Fractal: An Execution Model for Fine-Grain Nested Speculative Parallelism

Suvinay Subramanian∗  Mark C. Jeffrey∗  Maleen Abeydeera∗  Hyun Ryong Lee∗  Victor A. Ying∗  Joel Emer†  Daniel Sanchez∗
∗Massachusetts Institute of Technology †NVIDIA
{suvinay,mcj,maleen,hrlee,victory,emer,sanchez}@csail.mit.edu

ABSTRACT
Most systems that support speculative parallelization, like hardware transactional memory (HTM), do not support nested parallelism. This sacrifices substantial parallelism and precludes composing parallel algorithms. And the few HTMs that do support nested parallelism focus on parallelizing at the coarsest (shallower) levels, incurring large overheads that squander most of their potential.

We present Fractal, a new execution model that supports unordered and timestamp-ordered nested parallelism. Fractal lets programmers seamlessly compose speculative parallel algorithms, and lets the architecture exploit parallelism at all levels. Fractal can parallelize a broader range of applications than prior speculative execution models. We design a Fractal implementation that extends the Swarm architecture and focuses on parallelizing at the finest (deepest) levels. Our approach sidesteps the issues of nested parallel HTMs and uncovers abundant fine-grain parallelism. As a result, Fractal outperforms prior speculative architectures by up to 88× at 256 cores.

CCS CONCEPTS
• Computer systems organization → Multicore architectures;

KEYWORDS
Multicore, Speculative parallelization, Nested parallelism, Fine-grain parallelism, Transactional memory, Thread-level speculation

ACM Reference format:

1 INTRODUCTION
Systems that support speculative parallelization, such as hardware transactional memory (HTM) or thread-level speculation (TLS), have two major benefits over non-speculative systems: they uncover abundant parallelism in many challenging applications [32, 36] and simplify parallel programming [51]. But these systems suffer from limited support for nested speculative parallelism, i.e., the ability to invoke a speculative parallel algorithm within another speculative parallel algorithm. This causes three problems. First, it sacrifices substantial parallelism and limits the algorithms supported by these systems. Second, it disallows composing parallel algorithms, making it hard to write modular parallel programs. Third, it biases programmers to write coarse-grain speculative tasks, which are more expensive to support in hardware.

For example, consider the problem of parallelizing a transactional database. A natural approach is to use HTM and to make each database transaction a memory transaction. Each transaction executes on a thread, and the HTM system guarantees atomicity among concurrent transactions, detecting conflicting loads and stores on the fly, and aborting transactions to avoid serializability violations.

Unfortunately, this HTM approach faces significant challenges. First, each transaction must run on a single thread, but database transactions often consist of many queries or updates that could run in parallel. The HTM approach thus sacrifices this intra-transaction, fine-grain parallelism. Second, long transactions often have large read and write sets, which make conflicts and aborts more likely. These aborts often waste many operations that were not affected by the conflict. Third, supporting large read/write sets in hardware is costly. Hardware can track small read/write sets cheaply, e.g., using private caches [32, 56] or small Bloom filters [14, 65]. But these tracking structures have limited capacity and force transactions that overflow them to serialize, even when they have no conflicts [11, 32, 65]. Beyond these problems, HTM’s unordered execution semantics are insufficient for programs with ordered parallelism, where speculative tasks must appear to execute in a program-specified order [36].

The Swarm architecture [36, 37] can address some of these problems. Swarm programs consist of timestamped tasks. A task can create and enqueue child tasks with any timestamp equal to or greater than its own. Swarm guarantees that tasks appear to execute in timestamp order. To scale, Swarm executes tasks speculatively and out of order. Swarm’s microarchitecture focuses on supporting tasks as small as a few tens of instructions efficiently, including hardware support for speculative scheduling and a large speculation window.

By exposing timestamps to programs, Swarm can parallelize more algorithms than prior ordered speculation techniques, like TLS; Swarm also supports unordered, HTM-style execution. As a result, Swarm often uncovers abundant fine-grain parallelism. But Swarm’s software-visible timestamps can only convey very limited forms of nested parallelism, and they cause two key issues in this regard (Sec. 2). Timestamps make nested algorithms hard to compose, as algorithms at different nesting levels must agree on a common meaning for the timestamp. Timestamps also over-serialize nested algorithms, as they impose more order constraints than needed.

For instance, in the example above, Swarm can be used to break each database transaction into many small, ordered tasks. This exploits intra-transaction parallelism, and, at 256 cores, it is 21× faster than running operations within each transaction serially (Sec. 2.2).
However, to maintain atomicity among database transactions, the programmer must needlessly order database transactions and must carefully assign timestamps to tasks within each transaction. These problems are far from specific to database transactions. In general, large programs have speculative parallelism at multiple levels and often intermix ordered and unordered algorithms. Speculative architectures should support composition of ordered and unordered algorithms to convey all this nested parallelism without undue serialization.

We present two main contributions that achieve these goals. Our first contribution is Fractal, a new execution model for nested speculative parallelism. Fractal programs consist of tasks located in a hierarchy of nested domains. Within each domain, tasks can be ordered or unordered. Any task can create a new subdomain and enqueue new tasks in that subdomain. All tasks in a domain appear to execute atomically with respect to tasks outside the domain.

Fractal allows seamless composition of ordered and unordered nested parallelism. In the above example, each database transaction starts as a single task that runs in an unordered, root domain. Each of these unordered tasks creates an ordered subdomain in which it enqueues tasks for the different operations within the transaction. In the event of a conflict between tasks in two different transactions, Fractal selectively aborts conflicting tasks, rather than aborting all tasks in any one transaction. In fact, other tasks from the two transactions may continue to execute in parallel.

Our second contribution is a simple implementation of Fractal that builds on Swarm and supports arbitrary nesting levels cheaply (Sec. 4). Our implementation focuses on extracting parallelism at the finest (deepest) levels first. This is in stark contrast with current HTMs. Most HTMs only support serial execution of nested transactions, forgoing intra-transaction parallelism. A few HTMs support parallel nested transactions [6, 62], but they parallelize at the coarsest levels, suffer from subtle deadlock and livelock conditions, and impose large overheads because they merge the speculative state of nested transactions [5, 6]. The Fractal execution model lets our implementation avoid these problems. Beyond exploiting more parallelism, focusing on fine-grain tasks reduces the hardware costs of speculative execution.

We demonstrate Fractal’s performance and programmability benefits through several case studies (Sec. 2) and a broad evaluation (Sec. 6). Fractal uncovers abundant fine-grain parallelism on large programs. For example, ports of the STAMP benchmark suite to Fractal outperform baseline HTM implementations by up to $88 \times$ at 256 cores. As a result, while several of the original STAMP benchmarks cannot reach even $10 \times$ scaling, Fractal makes all STAMP benchmarks scale well to 256 cores.

2 MOTIVATION

We motivate Fractal through three case studies that highlight its key benefits: uncovering abundant parallelism, improving programmability, and avoiding over-serialization. Since Fractal subsumes prior speculative execution models (HTM, TLS, and Swarm), all case studies use the Fractal architecture (Sec. 4), and we compare applications written in Fractal vs. other execution models. This approach lets us focus on the effect of different Fractal features. Our implementation does not add overheads to programs that do not use Fractal’s features.

2.1 Fractal uncovers abundant parallelism

Consider the maxflow problem, which finds the maximum amount of flow that can be pushed from a source to a sink node in a network (a graph with directed edges labeled with capacities). Push-relabel is a fast and widely used maxflow algorithm [18], but it is hard to parallelize [8, 48]. Push-relabel tags each node with a height. It initially gives heights of 0 to the sink, $N$ (the number of nodes) to the source, and 1 to every other node. Nodes are temporarily allowed to have excess flow, i.e., have more incoming flow than outgoing flow. Nodes with excess flow are considered active and can push this flow to lower-height nodes. The algorithm processes one active node at a time, attempting to push flow to neighbor nodes and potentially making them active. When an active node cannot push its excess flow, it increases its height to the minimum value that allows pushing flow to a neighbor (this is called a relabel). The algorithm processes active nodes in arbitrary order until no active nodes are left.

To be efficient, push-relabel must use a heuristic that periodically recomputes node heights. Global relabeling [18] is a commonly used heuristic that updates many node heights by performing a breadth-first search on a subset of the graph. Global relabeling takes a significant fraction of the total work, typically 10–40% of instructions [3].

Since push-relabel can process active nodes in an arbitrary order, it can be parallelized using transactional tasks of two types [48, 49]. An active-node task processes a single node, and may enqueue other tasks to process newly-activated nodes. A global-relabel task performs a global relabel operation. Each task must run atomically, since tasks access data from multiple neighbors and must observe a consistent state. We call this implementation maxflow-flat.

We simulate maxflow-flat on systems of up to 256 cores. (See Sec. 5 for methodology details.) At 256 cores, maxflow-flat scales to 4.9× only. Fig. 1a illustrates the reason for this limited speedup: while active-node tasks are short, each global-relabel task is long, and queries and updates many nodes. When a global-relabel task runs, it conflicts with and serializes many active-node tasks.

Fortunately, each global-relabel task performs a breadth-first search, which has plentiful ordered speculative parallelism. Fractal lets us exploit this nested parallelism, running the breadth-first search...
search in parallel while maintaining its atomicity with respect to other active-node tasks. To achieve this, we develop a maxflow-fractal implementation where each global-label task creates an ordered subdomain, in which it executes a parallel breadth-first search using fine-grain ordered tasks, as shown in Fig. 2. A global-label task and its subdomain appear as a single atomic unit with respect to other tasks in the (unordered) root domain. Fig. 1b illustrates how this improves parallelism and efficiency. As a result, Fig. 3 shows that maxflow-fractal achieves a speedup of 322× at 256 cores (over maxflow-flat on one core).

FRACTAL is the first architecture that effectively exploits maxflow’s fine-grain nested parallelism: neither HTM, nor TLS, nor Swarm can support the combination of unordered and ordered parallelism maxflow has. Prior software-parallel push-relabel algorithms attempted to exploit this fine-grain parallelism [3, 48, 49], but the overheads of software speculation and scheduling negated the benefits of additional parallelism (in maxflow-fractal, each task is 373 cycles on average). We also evaluated two state-of-the-art software implementations: prsn [8] and Galois [48]. On 1–256 cores, they achieve maximum speedups of only 4.9× and 8.3× over maxflow-flat at one core, respectively.

2.2 Fractal eases parallel programming

Beyond improving performance, FRACTAL’s support for nested parallelism eases parallel programming because it enables parallel composition. Programmers can write multiple self-contained, modular parallel algorithms and compose them without sacrificing performance: when a parallel algorithm invokes another parallel algorithm, FRACTAL can exploit parallelism at both caller and callee.

In the previous case study, only FRACTAL was able to uncover nested parallelism. In some applications, prior architectures can also exploit the nested parallelism that FRACTAL uncovers, but they do so at the expense of composability.

Consider the transactional database example from Sec. 1. Conventional HTMs run each database transaction in a single thread, and exploit coarse-grain inter-transaction parallelism only. But each database transaction has plentiful ordered parallelism. FRACTAL can exploit both inter- and intra-transaction parallelism by running each transaction in its own ordered subdomain, just as each global relabel runs in its own ordered subdomain in Fig. 2. We apply both approaches to the silo in-memory database [61], Fig. 4 shows that, at 256 cores, silo-fractal scales to 206×, while silo-flat scales to 9.7× only, 21× slower than silo-fractal.

2.3 Fractal avoids over-serialization

Beyond forgoing composability, supporting fine-grain parallelism through manually-specified ordering can cause over-serialization.

Consider the maximal independent set algorithm (mis), which, given a graph, finds a set of nodes S such that no two nodes in S are adjacent, and each node not in S is adjacent to some node in S. mis can be easily parallelized with unordered, atomic tasks [54]. We call this implementation mis-flat. Each task operates on a node and its neighbors. If the node has not yet been visited, the task visits both the node and its neighbors, adding the node to the set and marking its neighbors as excluded from the set. mis-flat creates one task for each node in the graph, and finishes when all these tasks have executed. Fig. 6 shows that, on an R-MAT graph with 8 million nodes and 168 million edges, mis-flat scales to 98× at 256 cores.
mis-flat misses a source of nested parallelism: when a node is added to the set, its neighbors may be visited and excluded in parallel. This yields great benefits when nodes have many neighbors. mis-fractal defines two task types: include and exclude. An include task checks whether a node has already been visited. If it has not, it adds the node to the set and creates an unordered subdomain to run exclude tasks for the node’s neighbors. An exclude task permanently excludes a node from the set. Domains guarantee a node and its neighbors are visited atomically while allowing many tasks of both types to run in parallel. Fig. 6 shows that mis-fractal scales to 145 × at 256 cores, 48% faster than mis-flat.

Swarm cannot exploit this parallelism as effectively. Swarm can only guarantee atomicity for groups of tasks if the program specifies a total order among groups (as in silo). We follow this approach to implement mis-swarm: every include task is assigned a unique timestamp, and it shares its timestamp with any exclude tasks it enqueues. This imposes more order constraints than mis-fractal, where there is no order among tasks in the root domain. Fig. 6 shows that mis-swarm scales to 117 ×, 24% slower than mis-fractal, as unnecessary order constraints cause more aborted work.1

In summary, conveying the atomicity needs of nested parallelism through a fixed order limits parallel execution. FRACTAL allows programs to convey nested parallelism without undue order constraints.

3 FRACTAL EXECUTION MODEL

Fig. 7 illustrates the key elements of the FRACTAL execution model. FRACTAL programs consist of tasks in a hierarchy of nested domains. Each task may access arbitrary data, and may create child tasks as it finds new work to do. For example, in Fig. 7 task C creates children D and E. When each task is created, it is enqueued to a specific domain. Semantics within a domain: Each domain provides either unordered or timestamp-ordered execution semantics. In an unordered domain, FRACTAL chooses an arbitrary order among tasks that respects parent-child dependences, i.e., children are ordered after their parents. For example, in Fig. 7, task C’s children D and E must appear to run after C, but task D can appear to run either before or after task E. These semantics are similar to TM’s: all tasks execute atomically and in isolation.

In an ordered domain, each task has a program-specified timestamp. A task can enqueue child tasks to the same domain with any timestamp equal to or greater than its own. FRACTAL guarantees that tasks appear to run in increasing timestamp order. If multiple tasks have the same timestamp, FRACTAL arbitrarily chooses an order among them. This order always respects parent-child dependences. Timestamps let programs convey their specific order requirements, e.g., the order that events need to run in a simulator. For example, in Fig. 7, the timestamps of tasks F, G, L, and H ensure they appear to run in that fixed order. These semantics are the same as Swarm’s [36].

Semantics across domains: Each task can create a single subdomain and enqueue tasks into it. For example, in Fig. 7, task B creates a new subdomain and enqueues F and G into it. These tasks may themselves create their own subdomains. For example, F creates a subdomain and enqueues I into it.

FRACTAL provides strong atomicity guarantees across domains to allow parallel composition of speculative algorithms. All tasks in a domain appear to execute after the task that creates the domain and are not interleaved with tasks outside their domain. In other words, any non-root domain together with its creator appears to execute as a single atomic unit in isolation. For example, since F is ordered before G in B’s subdomain, all tasks in F’s subdomain (I, J, and K) must appear to execute immediately after F and before G. Furthermore, although no task in B’s subdomain is ordered with respect to any task in D’s subdomain, tasks in B’s and D’s subdomains are guaranteed not to be interleaved.

Tasks may also enqueue child tasks to their immediate enclosing domain, or superdomain. For example, in Fig. 7, K in F’s subdomain enqueues L to B’s subdomain. This lets a task delegate enqueuing future work to descendants within the subdomain it creates. A task cannot enqueue children to any domain beyond the domain it belongs to, its superdomain, and the single subdomain it may create.

3.1 Programming interface

We first expose FRACTAL’s features through a simple low-level C++ interface, then complement it with a high-level, OpenMP-style interface that makes it easier to write FRACTAL applications. Low-level interface: Listing 1 illustrates the key features of the low-level FRACTAL interface by showing the implementation of the mis-fractal tasks described in Sec. 2.3. A task is described by its function, arguments, and ordering properties. Task functions can take arbitrary arguments but do not return values. Tasks create children by calling one of three enqueue functions with the appropriate task function and arguments: fractal::enqueue places the child task in the same domain as the caller, fractal::enqueue_sub places the
void exclude(Node& n) {
  n.state = EXCLUDED;
}

void include(Node& n) {
  if (n.state == UNVISITED) {
    n.state = INCLUDED;
    fractal::create_subdomain(UNORDERED);
    for (Node& ngh: n.neighbors)
      fractal::enqueue_sub(exclude, ngh);
  }
}

Listing 1: FRACTAL implementation of mis tasks.

void include(Node& n) {
  if (n.state == UNVISITED) {
    n.state = INCLUDED;
    forall (Node& ngh: n.neighbors)
      ngh.state = EXCLUDED;
  }
}

Listing 2: Pseudocode for FRACTAL implementation of mis's include using the high-level interface.

child in the caller's subdomain, and fractal::enqueue_super places the child in the caller's superdomain. If the destination domain is ordered, the enqueuing function also takes the child task's timestamp. This isn't the case in Listing 1, as mis is unordered.

Before calling fractal::enqueue_sub to place tasks in a subdomain, a task must call fractal::create_subdomain exactly once to specify the subdomain's ordering semantics: unordered, or ordered with 32- or 64-bit timestamps. In Listing 1, each include task may create an unordered subdomain to atomically run exclude tasks for all its neighbors. The initialization code (not shown) creates an include task for every node in an unordered root domain.

Task enqueue functions also take one optional argument, a spatial hint [35], which is an integer that abstractly indicates what data the task is likely to access. Hints aid the system in performing locality-aware task mapping and load balancing. Hints are orthogonal to FRACTAL. We adopt them because we study systems of up to 256 cores, and several of our benchmarks suffer from poor locality without hints, which limits their scalability beyond tens of cores.

High-level interface: Although our low-level interface is simple, breaking straight-line code into many task functions can be tedious. To ease this burden, we implement a high-level interface in the style of OpenMP and OpenTM [7]. Table 1 details its main constructs, and Listing 2 shows it in action with pseudocode for include. Nested parallelism is expressed using forall, which automatically creates an unordered subdomain and enquesues each loop iteration as a separate task. This avoids breaking code into small functions like exclude. These constructs can be arbitrarily nested. Our actual syntax is slightly more complicated because we do not modify the compiler, and we implement these constructs using macros.2

4 FRACTAL IMPLEMENTATION

Our FRACTAL implementation seeks three desirable properties. First, the architecture should perform fine-grain speculation, carrying out conflict resolution and commits at the level of individual tasks, not complete domains. This avoids the granularity issues of nested parallel HTMs (Sec. 7). Second, creating a domain should be cheap, as domains with few tasks are common (e.g., mis in Sec. 2.3). Third, while the architecture should support unbounded nesting depth to enable software composition, parallelism compounds quickly with depth, so hardware only needs to support a few concurrent depths.

To meet these objectives, our FRACTAL implementation builds on Swarm, and dynamically chooses a task commit order that satisfies FRACTAL’s semantics. We first describe the Swarm microarchitecture, then introduce the modifications needed to support FRACTAL.

4.1 Baseline Swarm microarchitecture

Swarm uncovers parallelism by executing tasks speculatively and out of order. To uncover enough parallelism, Swarm can speculate thousands of tasks ahead of the earliest unfinished task. Swarm introduces modest changes to a tiled, cache-coherent multicore, shown in Fig. 8. Each tile has a group of simple cores, each with its own private L1 cache. All cores in a tile share an L2 cache, and each tile has a slice of a fully-shared L3 cache. Every tile is augmented with a task unit that queues, dispatches, and commits tasks.

Swarm hardware efficiently supports fine-grain tasks and a large speculation window through four main mechanisms: low-overhead hardware task management, large task queues, scalable data-dependence speculation techniques, and high-throughput ordered commits.

---

2 The difference between the pseudocode in Listing 2 and our actual code is that we have to tag the end of control blocks, i.e., using forall_begin(...) {...} forall_end(); This could be avoided with compiler support, as in OpenMP.

---
Hardware task management: Tasks create child tasks and enqueue them to a tile using an enqueue instruction with arguments stored in registers. Each tile’s task queue runsnable tasks and maintains the speculative state of finished tasks that cannot yet commit. Each task is represented by a task descriptor that contains its function pointer, arguments, timestamp, and spatial hint [35]. Cores dequeue tasks for execution from the local task unit. Task units can dispatch any available task to cores, however distant in program order. Dequeueing initiates speculative execution at the task’s function pointer and makes the task’s timestamp and arguments available in registers. A core stalls if there is no task available.

Large task queues: The task unit has two main structures: (i) a task queue that holds task descriptors for every task in the tile, and (ii) a commit queue that holds the speculative state of tasks that have finished execution but cannot yet commit. Together, these queues implement a task-level reorder buffer.

Task and commit queues support tens of speculative tasks per core (e.g., 64 task queue entries and 16 commit queue entries per core) to implement a large window of speculation (e.g., 16384 tasks in the 256-core chip in Fig. 8). Nevertheless, because programs can enqueue tasks with arbitrary timestamps, task and commit queues can fill up. Tasks that have not been dequeued and whose parent has committed can be spilled to memory to free task queue entries. Queue resource exhaustion can also be handled by either stalling the enqueue or aborting higher-timestamp tasks to free space [36].


High-throughput ordered commits: Finally, Swarm adapts the virtual time algorithm [34] to achieve high-throughput ordered commits. Tiles periodically communicate with a central arbiter (e.g., every 200 cycles) to discover the earliest unfinished task in the system. All tasks that precede this earliest unfinished task can safely commit. This scheme can sustain multiple task commits per cycle on average, efficiently supporting ordered tasks as short as a few tens of cycles.

Swarm maintains a consistent order among tasks by giving a unique virtual time (VT) to each task when it is dispatched. Swarm VTs are 128-bit integers that extend the 64-bit program-assigned timestamp with a 64-bit tiebreaker. This tiebreaker is the concatenation of the dispatch cycle and tile id, as shown in Fig. 9. Thus, Swarm VTs break ties among same-timestamp tasks sensibly (prioritizing older tasks), and they satisfy Swarm’s semantics (they order a child task after its parent, since the child is always dispatched at a later cycle). However, FRAC TAL needs a different schema to match its semantics.

4.2 Fractal virtual time

FRACTAL assigns a fractal virtual time (fractal VT) to each task. This fractal VT is the concatenation of one or more domain virtual times (domain VTs).

Figure 10: Domain VT formats.

Domain VTs order all tasks in a domain and are constructed similarly to Swarm VTs. In an ordered domain, each task’s domain VT is the concatenation of its 32- or 64-bit timestamp and a tiebreaker. In an unordered domain, tasks do not have timestamps, so each task’s domain VT is just a tiebreaker, assigned at dispatch time.

FRACTAL uses 32-bit rather than 64-bit tiebreakers for efficiency. As in Swarm, each tiebreaker is the concatenation of dispatch cycle and tile id, which orders parents before children. While 32-bit tiebreakers are efficient, they can wrap around. Sec. 4.4 discusses how FRACTAL handles wrap-arounds. Fig. 10 illustrates the possible formats of a domain VT, which can take 32, 64, or 96 bits.

Fractal VTs enforce a total order among tasks in the system. This order satisfies FRACTAL’s semantics across domains: all tasks within each domain are ordered immediately after the domain’s creator and before any other tasks outside the domain. These semantics can be implemented with two simple rules. First, the fractal VT of a task in the root domain is just its root domain VT. Second, the fractal VT of any other task is equal to its domain VT appended to the fractal VT of the task that created its domain. Fig. 11 shows some example fractal VT formats. A task’s fractal VT is thus made up of one domain VT for each enclosing domain. Two fractal VTs can be compared with a natural lexicographic comparison.

FRACTAL VTs are easy to support in hardware. We use a fixed-width field in the task descriptor to store each fractal VT, 128 bits in our implementation. Fractal VTs smaller than 128 bits are right-padded with zeros. This fixed-width format makes comparing fractal VTs easy, requiring conventional 128-bit comparators. With a 128-bit budget, FRACTAL hardware can support up to four levels of nesting, depending on the sizes of domain VTs. Sec. 4.3 describes how to support levels beyond those that can be represented in 128 bits.

Fig. 12 shows fractal VTs in a system with three domains: an unordered root domain, B’s subdomain (ordered with 64-bit timestamps), and D’s subdomain (unordered). Idle tasks do not have tiebreakers, which are assigned on dispatch. Any two dispatched tasks can be ordered by comparing their fractal VTs. For example, F (in B’s subdomain) is ordered after B, but before M (in D’s subdomain). FRACTAL performs fine-grain speculation by using the
Figure 13: Starting from Fig. 12, zooming in allows F to create and enqueue to a subdomain by shifting fractal VTs.
base domain timestamp, if any. The restored base domain VT has its tiebreaker set to zero, but this does not change any order relations because the domain from which we are zooming out contains all the earliest active tasks.

**Avoiding quiescence:** As explained so far, the system would have to be completely quiesced while fractal VTs are being shifted. This overhead is small—a few hundred cycles—but introducing mechanisms to quiesce the whole system would add complexity. Instead, we use an alternating-bit protocol [60] to let tasks continue running while fractal VTs are modified. Each fractal VT entry in the system has an extra bit that is flipped on each zoom in/out operation. When the bits of two fractal VTs being compared differ, one of them is shifted appropriately to perform the comparison.

### 4.4 Handling tiebreaker wrap-arounds

Using 32-bit tiebreakers makes fractal VTs compact, but causes tiebreakers to wrap around every few tens of milliseconds. Since domains can exist for long periods of time, the range of existing tiebreakers must be compacted to make room for new ones. When tiebreakers are about to wrap around, the system walks every fractal VT and performs the following actions:

1. Subtract \(2^{31}\) (half the range) with saturate-to-0 from each tiebreaker in the fractal VT (i.e., flip the MSB from 1 to 0, or zero all the bits if the MSB was 0).
2. If a task’s final tiebreaker is 0 after subtraction and the task is not the earliest unfinished task, abort it.

When this process finishes, all tiebreakers are \(< 2^{31}\), so the system continues assigning tiebreakers from \(2^{31}\).

This exploits the property that, if the task that created a domain precedes all other active tasks, its tiebreaker can be set to zero without affecting order relations. If the task is aborted because its tiebreaker is set to 0, any subdomain it created will be squashed. In practice, we find this has no effect on performance, because, to be aborted, a task would have to remain speculative for far longer than we observe in any benchmark.

### 4.5 Putting it all together

Our FRACTAL implementation adds small hardware overheads over Swarm. Swarm itself imposes modest overheads to implement speculative execution [36].) Each fractal VT consumes five additional bits beyond Swarm’s 128: four to encode its format (14 possibilities), and one for the alternating-bit protocol. This adds storage overheads of 240 bytes per 4-core tile. FRACTAL also adds simple logic to each subdomain to save and restore base domain timestamps.

FRACTAL makes small changes to the ISA: it modifies the enqueue instruction and adds a create_subdomain instruction. Task enqueue messages carry a fractal VT without the final tiebreaker (up to 96+5 bits) compared to the 64-bit timestamp in Swarm.

Finally, in our implementation, zoom-in/out requests and tiebreaker wrap-arounds are handled by the global virtual time arbiter (the unit that runs the ordered-commit protocol). This adds a few message types between this arbiter and the tiles to carry out the steps in each of these operations. The arbiter must manage a simple in-memory stack to save and restore base domain timestamps.

<table>
<thead>
<tr>
<th>Application</th>
<th>Input</th>
<th>1-core runtime (B cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>color [33]</td>
<td>com-youtube [39]</td>
<td>0.968</td>
</tr>
<tr>
<td>msf [54]</td>
<td>kron_g500-log16 [4, 21]</td>
<td>0.717</td>
</tr>
<tr>
<td>silo [61]</td>
<td>TPC-C, 4 whs, 2 GHz, x86-64 ISA; 8B-wide ifetch, 2-level bprwd with 256×9-bit BHSRs + 512×2-bit PHT, single-issue in-order scoreboarding (stub-on-use), functional unit latencies as in Nehalem [52], 4-entry load and store buffers</td>
<td>2.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Application</th>
<th>Input</th>
<th>1-core runtime (B cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>vacation</td>
<td>-n4 -g06 -s90 +r1048576 +e262144</td>
<td>10.6</td>
</tr>
<tr>
<td>genome</td>
<td>-g4096 -s48 -n1048576</td>
<td>4.31</td>
</tr>
<tr>
<td>kmeans</td>
<td>-m40 -n40 -i rand-n16384 -d24 -c16</td>
<td>8.75</td>
</tr>
<tr>
<td>intruder</td>
<td>-a10 -i 164 -c32768</td>
<td>2.12</td>
</tr>
<tr>
<td>yada</td>
<td>-a15 -i timeun000000.2</td>
<td>3.41</td>
</tr>
<tr>
<td>labyrinth</td>
<td>random-x128-y128-z5-n128</td>
<td>4.41</td>
</tr>
<tr>
<td>bayes</td>
<td>-v32 -e4096 -n10 -p40 -c2 -e8 -s1</td>
<td>8.81</td>
</tr>
</tbody>
</table>

### 5 EXPERIMENTAL METHODOLOGY

**Modeled system:** We use a cycle-accurate, event-driven simulator based on Pin [40, 47] to model FRACTAL systems of up to 256 cores, as shown in Fig. 8, with parameters in Table 2. Swarm parameters (task and commit queue sizes, etc.) match those from prior work [35, 36, 37]. We use detailed core, cache, network, and main memory models, and faithfully simulate all speculation overheads (e.g., running misspeculating tasks until they abort, simulating conflict check and rollback delays and traffic, etc.). Our 256-core configuration is similar to the Kalray MPPA [22]. We also simulate smaller systems with square meshes (\(K \times K\) tiles for \(K \leq 8\)). We keep per-core L2/L3 sizes and queue capacities constant across system sizes. This captures performance per unit area. As a result, larger systems have higher queue and cache capacities, which sometimes cause superlinear speedups.

**Benchmarks:** Table 3 reports the benchmarks we evaluate. Benchmarks have 1-core run-times of about 1 B cycles or longer. We

<table>
<thead>
<tr>
<th>Cores</th>
<th>256 cores in 64 tiles (4 cores/tile), 2 GHz, x86-64 ISA; 8B-wide ifetch, 2-level bprwd with 256×9-bit BHSRs + 512×2-bit PHT, single-issue in-order scoreboarding (stub-on-use), functional unit latencies as in Nehalem [52], 4-entry load and store buffers</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 caches</td>
<td>16 KB, per-core, split D/I, 8-way, 2-cycle latency</td>
</tr>
<tr>
<td>L2 caches</td>
<td>256 KB, per-tile, 8-way, inclusive, 7-cycle latency</td>
</tr>
<tr>
<td>L3 cache</td>
<td>64 MB, shared, static NUCA [38] (1 MB bank/tile), 16-way, inclusive, 9-cycle bank latency</td>
</tr>
<tr>
<td>Coherence</td>
<td>MESI, 64 B lines, in-cache directories</td>
</tr>
<tr>
<td>NoC</td>
<td>4 controllers at chip edges, 120-cycle latency</td>
</tr>
<tr>
<td>Main mem</td>
<td>64 task queue entries/core (16384 total), 16 commit queue entries/core (4096 total), 128-bit fractal VTs</td>
</tr>
<tr>
<td>FRACTAL instructions</td>
<td>5 cycles per enqueue/dequeue/finish_task, 2 cycles per create_subdomain instruction</td>
</tr>
<tr>
<td>Scheduler</td>
<td>Spatial hints with load balancing [35]</td>
</tr>
<tr>
<td>Conflicts</td>
<td>2 Kbit 8-way Bloom filters, H, hash functions [13]</td>
</tr>
<tr>
<td>Commits</td>
<td>Tiles send updates to GVT arbiter every 200 cycles</td>
</tr>
<tr>
<td>Spills</td>
<td>Coalescers fire when a task queue is 85% full</td>
</tr>
</tbody>
</table>

---

Table 2: Configuration of the 256-core system.

Table 3: Benchmark information: source implementations, inputs, and execution time on a single-core system.
begin with applications where FRACtAL uncovers abundant parallelism. On all benchmarks except bayes, FRACtAL includes an optimization to filter out non-spanning edges efficiently. This optimization improves absolute performance but reduces the amount of highly parallel work, so msf has lower scalability than the unoptimized Swarm version [36].

STAMP benchmarks use inputs between the recommended “+” and “++” sizes, to achieve a run-time large enough to evaluate 256-core systems, yet small enough to be simulated in reasonable time. maxflow uses rmf-wide [29], one of the harder graph families from the DIMACS maxflow challenge [8]. mis uses an R-MAT graph [15], which has a power-law distribution.

We fast-forward each benchmark to the start of its parallel region (skipping initialization), and report results for the full parallel region. On all benchmarks except bayes, we perform enough runs to achieve 95% confidence intervals ≤ 1%. bayes is highly nondeterministic, so we report its average results with 95% confidence intervals over 50 runs.

### 6 EVALUATION

We now analyze the benefits of FRACtAL in depth. As in Sec. 2, we begin with applications where FRACtAL uncovers abundant fine-grain parallelism through nesting. We then discuss FRACtAL’s benefits from avoiding over-serialization. Finally, we characterize the performance overheads of zooming to support deeper nesting.

#### 6.1 Fractal uncovers abundant parallelism

FRACtAL’s support for nested parallelism greatly benefits three benchmarks: maxflow, as well as labyrinth and bayes, the two least scalable benchmarks from STAMP. maxflow, as discussed in Sec. 2.1, is limited by long global-relabel tasks. Our fractal version performs the breadth-first search nested within each global relabel in parallel. labyrinth finds non-overlapping paths between pairs of (start, end) cells on a 3D grid. Each transaction operates on one pair: it finds the shortest path on the grid and claims the cells on the path for itself. In the STAMP implementation, each transaction performs this shortest-path search sequentially. Our fractal version runs the shortest-path search nested within each transaction in parallel, using an ordered subdomain.

<table>
<thead>
<tr>
<th>Perf. vs serial</th>
<th>Avg task length</th>
<th>Nesting type</th>
</tr>
</thead>
<tbody>
<tr>
<td>@ 1-core</td>
<td>(cycles)</td>
<td></td>
</tr>
<tr>
<td>flat</td>
<td>fractal</td>
<td>flat</td>
</tr>
<tr>
<td>maxflow</td>
<td>0.92×</td>
<td>0.68×</td>
</tr>
<tr>
<td>labyrinth</td>
<td>1×</td>
<td>0.62×</td>
</tr>
<tr>
<td>bayes</td>
<td>1×</td>
<td>1.11×</td>
</tr>
<tr>
<td>silo</td>
<td>1.14×</td>
<td>1.10×</td>
</tr>
<tr>
<td>mis</td>
<td>0.79×</td>
<td>0.26×</td>
</tr>
<tr>
<td>color</td>
<td>1.06×</td>
<td>0.80×</td>
</tr>
<tr>
<td>msf</td>
<td>3.1×</td>
<td>1.73×</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nesting type</th>
<th>ord-32b</th>
<th>ord-64b</th>
<th>unord</th>
</tr>
</thead>
<tbody>
<tr>
<td>maxflow</td>
<td>→</td>
<td>→</td>
<td>→</td>
</tr>
<tr>
<td>labyrinth</td>
<td>→</td>
<td>→</td>
<td>→</td>
</tr>
<tr>
<td>bayes</td>
<td>→</td>
<td>→</td>
<td>→</td>
</tr>
</tbody>
</table>

| (a) Speedup from 1 to 256 cores relative to 1-core flat. |

| (b) Breakdown of core cycles at 256 cores, with speedups on top. |

Figure 14: Performance of flat and fractal versions of applications with abundant nested parallelism, using Bloom filter-based or Precise conflict detection.

bayes learns the structure of a Bayesian network, a DAG where nodes denote random variables and edges denote conditional dependencies among variables. bayes spends most time deciding whether to insert, remove, or reverse network edges. Evaluating each decision requires performing many queries to an ADTree data structure, which efficiently represents probability estimates. In the STAMP implementation, each transaction evaluates and applies an insert/remove/reverse decision. Since the ADTree queries performed depend on the structure of the network, transactions serialize often. Our fractal version runs ADTree queries nested within each transaction in parallel, using an unordered subdomain.

Table 4 compares the 1-core performance and average task lengths of flat and fractal versions. flat versions of these benchmarks have long, unordered transactions (up to 16 M cycles). fractal versions have much smaller tasks (up to 3590 cycles on average in bayes). These short tasks hurt serial performance (by up to 38% in labyrinth), but expose plentiful intra-domain parallelism (e.g., a parallel breadth-first search), yielding great scalability.

Beyond limiting parallelism, the long transactions of flat versions have large read/write sets that often overflow FRACtAL’s Bloom filters, causing false-positive aborts. Therefore, we also present results under an idealized, precise conflict detection scheme that does not incur false positives. High false positive rates are not specific to FRACtAL—prior HTMs used similarly-sized Bloom filters [14, 43, 53, 65].

Fig. 14a shows the performance of the flat and fractal versions when scaling from 1- to 256-core systems. All speedups reported are over the 1-core flat version. Solid lines show speedups when using Bloom filters, while dashed ones show speedups under precise conflict detection. flat versions scale poorly, especially when using Bloom filters: the maximum speedups across
all system sizes range from $1.0 \times$ (labyrinth at 1 core) to $4.9 \times$ (maxflow). By contrast, fractal versions scale much better, from $88 \times$ (labyrinth) to $322 \times$ (maxflow).3

Fig. 14b gives more insight into these differences by showing the percentage of cycles that cores spend on different activities: (i) running tasks that are ultimately committed, (ii) running tasks that are later aborted, (iii) spilling tasks from the hardware task queues, (iv) stalled on a full task or commit queue, or (v) stalled due to lack of tasks. Each group of bars shows results for a different application at 256 cores.

Fig. 14b shows that flat versions suffer from lack of work caused by insufficient parallelism, and stalls caused by long tasks that eventually become the earliest active task and prevent others from committing. Moreover, most of the work performed by flat versions is aborted as tasks have large read/write sets and frequently conflict. labyrinth-flat and bayes-flat also suffer frequent false-positive aborts that hurt performance with Bloom filter conflict detection. Although precise conflict detection helps labyrinth-flat and bayes-flat, both benchmarks still scale poorly (to 4.3 \times and 6.8 \times, respectively) due to insufficient parallelism.

By contrast, fractal versions spend most cycles executing useful work, and aborted cycles are relatively small, from 7% (bayes) to 24% (maxflow). fractal versions perform just as well with Bloom filters as with precise conflict detection. These results show that exploiting fine-grain nested speculative parallelism is an effective way to scale challenging applications.

### 6.2 Fractal avoids over-serialization

Fractal’s support for nested parallelism avoids over-serialization on four benchmarks: silo, mis, color, and msf. Swarm can exploit nested parallelism in these benchmarks by imposing a total order among coarse-grain operations or groups of tasks (Sec. 2.3). Sec. 2 showed that this has a negligible effect on silo, so we focus on the other three applications.

mis, color, and msf are graph-processing applications. Their flat versions perform operations on many graph nodes that can be parallelized but must remain atomic—e.g., in mis, adding a node to the independent set and excluding its neighbors (Sec. 2.3). mis-flat is unordered, while color-flat and msf-flat visit nodes in a partial order (e.g., color visits larger-degree nodes first). Our fractal versions use one subdomain per coarse-grain operation to exploit this nested parallelism (Table 4). The swarm-fg versions of these benchmarks use the same fine-grain tasks as fractal but use a unique timestamp or timestamp range per coarse-grain operation to guarantee atomicity, imposing a fixed order among coarse-grain operations.

Fig. 15 shows the scalability and cycle breakdowns for these benchmarks. flat versions achieve the lowest speedups, from $26 \times$ (msf at 64 cores) to $98 \times$ (mis). Fig. 15b shows that they are dominated by aborts, which take up to 73% of cycles in color-flat, and empty cycles caused by insufficient parallelism in msf and mis. In msf-flat, frequent aborts hurt performance beyond 64 cores.

By contrast, fractal versions achieve the highest performance, from $40 \times$ (msf) to $145 \times$ (mis). At 256 cores, the majority of time is spent on committed work, although aborts are still noticeable (up to 30% of cycles in color). While fractal versions perform better at 256 cores, their tiny tasks impose higher overheads, so they underperform flat on small core counts. This is most apparent in msf, where fractal tasks are just 49 cycles on average (Table 4).

Finally, swarm-fg versions follow the same scaling trends as fractal ones, but over-serialization makes them 6% (color), 24% (mis), and 93% (msf) slower. Fig. 15b shows that these slowdowns primarily stem from more frequent aborts. This is because in swarm-fg versions, conflict resolution priority is static (determined by timestamps), while in fractal versions, it is based on the dynamic execution order (determined by tiebreakers). In summary, these results show that Fractal makes fine-grain parallelism more attractive by avoiding needless order constraints.

### 6.3 Zooming overheads

Although our Fractal implementation supports unbounded nesting (Sec. 4.3), two nesting levels suffice for all the benchmarks we evaluate. Larger programs should require deeper nesting. Therefore, we use a microbenchmark to characterize the overheads of Fractal’s zooming technique.

Our microbenchmark stresses Fractal by creating many nested domains that contain few tasks each. Specifically, it generates a depth-8 tree of nested domains with fanout $F$. All tasks perform a small, fixed amount of work (1500 cycles). Non-leaf tasks then create an unordered subdomain and enqueue $F$ children into it. We sweep both the fanout ($F = 4$ to 12) and the maximum number of concurrent levels $D$ in Fractal, from 2 (64-bit fractal VTs) to 8 (256-bit fractal VTs). At $D = 8$, the system does not perform any zooming. Our default hardware configuration supports up to 4 concurrent levels.
We considered 18 benchmarks to evaluate Fractal: all eight from Swarm [35, 36], all eight from STAMP [42], as well as maxflow and m&is. We looked for opportunities to exploit nested parallelism, focusing on benchmarks with limited speedups. In summary, Fractal benefits 7 out of these 18 benchmarks. We did not find opportunities to exploit nested parallelism in the five Swarm benchmarks not presented here (bfs, sssp, astart, des, and nocsim). These benchmarks already use fine-grain tasks and scale well to 256 cores.

Fig. 17 shows how each STAMP benchmark scales when using different Fractal features. All speedups reported are over the 1-core TM version. The TM lines show the performance of the original STAMP transactions ported to Swarm tasks. Three applications (intruder, labyrinth, and bayes) barely scale, while two (yada and kmeans) scale well at small core counts but suffer on larger systems. By contrast, Fractal’s features make all STAMP applications scale, although speedups are not only due to nesting. First, the TM versions of intruder and yada use software task queues that limit their scalability. Refactoring them to use Swarm/Fractal hardware task queues [36] makes them scale. Second, spatial hints [35] improves genome and makes kmeans scale.

6.4 Discussion

We considered 18 benchmarks to evaluate Fractal: all eight from Swarm [35, 36], all eight from STAMP [42], as well as maxflow and m&is. We looked for opportunities to exploit nested parallelism, focusing on benchmarks with limited speedups. In summary, Fractal benefits 7 out of these 18 benchmarks. We did not find opportunities to exploit nested parallelism in the five Swarm benchmarks not presented here (bfs, sssp, astart, des, and nocsim). These benchmarks already use fine-grain tasks and scale well to 256 cores.

Fig. 17 shows how each STAMP benchmark scales when using different Fractal features. All speedups reported are over the 1-core TM version. The TM lines show the performance of the original STAMP transactions ported to Swarm tasks. Three applications (intruder, labyrinth, and bayes) barely scale, while two (yada and kmeans) scale well at small core counts but suffer on larger systems. By contrast, Fractal’s features make all STAMP applications scale, although speedups are not only due to nesting. First, the TM versions of intruder and yada use software task queues that limit their scalability. Refactoring them to use Swarm/Fractal hardware task queues [36] makes them scale. Second, spatial hints [35] improves genome and makes kmeans scale.

Finally, as we saw in Sec. 6.1, Fractal’s support for nesting makes

7 RELATED WORK

7.1 Nesting in transactional memory

Serial nesting: Most HTMs support serial execution of nested transactions, which makes transactional code easy to compose but forgoes intra-transaction parallelism. Nesting can be trivially supported by ignoring the boundaries of all nested transactions, treating them as part of the top-level one. Some HTMs exploit nesting to implement partial aborts [44]: they track the speculative state of a nested transaction separately while it executes, so conflicts that occur while the nested transaction runs do not abort the top-level one.

Even with partial aborts, HTMs ultimately merge nested speculative state into the top-level transaction, resulting in large atomic regions that are hard to support in hardware [2, 12, 19, 20] and make conflicts more likely.

Prior work has explored relaxed nesting semantics, like open nesting [41, 44, 46] and early release [55], which relax isolation to improve performance. Fractal is orthogonal to these techniques and could be extended to support them, but we do not see the need on the applications we study.

Parallel nesting: Some TM systems support running nested transactions in parallel [45]: a transaction can launch multiple nested transactions and wait for them to finish. Nested transactions may run in parallel and can observe updates from their parent transaction. As in serial nesting, when a nested transaction finishes, its speculative state is merged with its parent’s. When all nested transactions finish, the parent transaction resumes execution.

Most of this work has been in software TM (STM) implementations [1, 5, 24, 63], but these suffer from even higher overheads.
than flat STMs. Vachharajani [62, Ch. 7] and FaNTM [6] introduce hardware support to reduce parallel nesting overheads. Even with hardware support, parallel-nesting HTMs yield limited gains—e.g., FaNTM is often slower than a flat HTM, and moderately outperforms it (by up to 40%) only on a microbenchmark.

Parallel-nesting TMs suffer from three main problems. First, nested transactions merge their speculative state with their parent’s, and only the coarse, top-level transaction can commit. This results in large atomic blocks that are as expensive to track and as prone to abort as large serial transactions. By contrast, FRACTAL performs fine-grain speculation, at the level of individual tasks. It never merges the speculative state of tasks, and relies on ordering tasks to guarantee the atomicity of nested domains.

Second, because the parent transaction waits for its nested transactions to finish, there is a cyclic dependence between the parent and its nested transactions. This introduces many subtle problems, including data races with the parent, deadlock, and livelock [6]. Workarounds for these issues are complex and sacrifice performance (e.g., a nested transaction eventually aborts all its ancestors for livelock avoidance [6]). By contrast, all dependences in FRACTAL are acyclic, from parents to children, which avoids these issues. FRACTAL supports the fork-join semantics of parallel-nesting TMs by having nested transactions enqueue their parent’s continuation.

Finally, parallel-nesting TMs do not support ordered speculative parallelism. By contrast, FRACTAL supports arbitrary nesting of ordered and unordered parallelism, which accelerates a broader range of applications.

7.2 Thread-level speculation
Thread-level speculation (TLS) schemes [28, 31, 50, 56, 58] ship tasks from function calls or loop iterations to different cores, run them speculatively, and commit them in program order. Prior TLS systems scale poorly beyond few cores, cannot support large speculation windows, and are less general than Swarm’s timestamp-ordered execution model [36].

A few TLS systems use timestamps internally, but do not let programs control them [32, 50, 57]. Renau et al. [50] use timestamps to allow out-of-order task spawn. Each task carries a timestamp range, and splits it in half when it spawns a successor. This approach could be adapted to support the order constraints required by nesting. However, while this technique works well at the scale it was evaluated (4 speculative tasks), it would require an impractical number of timestamp bits at the scale we consider (4096 speculative tasks). Moreover, this technique would cause over-serialization and does not support exposing timestamps to programs.

7.3 Nesting with non-speculative parallelism
Nesting is supported by most parallel programming languages, such as OpenMP [26]. In many languages, such as NESL [10], Cilk [27], and X10 [16], nesting is the natural way to express parallelism. Supporting nested parallelism in these non-speculative systems is easy because parallel tasks have no atomicity requirements: they either operate on disjoint data or use explicit synchronization, such as locks [17] or dataflow annotations [25], to avoid data races. Though nested non-speculative parallelism is often sufficient, many algorithms need speculation to be parallelized efficiently [48]. By making nested speculative parallelism practical, FRACTAL brings the benefits of composability and fine-grain parallelism to a broader set of programs.

8 CONCLUSION
We have presented FRACTAL, a new execution model for fine-grain nested speculative parallelism. FRACTAL lets programmers compose ordered and unordered algorithms without undue serialization. Our FRACTAL implementation builds on the Swarm architecture and relies on a dynamically chosen task order to perform fine-grain speculation, operating at the level of individual tasks. Our implementation sidesteps the scalability issues of parallel-nesting HTMs and requires simple hardware. We have shown that FRACTAL can parallelize a broader range of applications than prior work, and outperforms prior speculative architectures by up to 88×.

ACKNOWLEDGMENTS
We sincerely thank Nathan Beckmann, Nosayba El-Sayed, Harshad Kasture, Po-An Tsai, Guowei Zhang, Anurag Mukkara, Yee Ling Gan, and the anonymous reviewers for their helpful feedback. Niklas Baumstark and Julian Shun graciously shared their serial and parallel implementations of max-flow.

This work was supported in part by C-FAR, one of six SRC STARNet centers by MARCO and DARPA, and by NSF grants CAREER-1452994 and CCF-1318384. Mark C. Jeffrey was partially supported by an NSERC postgraduate scholarship; Hyun Ryong Lee was partially supported by a Kwanjeong Educational Foundation scholarship; and Victor A. Ying was supported by an MIT EECS Edwin S. Webster Graduate Fellowship.

REFERENCES