T_{i-1}^{(m)})$   $\triangleright$  Sample part's start location
     $T_i^{(m)} \leftarrow f(L_i^{(m)}, S_i^{(m)})$   $\triangleright$  Compose a part's trajectory
  end for
   $A^{(m)} \leftarrow P(A^{(m)})$   $\triangleright$  Sample affine transform
   $I^{(m)} \leftarrow P(I^{(m)}|T^{(m)}, A^{(m)})$   $\triangleright$  Sample image
  return  $I^{(m)}$ 

```

human-level concept learning

the speed of learning

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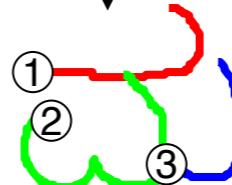
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the richness of representation

parsing

ಮು



generating
new concepts

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generating
new examples

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generating
new examples

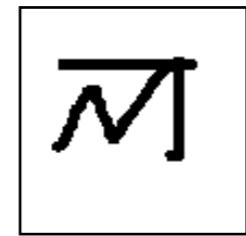
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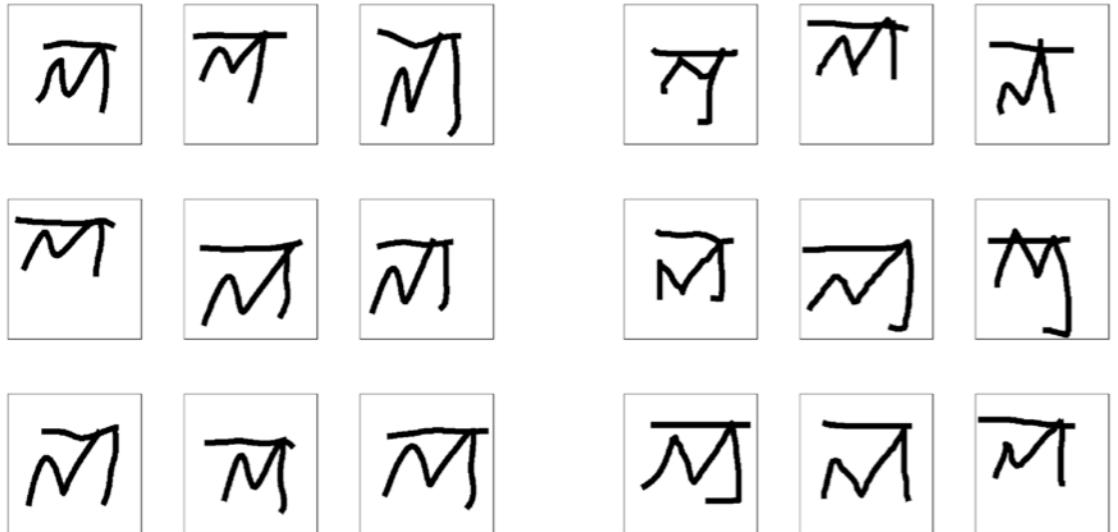
“Draw a new example”

Which grid is produced by the model?

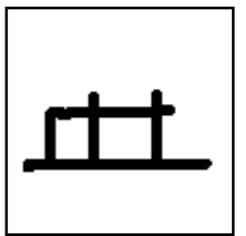
A



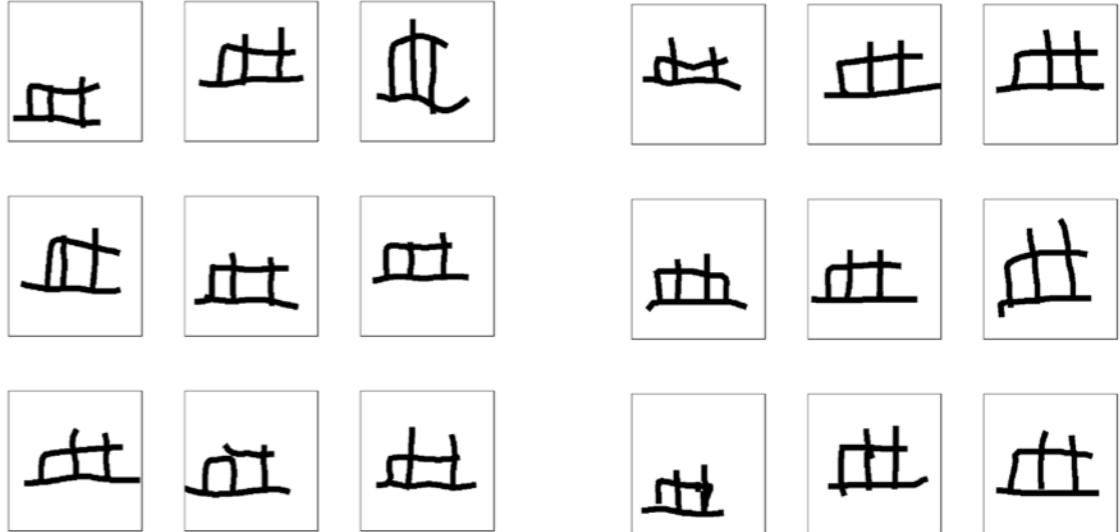
B



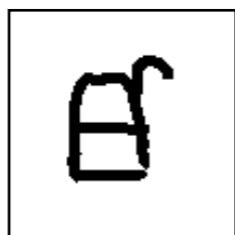
A



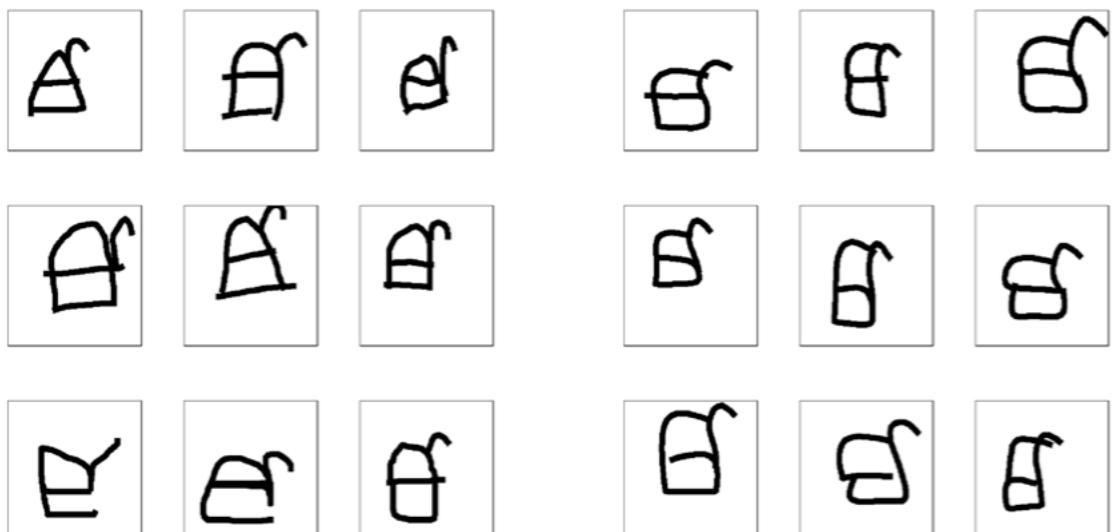
B



A



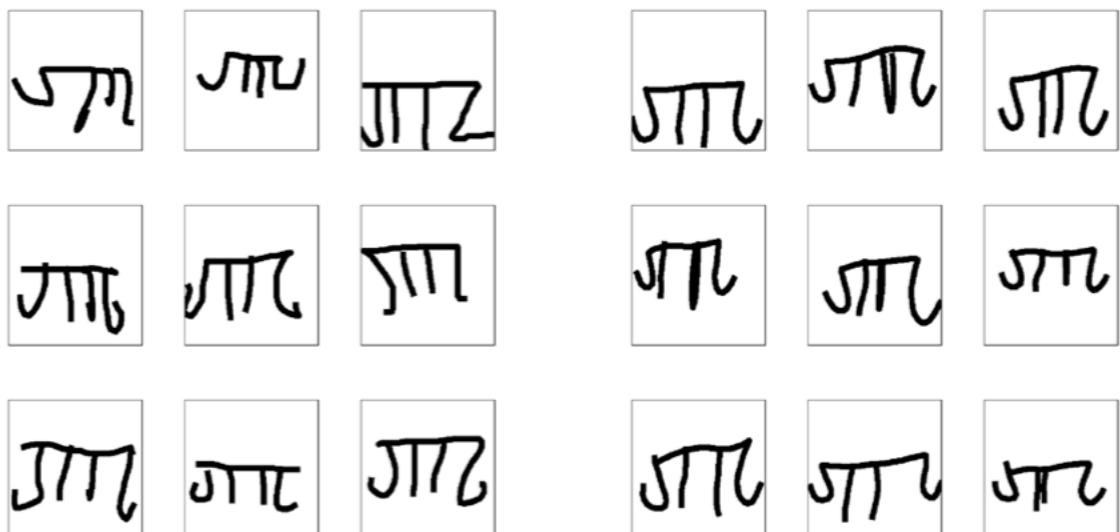
B



A

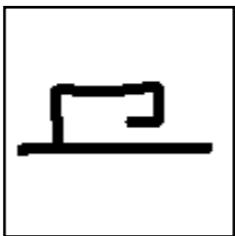


B



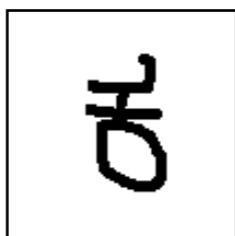
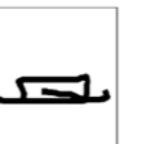
“Draw a new example”

Which grid is produced by the model?



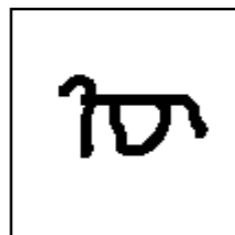
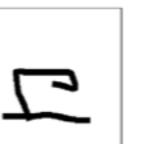
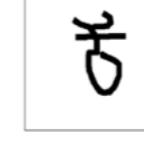
A

B



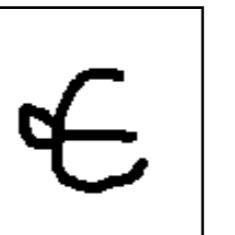
A

B



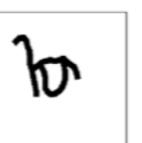
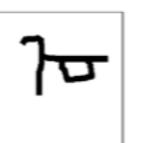
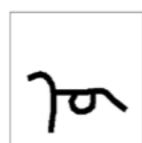
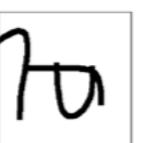
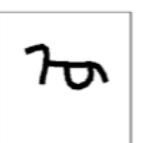
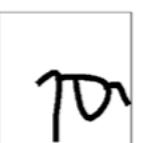
A

B



A

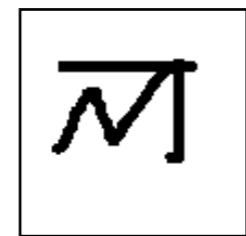
B



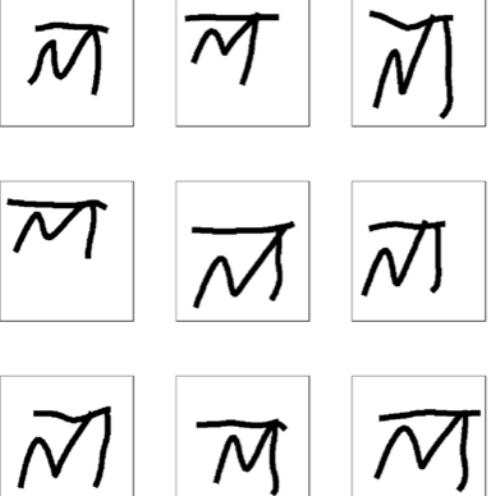
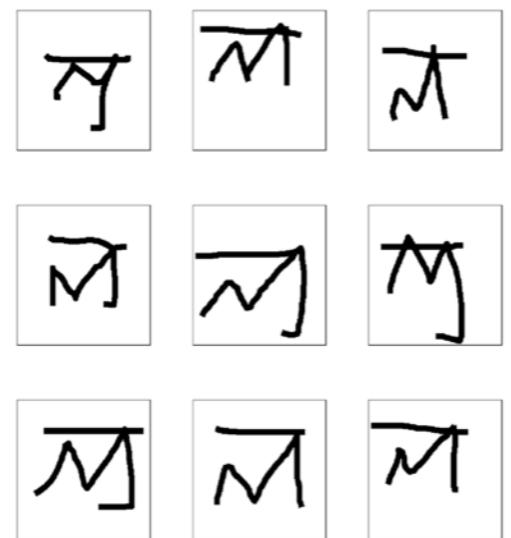
“Draw a new example”

Which grid is produced by the model?

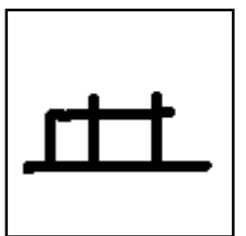
A



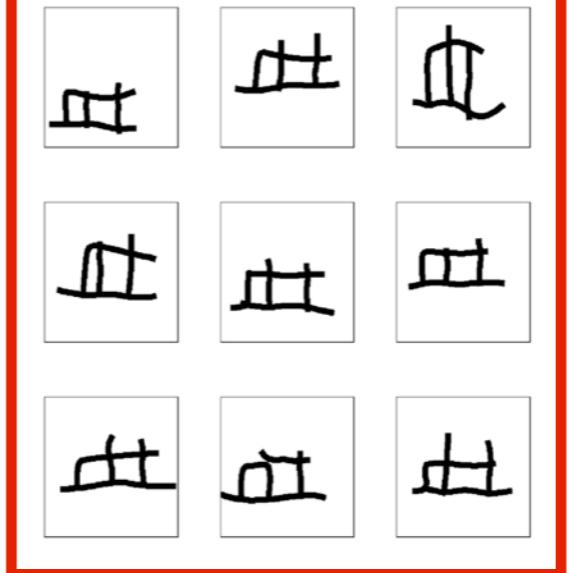
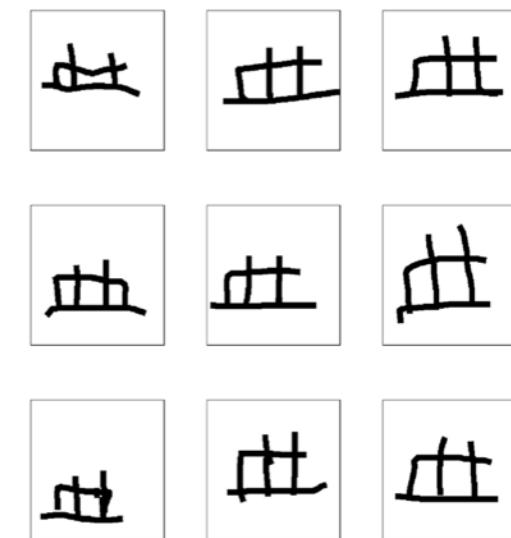
B



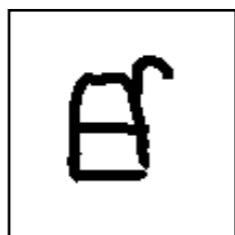
A



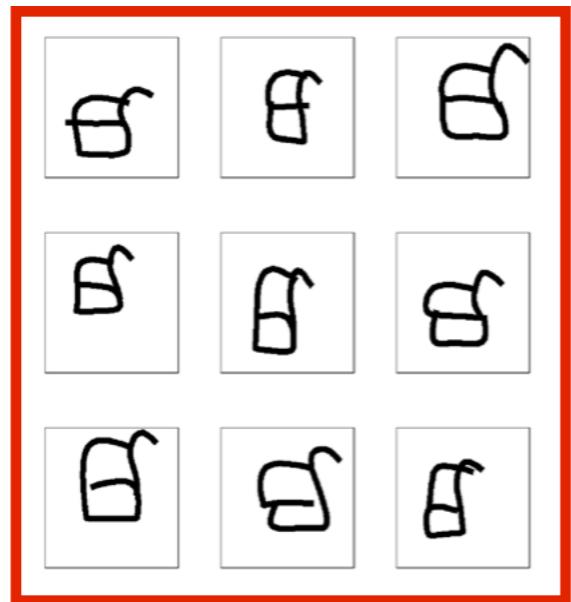
B



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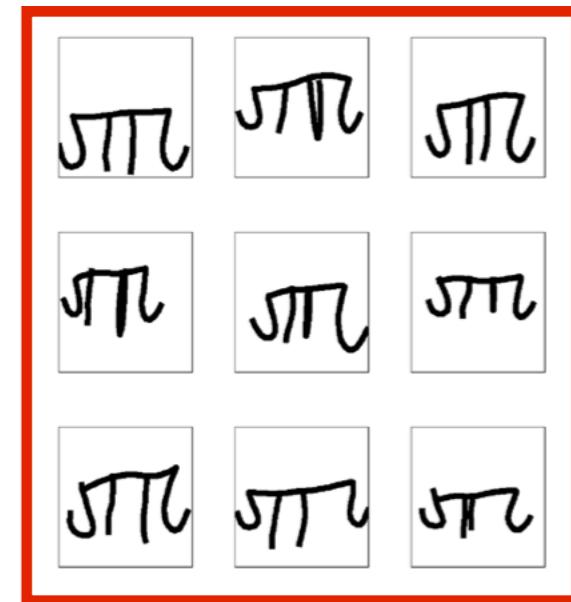
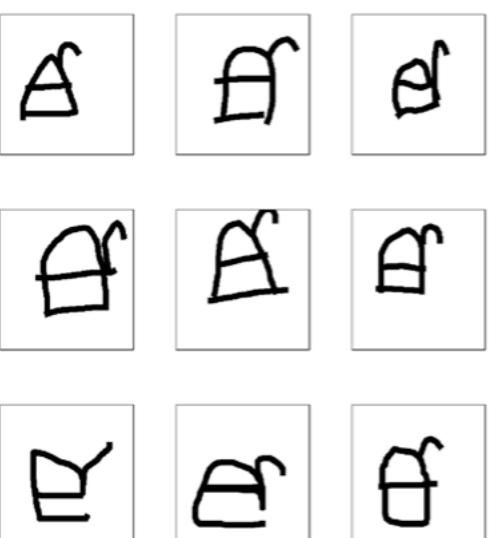
B



A

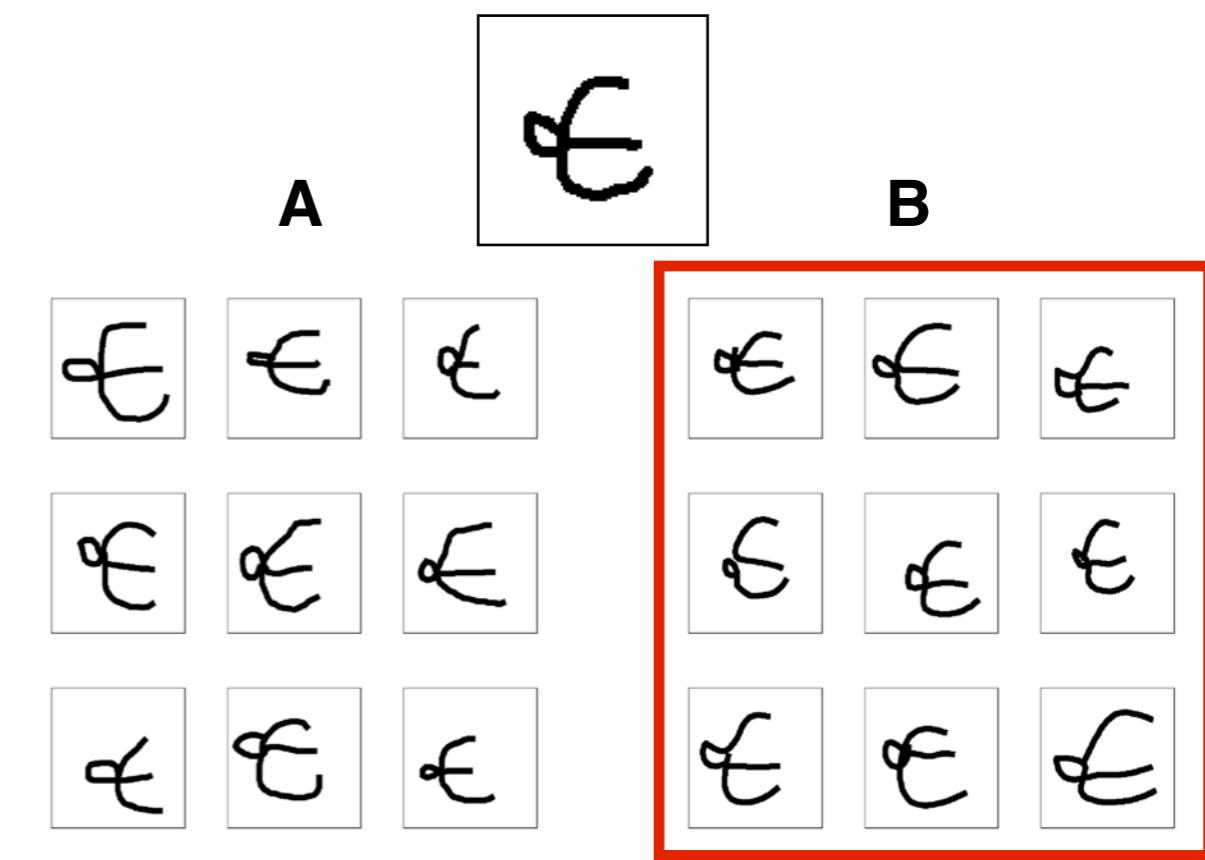
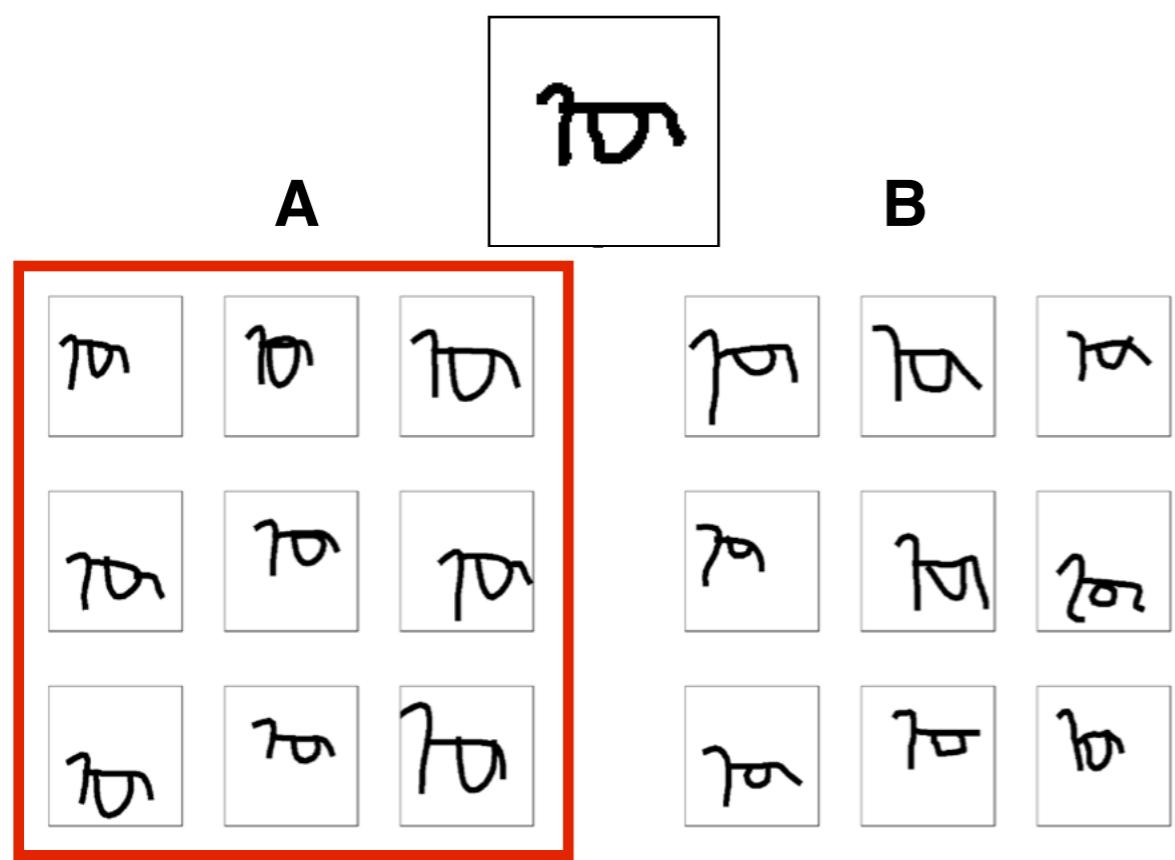
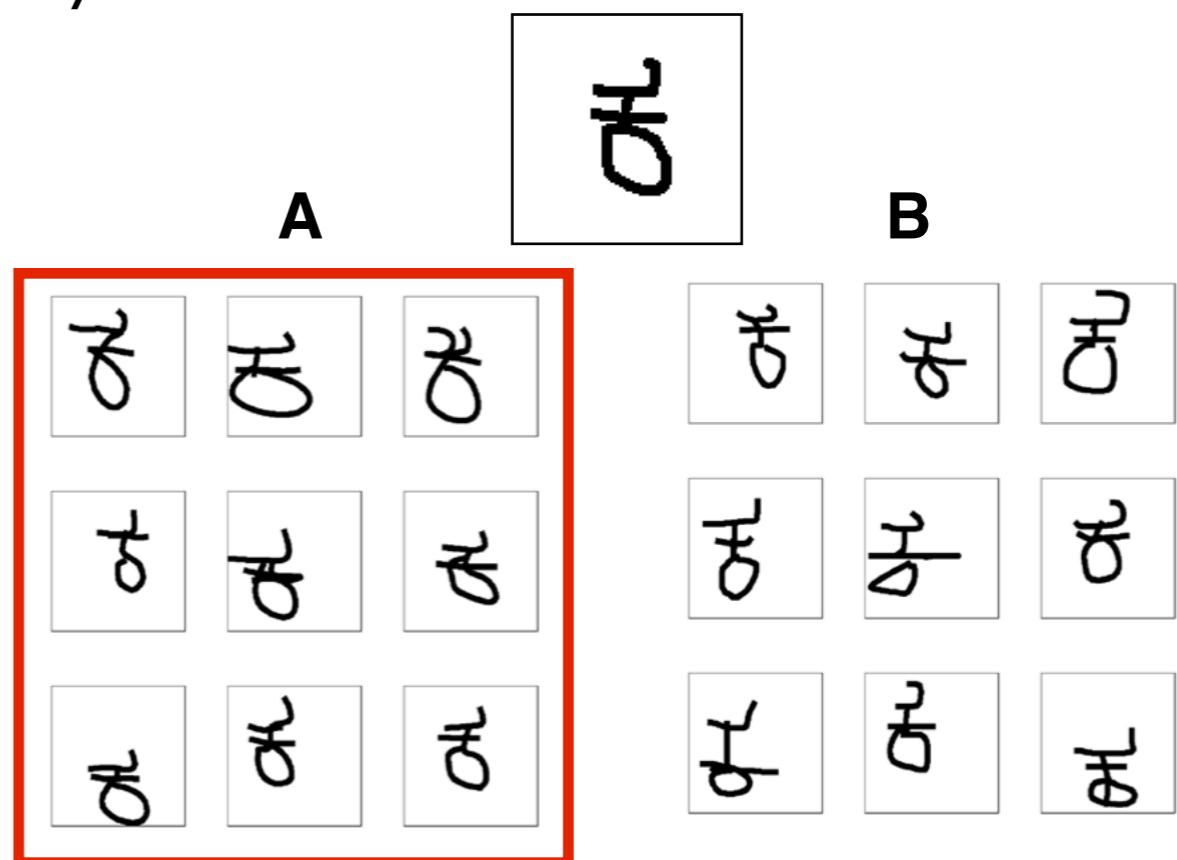
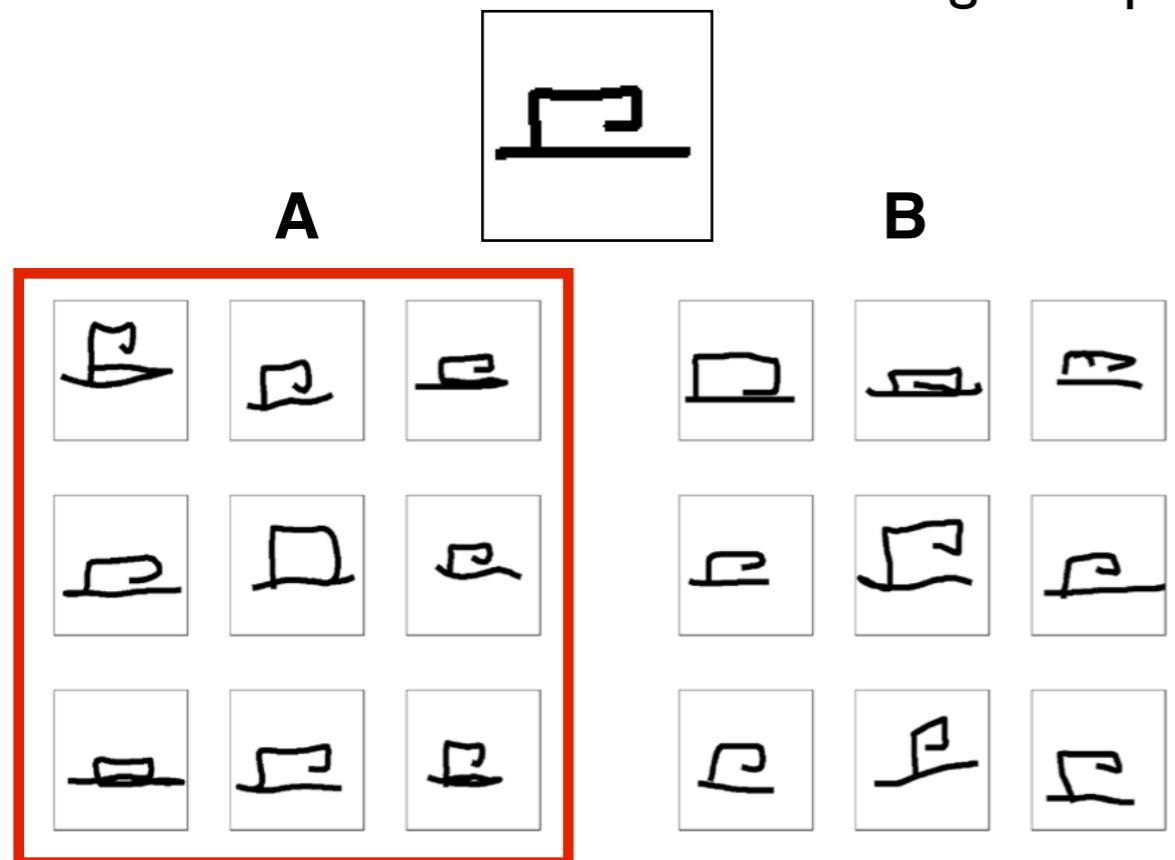


B



“Draw a new example”

Which grid is produced by the model?



human-level concept learning

the speed of learning



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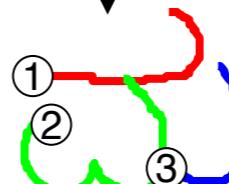


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the richness of representation

parsing

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generating
new examples

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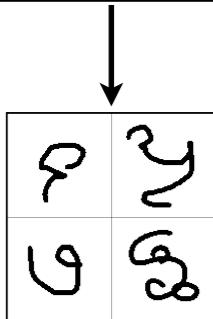
generating
new concepts

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generating
new concepts

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Task: “Design a new character from the same alphabet”

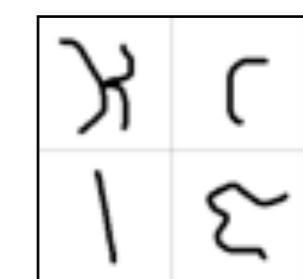
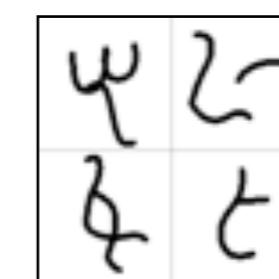
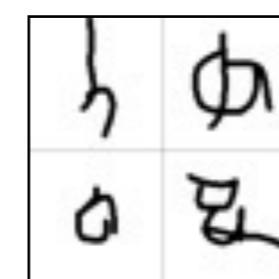
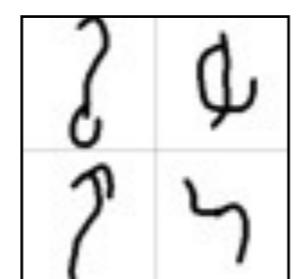
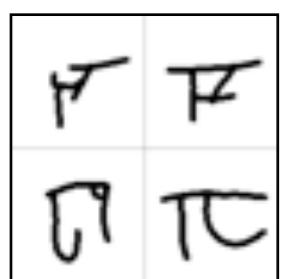
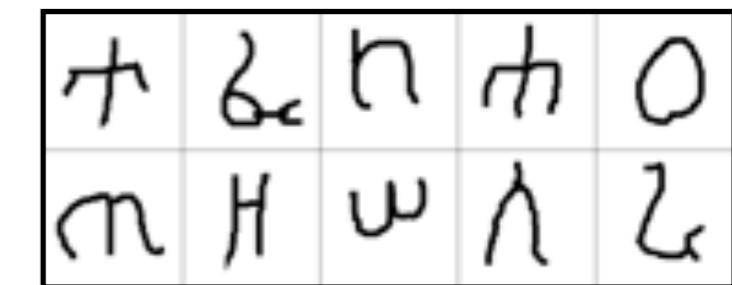
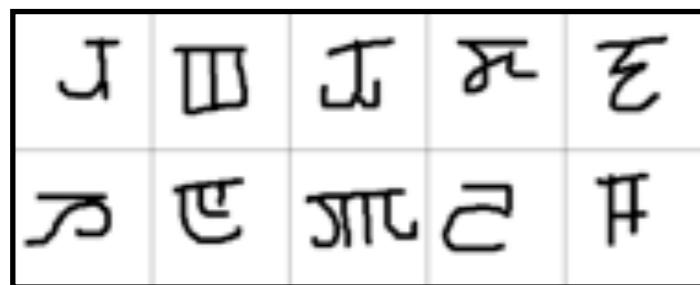
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3 seconds
remaining

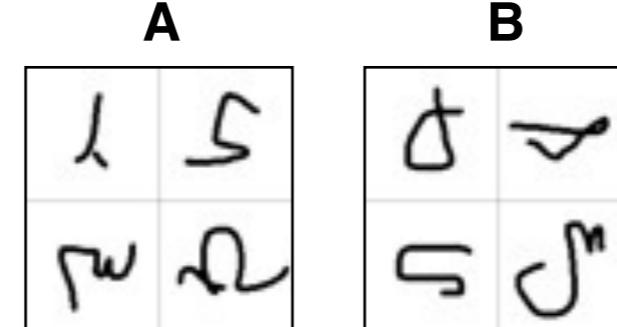
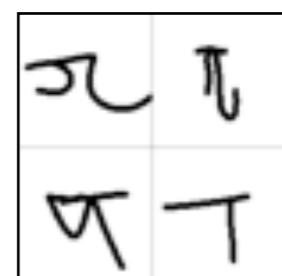
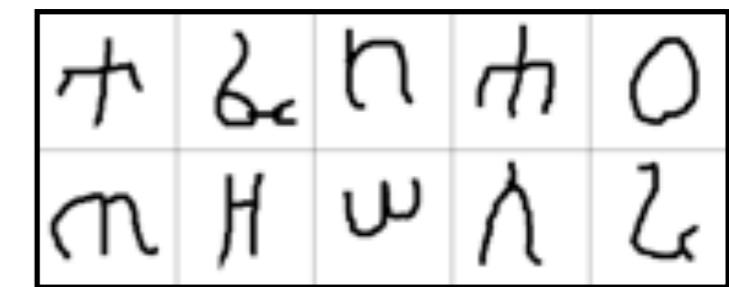
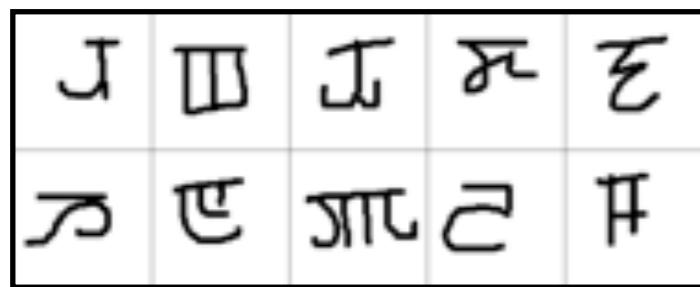
Task: “Design a new character from the same alphabet”

Which grid is produced by the model?



Task: “Design a new character from the same alphabet”

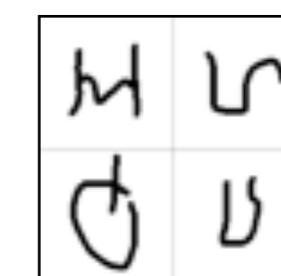
Which grid is produced by the model?



ㅓ ㅗ



ㅓ ㅗ



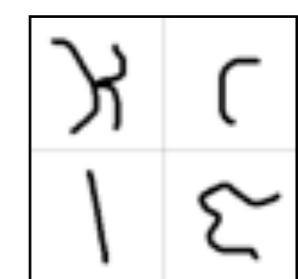
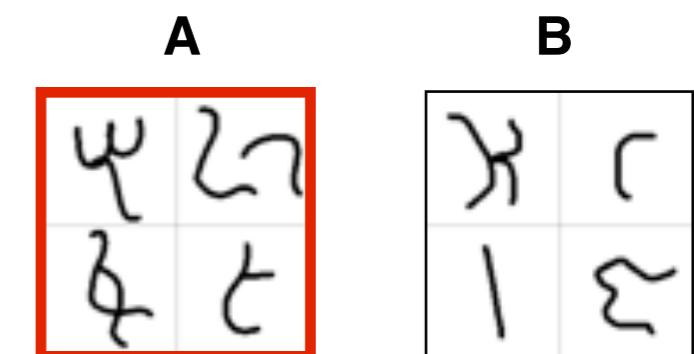
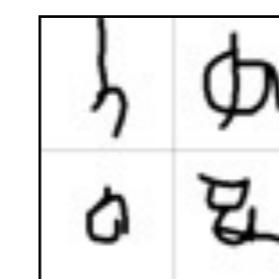
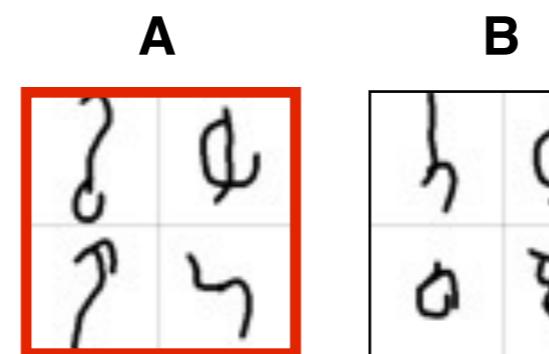
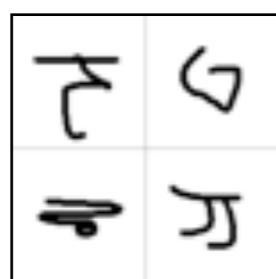
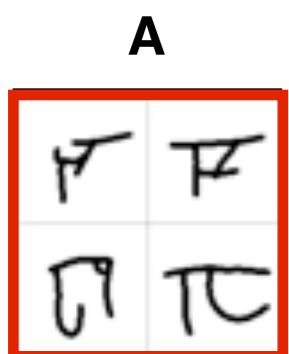
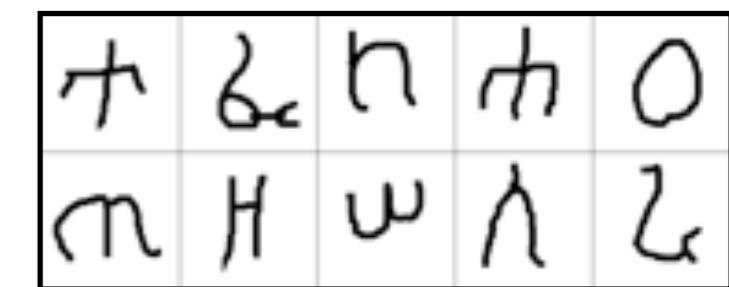
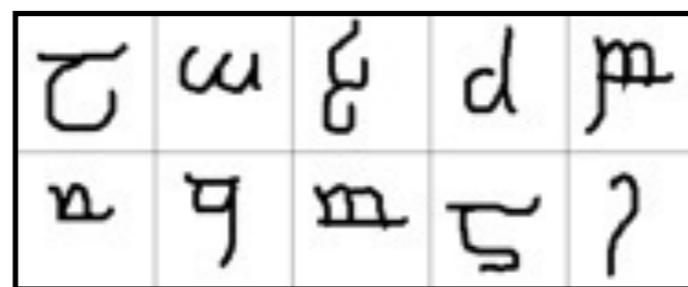
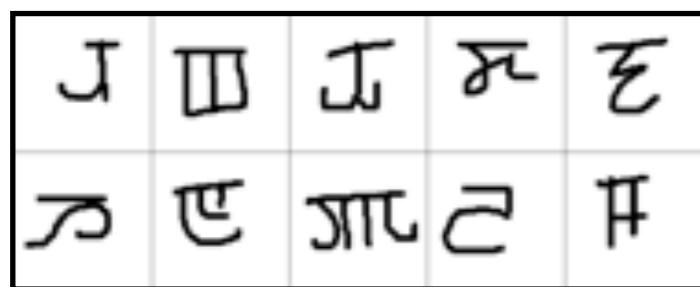
ㅓ ㅗ



ㅓ ㅗ

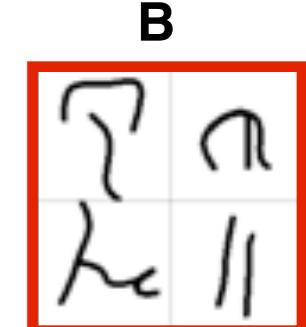
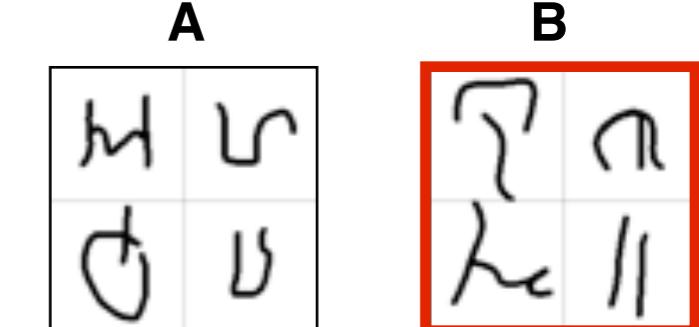
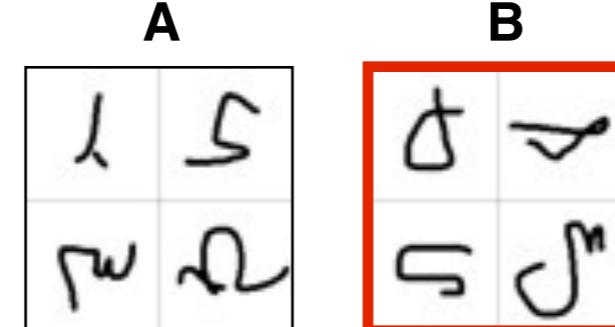
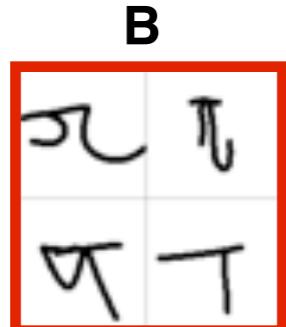
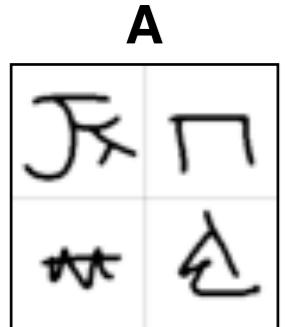
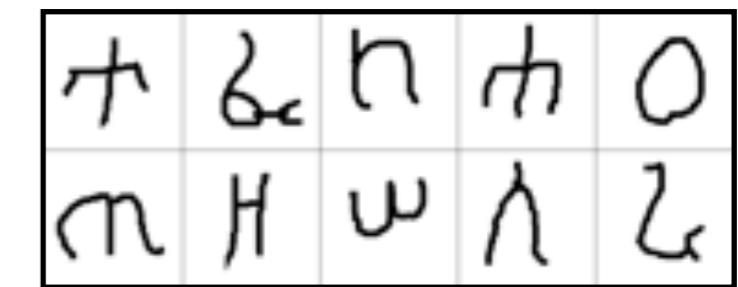
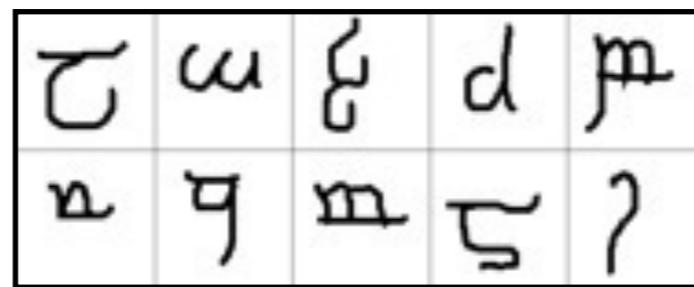
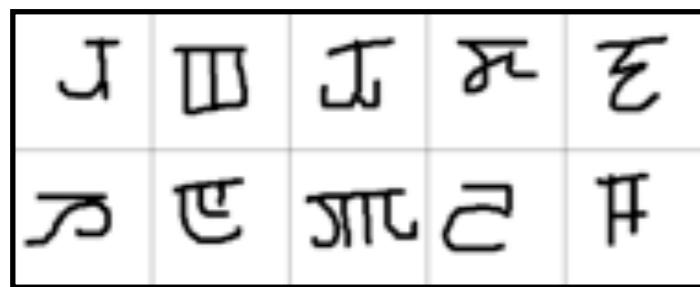
Task: “Design a new character from the same alphabet”

Which grid is produced by the model?



Task: “Design a new character from the same alphabet”

Which grid is produced by the model?



Generate a new characters from the same alphabet

Alphabet of characters

ଜ	ଣ	ହୁ	ମୁ	କ
ର	ଏ	ଗା	ର୍ଲ	ମ

தென்டோ
நூற்கு

ମ	ଓ	ବ୍	ହାଯ
S	B	E	H

New machine-generated characters in each alphabet

ପ	ମ	ର	ଟ	ଳ	ତ
ନ	ଗ	ଭ	ଟ୍ଟ	ଳ୍ଳ	ତ୍ତ
କ	ଚ	କ	ଣ୍ଟ	ଳୁ	ତୁ
ଶ	ତ	ଶ	ଦ୍ର	ଦୁ	ଦୁ
ର୍ମ	ତ୍ର	ର୍ମ	ଦ୍ର୍ମ	ଦୁର୍ମ	ଦୁର୍ମ
ର୍ମ୍ଭ	ତ୍ର୍ବ	ର୍ମ୍ଭ	ଦ୍ର୍ବ	ଦୁର୍ବ	ଦୁର୍ବ

॥	ଫ	ନ୍ତ	ପ୍ର	କୁ	ଟା
କୁ	ନ୍ତ	ଫ	ଟ	ପ୍ର	ନ୍ତ
ନ୍ତ	କୁ	ଟା	ଫ	ନ୍ତ	ପ୍ର
ପ୍ର	ନ୍ତ	ଫ	ଟା	କୁ	ନ୍ତ
କୁ	ଟା	ନ୍ତ	ଫ	ପ୍ର	ନ୍ତ
ନ୍ତ	ଫ	ଟା	କୁ	ନ୍ତ	ପ୍ର
ଫ	କୁ	ନ୍ତ	ଟା	ପ୍ର	ନ୍ତ
ଟା	ନ୍ତ	ଫ	କୁ	ନ୍ତ	ପ୍ର

ର କୁ ହ ଶୁ ଟି ନ୍ତ
 ପୁ ର ର ଏ ଫୁ ଟି
 କ ଲ ଗ ପିଣ୍ଡି ନ
 ଲ ଏ ବ ପ ଙ୍କା
 ଦ ଲାଭ ଏ ଚା ର
 ଥ ଲ ଅନ୍ଧ ନ କା

Alphabet of characters

ଶ	ରେ	ତେ	କୁ	ମୁ
ଖ	ରୀ	ତୀ	କୁ	ମୁ

ଯେଣି ଦେନାଲ
ଯେତେ ପାହାଲ

New machine-generated characters in each alphabet

፩	፪	፫	፬	፭	፮	፯
፻	፼	፽	፾	፷	፸	፹
፻	፻	፻	፻	፻	፻	፻
፻	፻	፻	፻	፻	፻	፻
፻	፻	፻	፻	፻	፻	፻

ଭ	ଟୁଅ	ଟ୍ରୀ	ଇ	ଦ
ବ	ପ୍ରାଣ	ବି	ପାଇଁ	ନ୍ତର
ଶ	ଶ୍ରୀମଦ୍ଭଗବତ	ଶବ୍ଦ	ଶବ୍ଦିକା	ଶବ୍ଦିକା
ହ	ହୃଦୟ	ହୃଦୟ	ହୃଦୟ	ହୃଦୟ
କୁ	କୁଳ	କୁଳ	କୁଳ	କୁଳ
କୁଳ	କୁଳ	କୁଳ	କୁଳ	କୁଳ
କୁଳ	କୁଳ	କୁଳ	କୁଳ	କୁଳ

Bayesian Program Learning

primitives
(1D curvelets, 2D patches, 3D geons, actions, sounds, etc.)

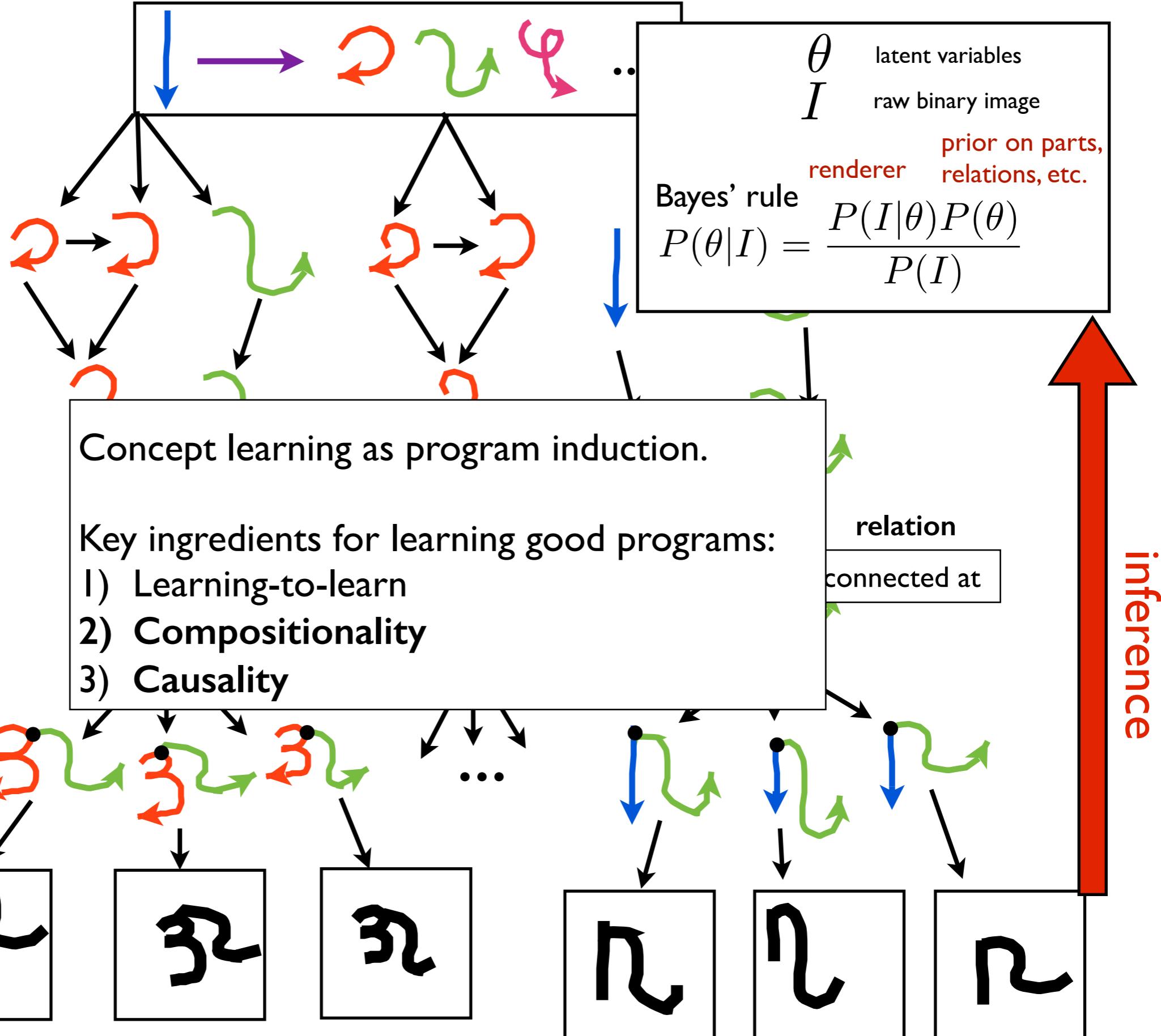
sub-parts

parts

object template

exemplars

raw data



Bayesian Program Learning

primitives
 (1D curvelets, 2D
 patches, 3D geons,
 actions, sounds, etc.)

procedure GENERATETYPE

$\kappa \leftarrow P(\kappa)$ Sample number of parts

for $i = 1 \dots \kappa$ **do**

$n_i \leftarrow P(n_i|\kappa)$ Sample number of sub-parts

$S_i \leftarrow P(S_i|n_i)$ Sample sequence of sub-parts

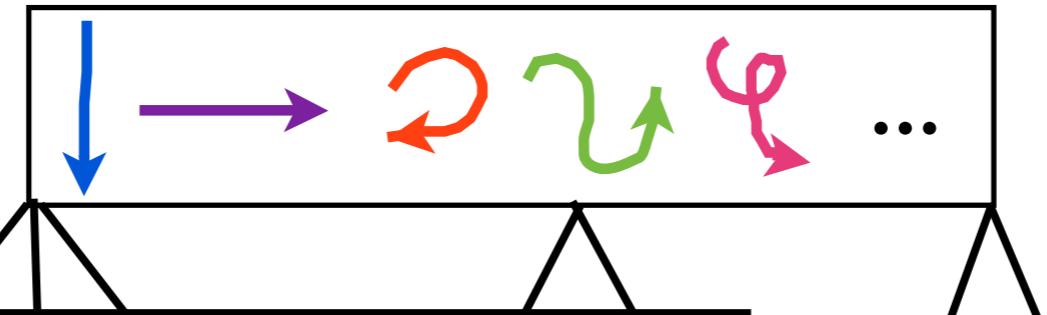
$R_i \leftarrow P(R_i|S_1, \dots, S_{i-1})$ Sample relation

end for

$\psi \leftarrow \{\kappa, R, S\}$

return @GENERATETOKEN

end procedure Return handle to a s



procedure GENERATETOKEN(ψ)

for $i = 1 \dots \kappa$ **do**

$S_i^{(m)} \leftarrow P(S_i^{(m)}|S_i)$ Add motor variance

$L_i^{(m)} \leftarrow P(L_i^{(m)}|R_i, T_1^{(m)}, \dots, T_{i-1}^{(m)})$ Sample part's start location

$T_i^{(m)} \leftarrow f(L_i^{(m)}, S_i^{(m)})$ Compose a part's pen trajectory

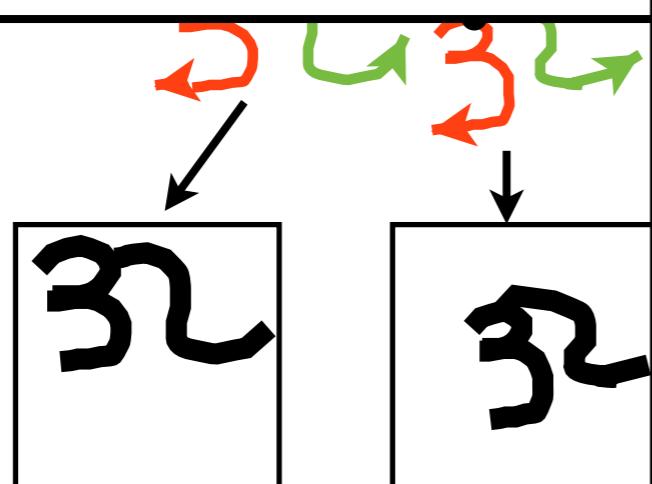
end for

$A^{(m)} \leftarrow P(A^{(m)})$ Sample affine transform

$I^{(m)} \leftarrow P(I^{(m)}|T^{(m)}, A^{(m)})$

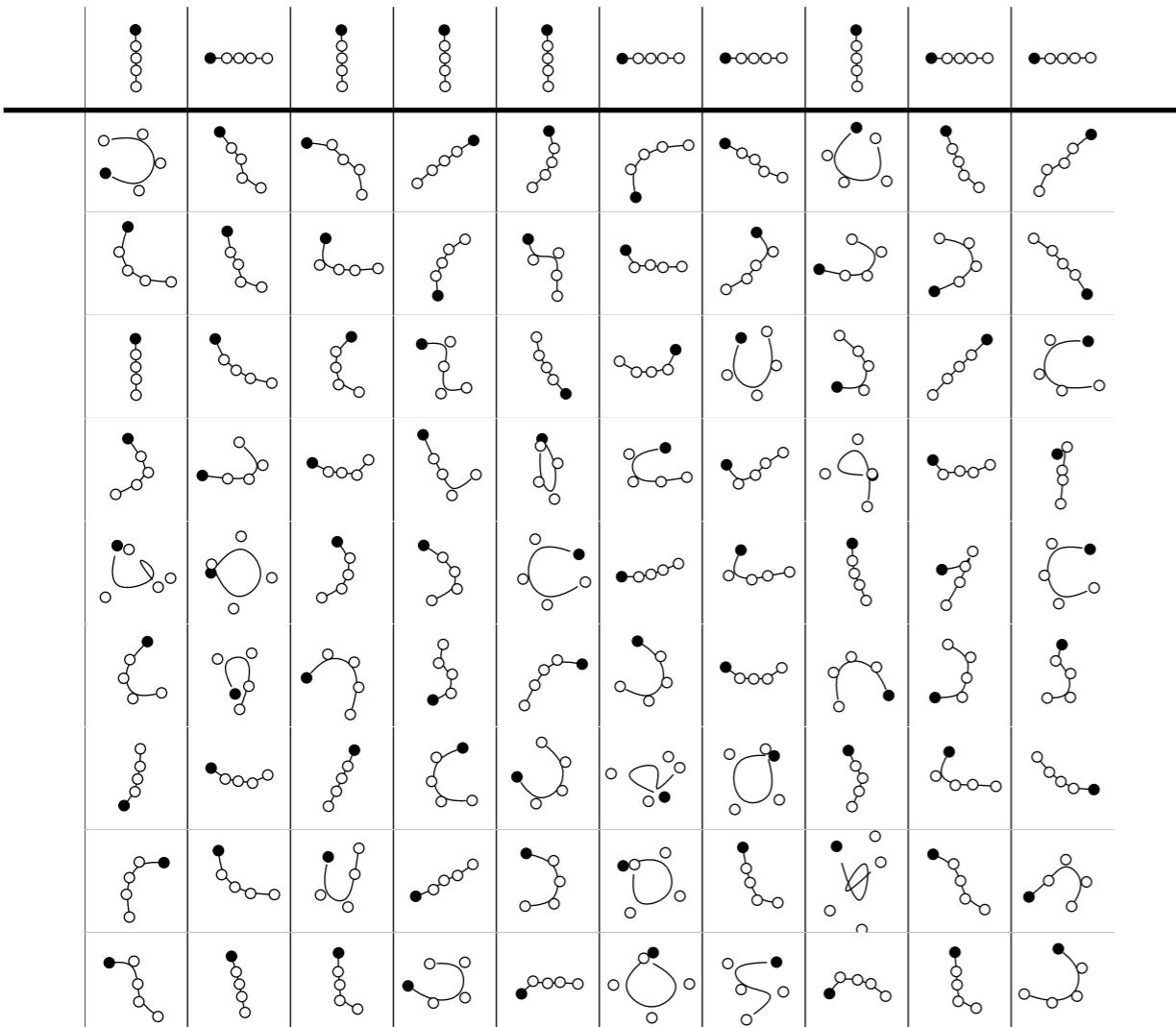
return $I^{(m)}$ Render and sample the binary image

end procedure



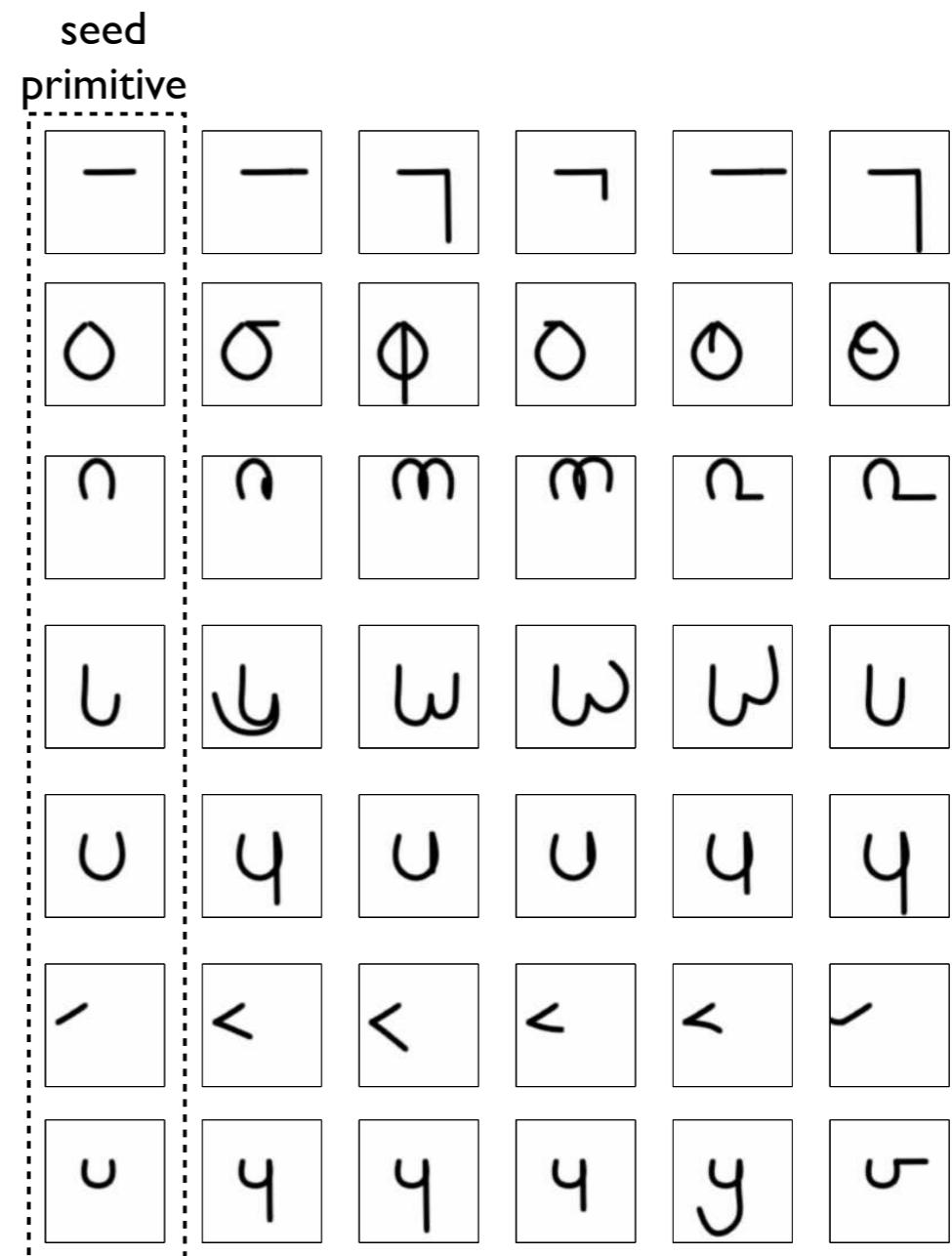
Learning-to-learn programs

learned action primitives



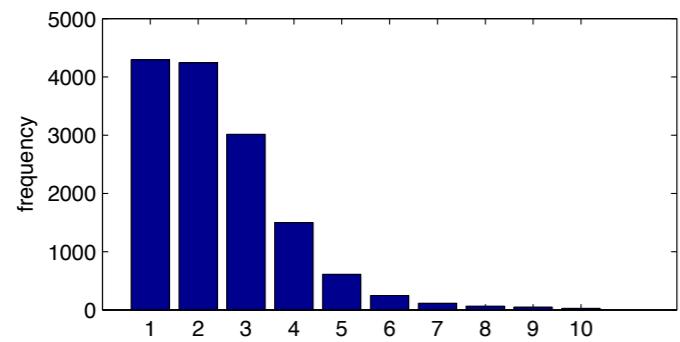
1250 primitives
scale selective
translation invariant

learned primitive transitions

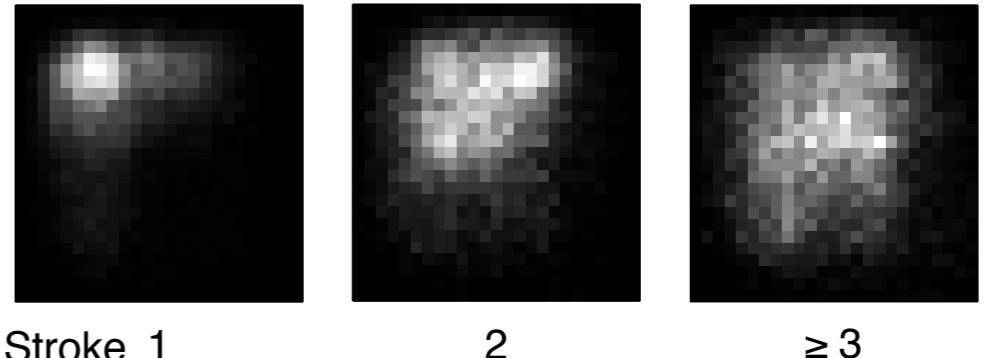


Learning-to-learn programs

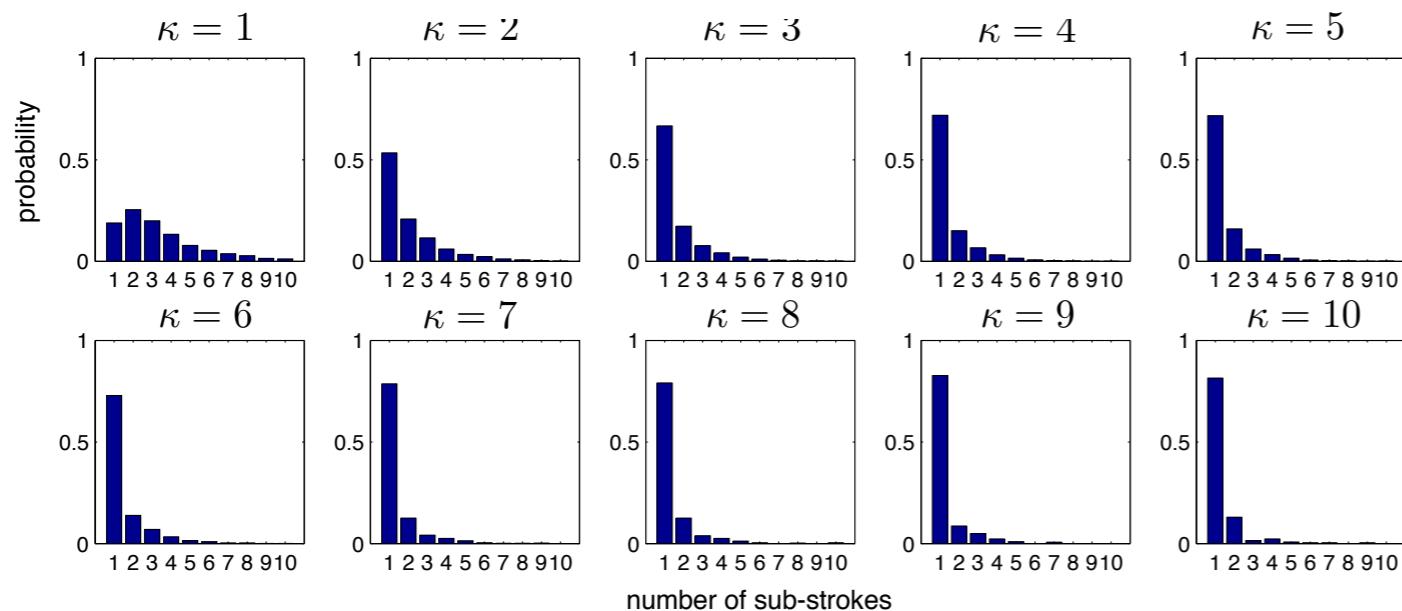
number of strokes



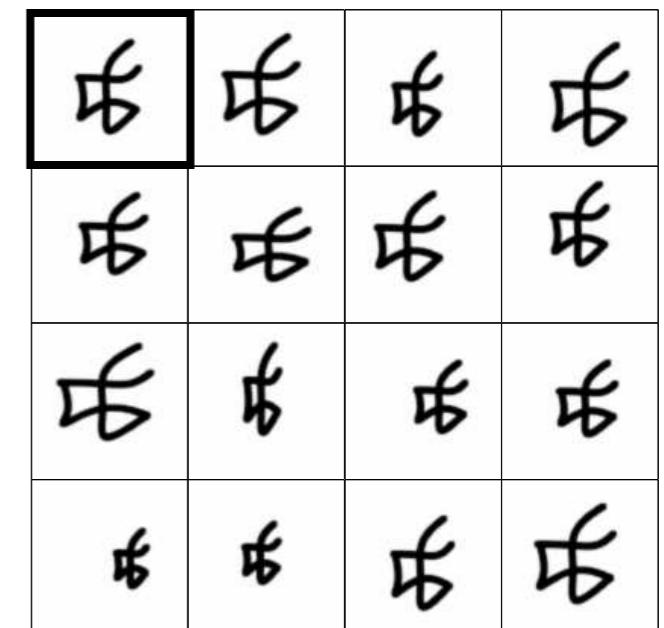
stroke start positions



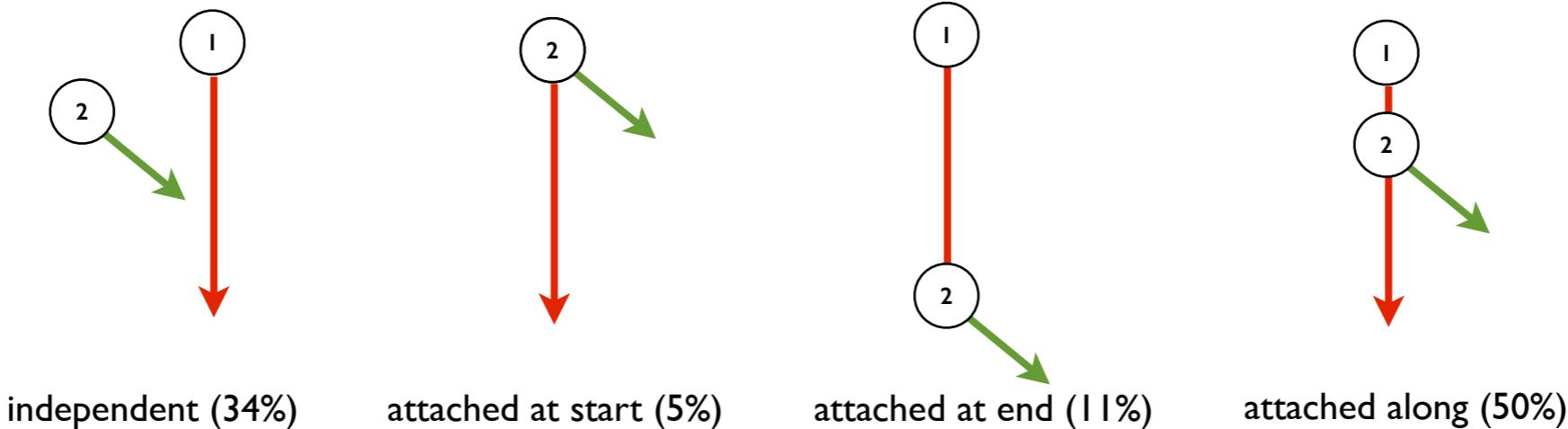
number of sub-strokes for a character with κ strokes



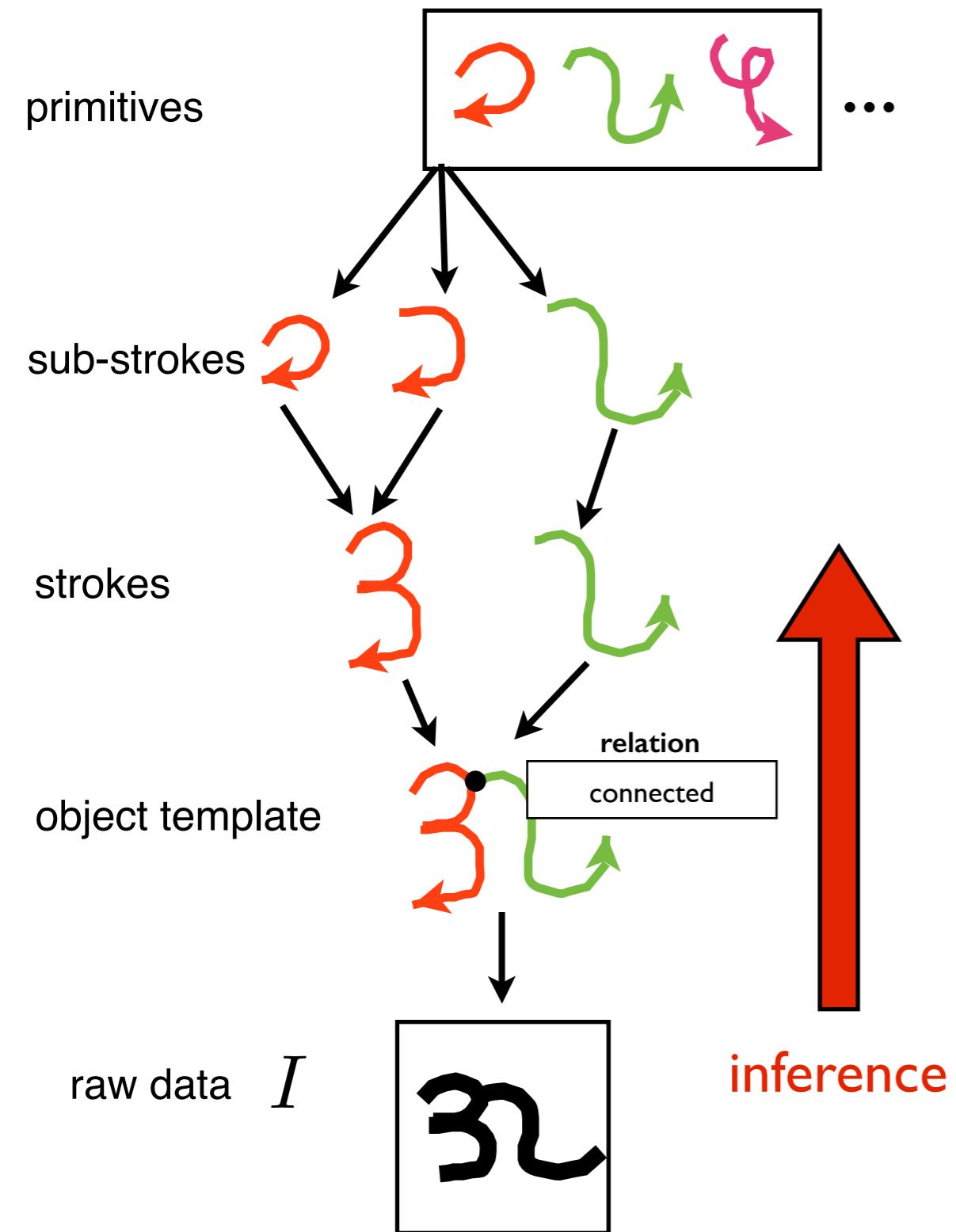
global transformations



relations between strokes



Inferring latent motor programs



θ latent variables

I raw binary image

Bayes' rule

$$P(\theta|I) = \frac{P(I|\theta)P(\theta)}{P(I)}$$

renderer prior on programs

Discrete ($K=5$) approximation to posterior

$$P(\theta|I) \approx \frac{\sum_{i=1}^K w_i \delta(\theta - \theta^{[i]})}{\sum_{i=1}^K w_i}$$

such that

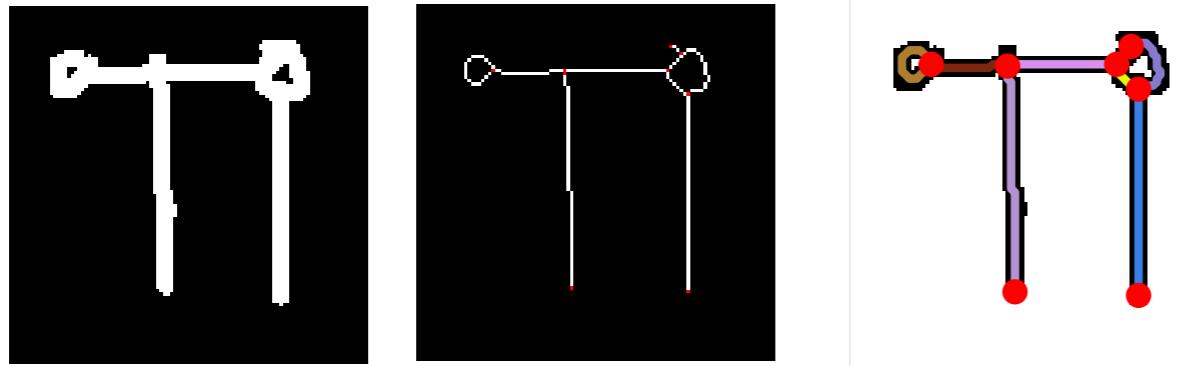
$$w_i \propto P(\theta^{[i]}|I)$$

Intuition: Fit strokes to the observed pixels as closely as possible, with these constraints:

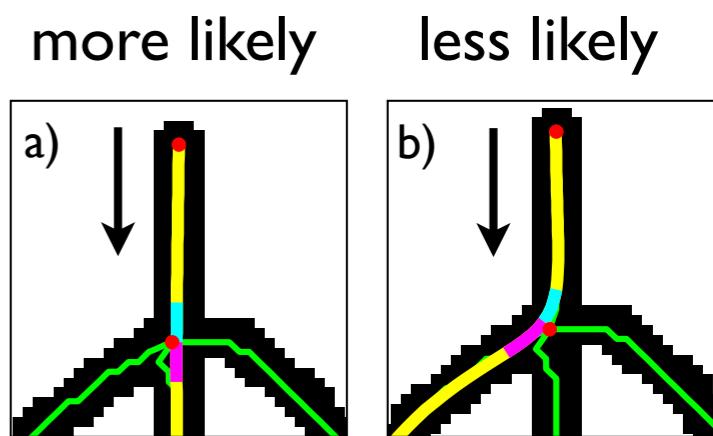
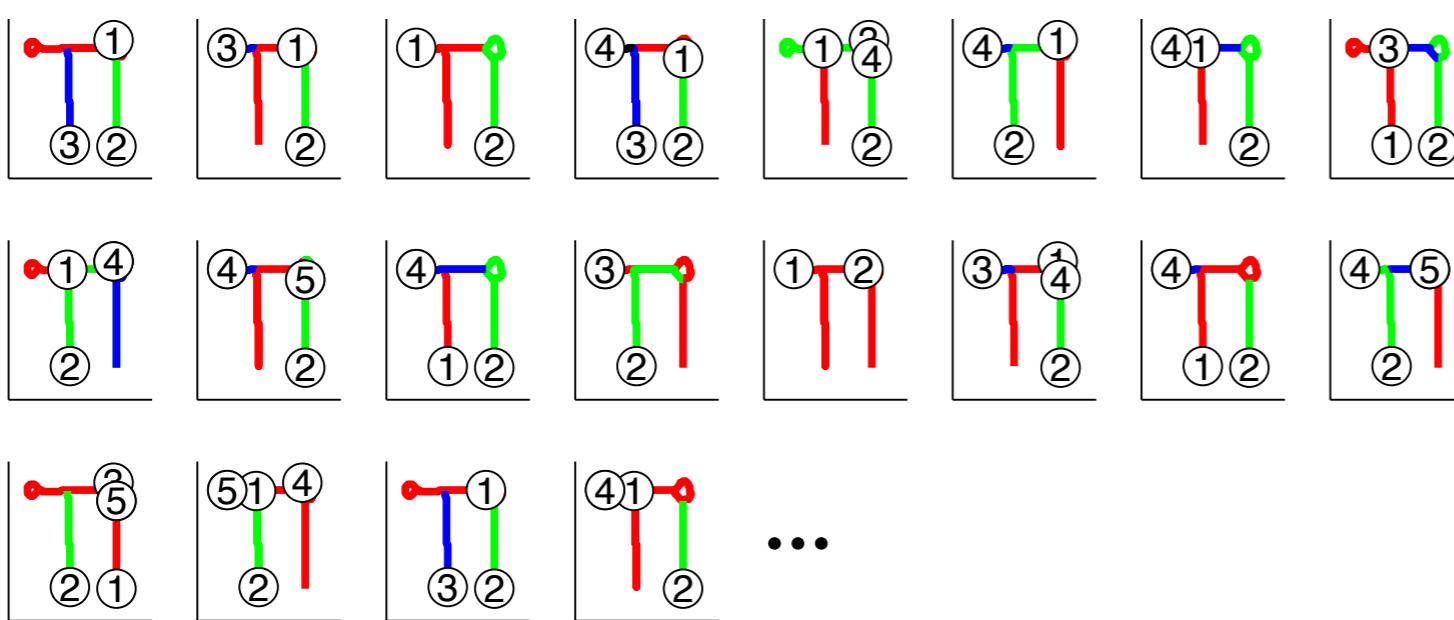
- fewer strokes
- high-probability primitive sequence
- use relations
- stroke order
- stroke directions

Inference

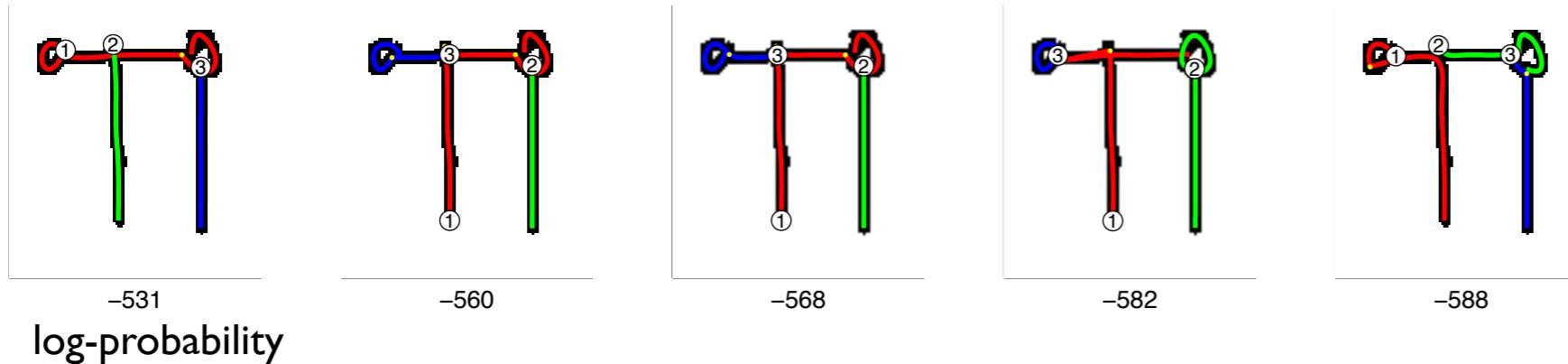
Step 1: characters as undirected graphs



Step 2: guided random parses



Step 3: Top-down fitting with gradient-based optimization



One-shot classification



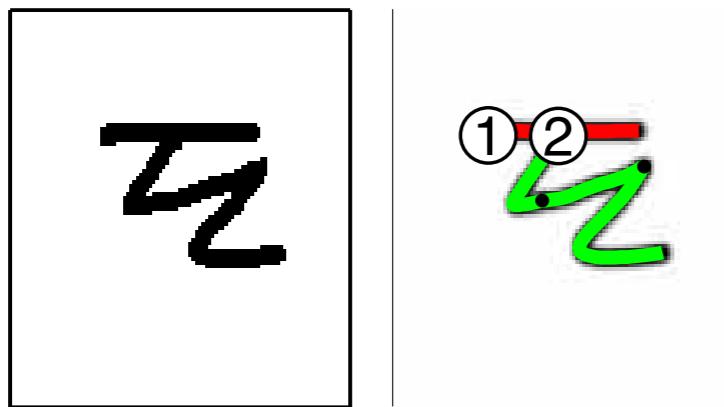
Target character

Click the image below that shows the same character.

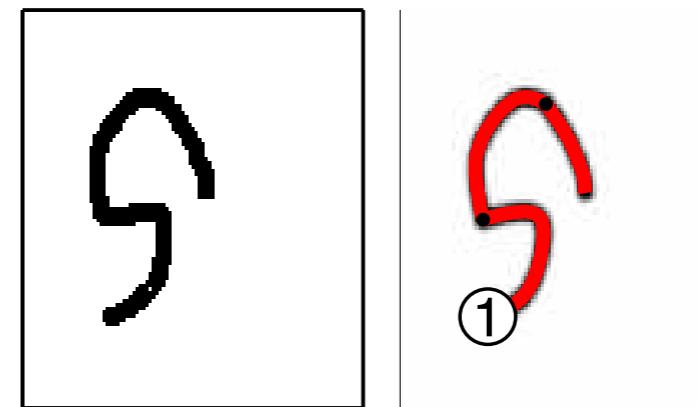
Ⓐ	Ⓑ	ⓐ	Ⓓ	Ⓔ
Ⓛ	Ⓜ	Ⓛ	Ⓜ	Ⓛ
Ⓛ	Ⓜ	Ⓛ	Ⓜ	Ⓛ
Ⓛ	Ⓜ	Ⓛ	Ⓜ	Ⓛ

HBPL: Computing the classification score

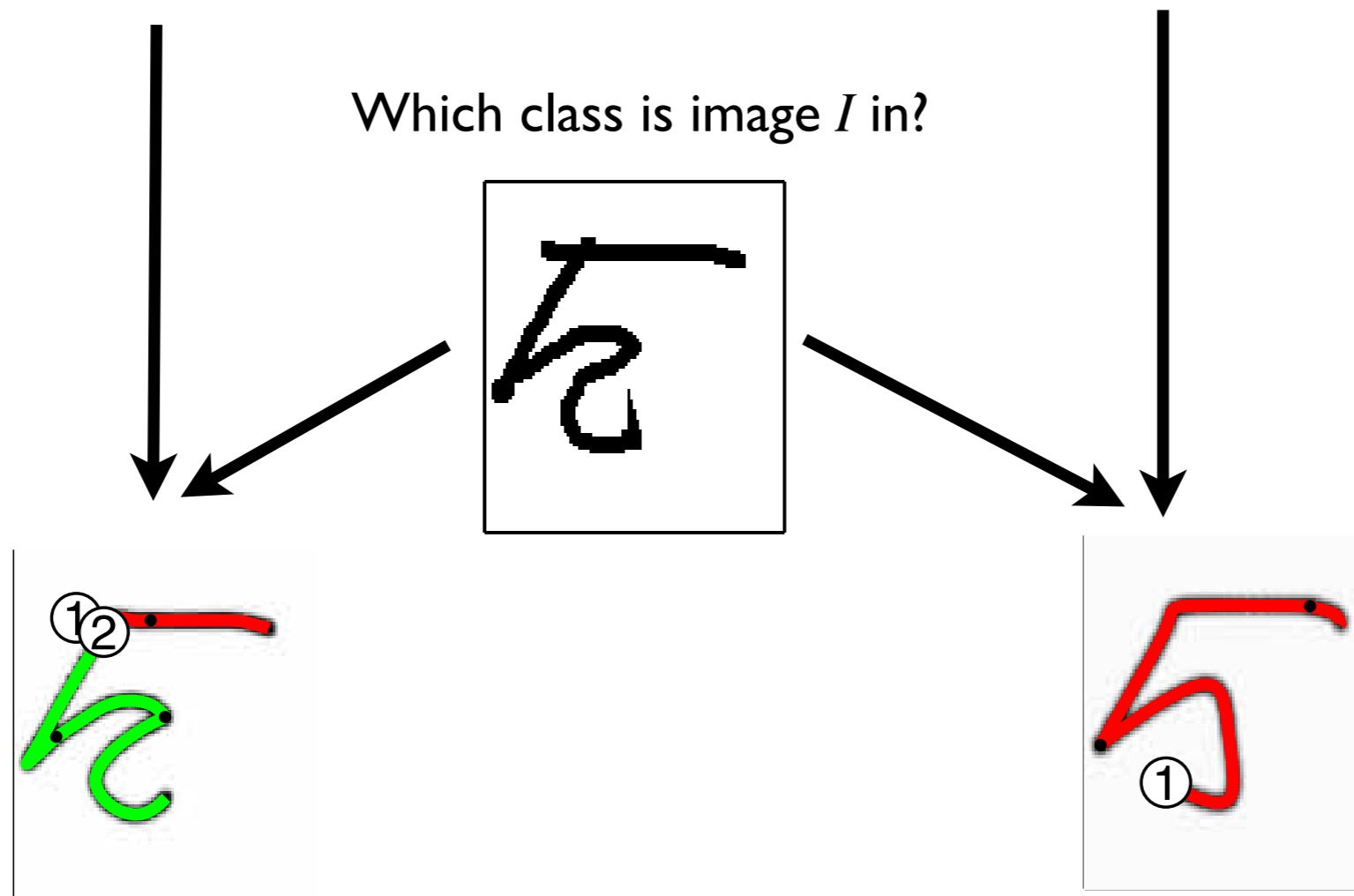
Class 1



Class 2



Which class is image I in?



$$\log P(I|\text{class 1}) \approx -758$$

$$\log P(I|\text{class 2}) \approx -1880$$

Comparing human and machine performance on five tasks

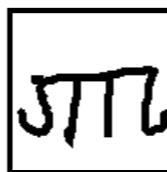
One-shot classification (20-way)



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କ	ର	ମ	ପ	ତ
କ	ର	ମ	ପ	ତ
କ	ର	ମ	ପ	ତ

4.5% human error rate
3.2% machine error rate

Generating new examples



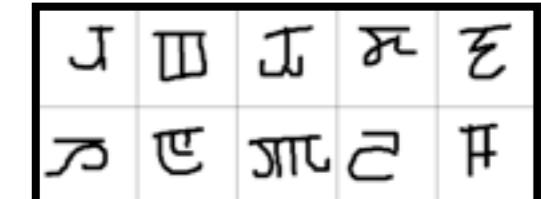
Human or Machine?



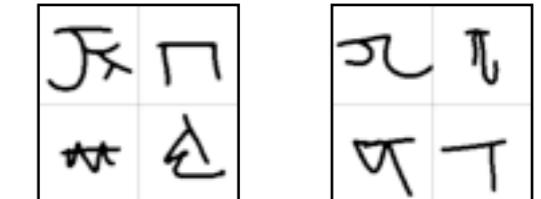
51% Identification (ID) Level
[% judges who correctly ID machine vs. human]

Generating new concepts (from type)

Alphabet

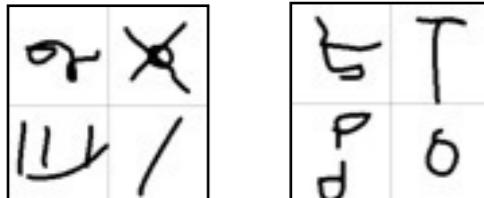


Human or Machine?

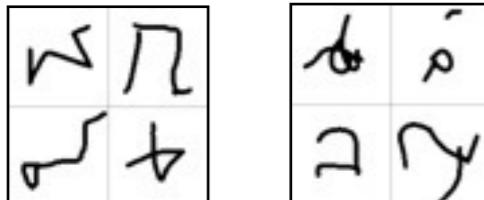


Generating new concepts (unconstrained)

Human or Machine?

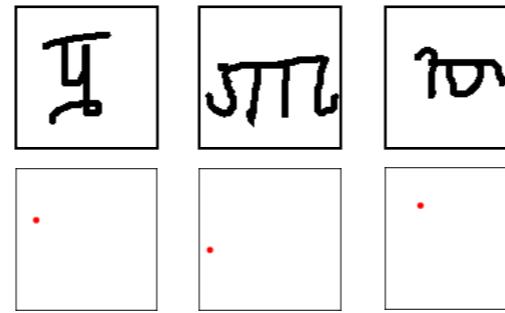


Human or Machine?

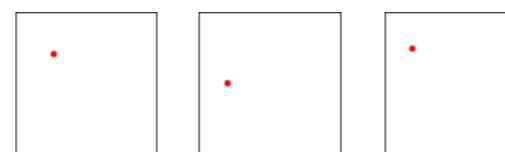


51% ID Level

Generating new examples (dynamic)

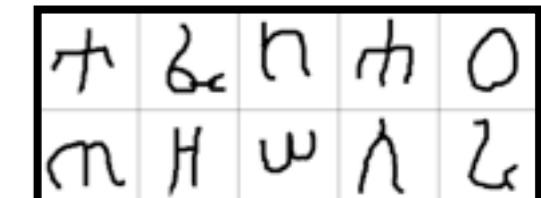


Human
or Machine?

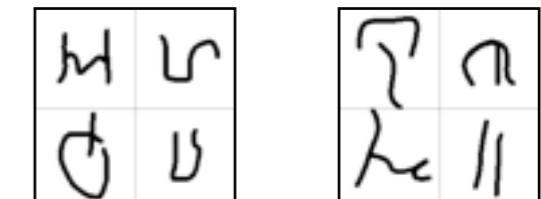


59% ID Level

Alphabet

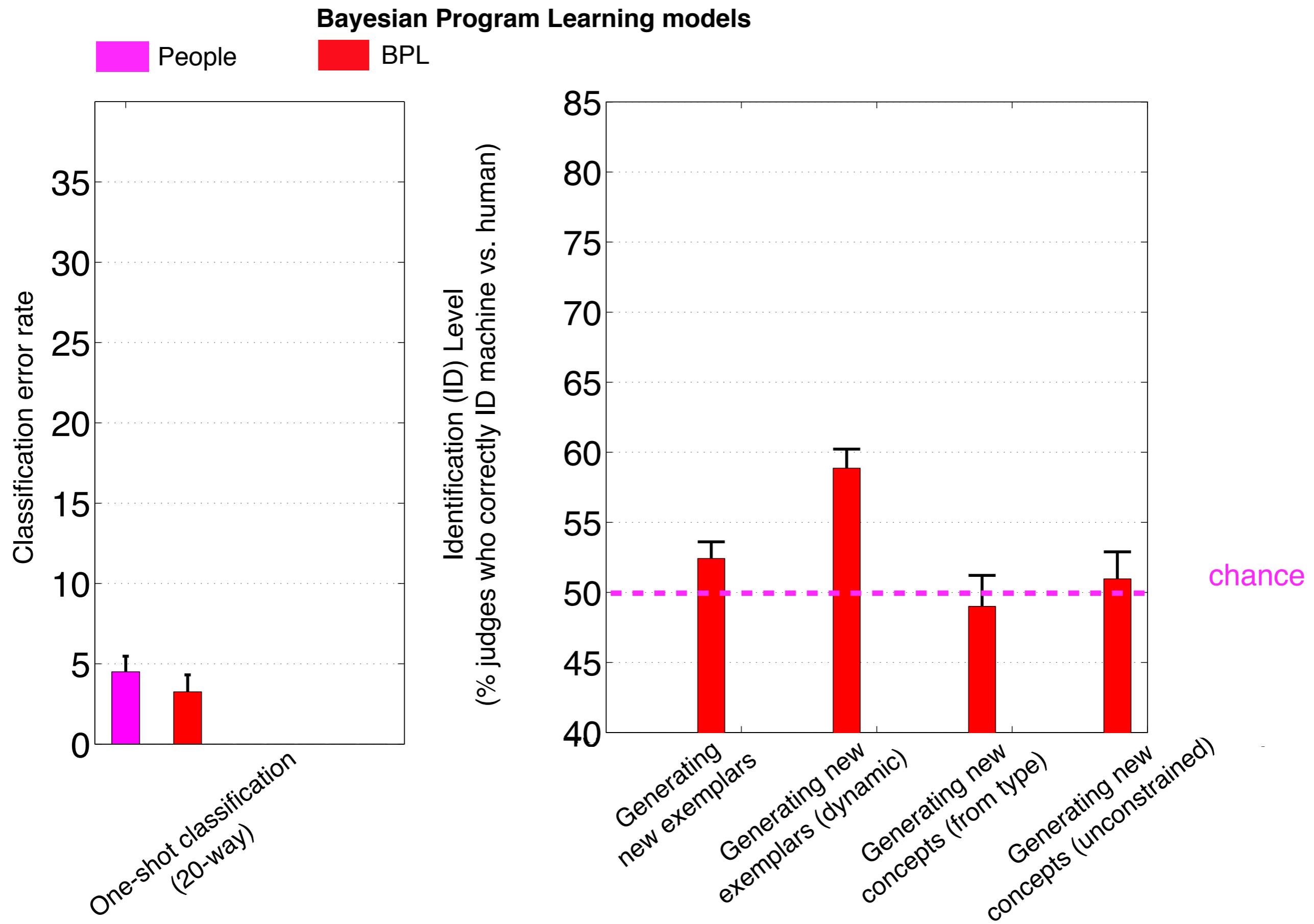


Human or Machine?

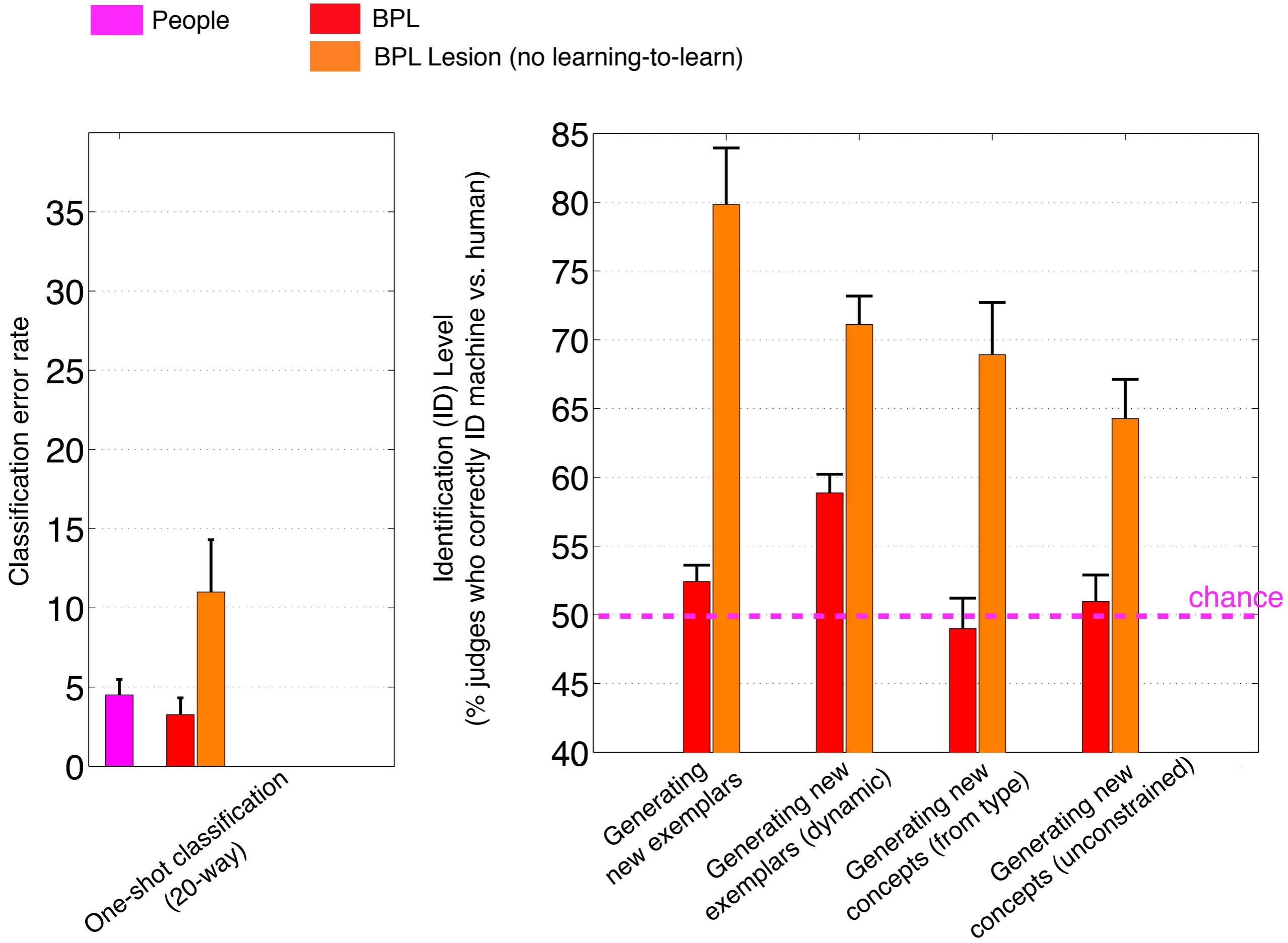


49% ID Level

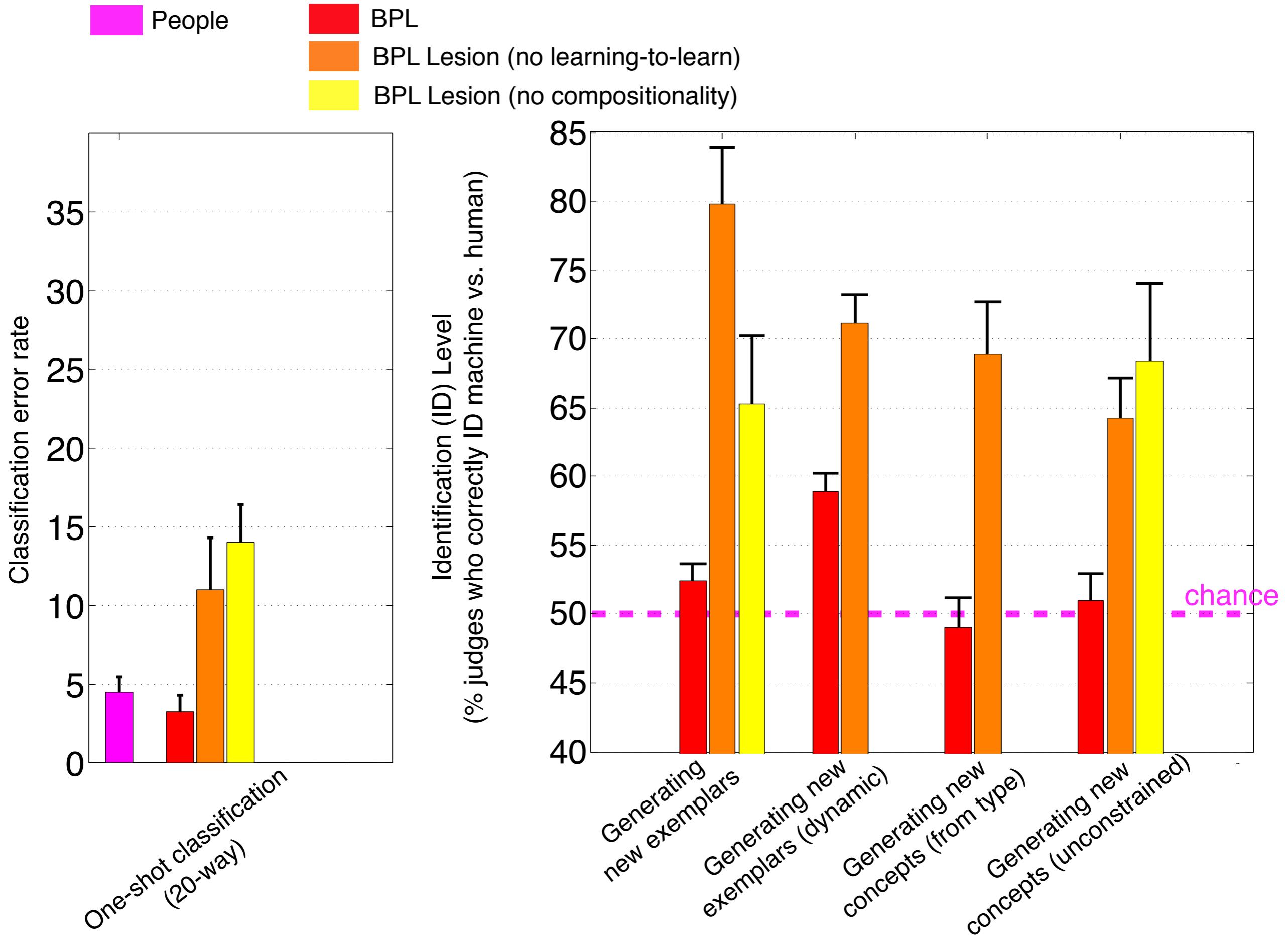
Analyzing the core ingredients

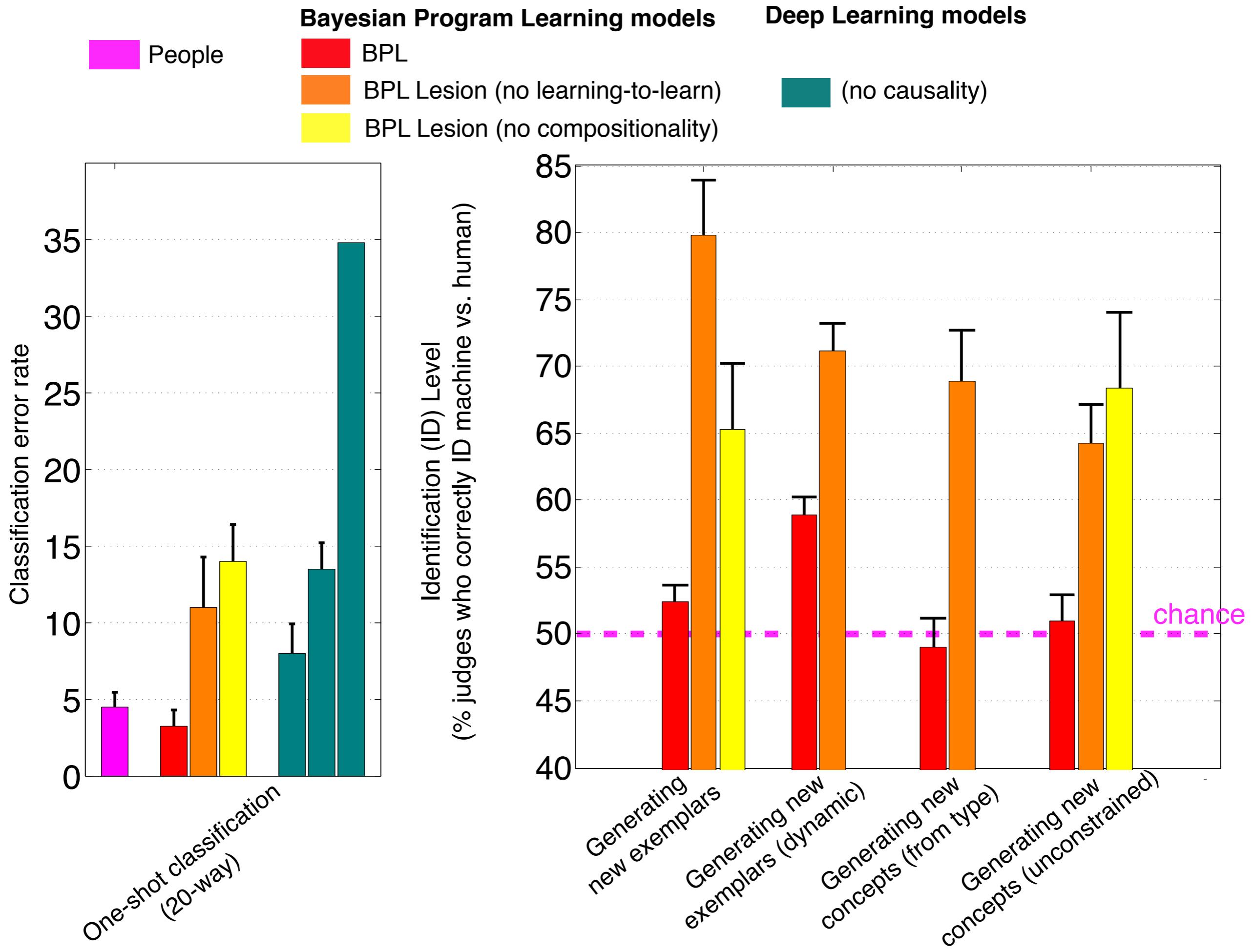


Bayesian Program Learning models



Bayesian Program Learning models





How can people acquire such *rich concepts* from only *one or a few examples*?

the speed of learning

the richness of representation

Conclusion

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Probabilistic programs can help us understand how people learn rich concepts from sparse data.

ss

எ¹
எ²

Programs can represent abstract causal processes.

Probability allows models to handle noise and produce creative outputs.

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கு
மீ
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Many challenges for future work, including developing new inference algorithms and extending approach to other real world tasks.

ந மு லக் டு

மு