Image Fusion for MR Bias Correction

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Magnetic Resonance

- Nuclear magnetic resonance (NMR) effect: nuclei resonate due to applied magnetic field. They emit radio frequency (RF) pulses at the resonant frequency.
- Larmour equation: resonant frequency proportional to the applied magnetic field strength
- Spatially varying field strength encodes spatial location in the frequency domain

MR Imaging

- The image generation is controlled by three intrinsic tissue properties and two user definable parameters. For example, for fast-spin echo (FSE) pulse sequences, the MR signal is given by this equation:

$$I = \rho \exp(-\mathrm{TE}/\mathrm{T2})(1 - \exp(-\mathrm{TR}/\mathrm{T1}))$$

- Target T1 and T2 through appropriate selection of TE and TR

TE: time echo (time we measure signal)

TR: time repeat (time between pulse sequences)

T1: spin-lattice relaxation (recovery of z-magnetization)

T2: spin-spin relaxation (loss of xy-magnetization)

ρ: proton density

Image Reconstruction

- Received MR signal is converted from the frequency domain using inverse Fourier transform
- Due to imperfections in acquisition process, the result of the IFFT will be complex. Generally, the final reconstructed image is the absolute value of the complex image (to eliminate phase effects)
- Noise in acquisition is Gaussian, so reconstruction results in Rician noise. Rician noise is similar to Rayleigh at low SNR, Gaussian at high SNR

Bias Field

- Typically transmit RF pulses with body coil due to good spatial homogeneity. Receive with surface coil due to high local SNR (bird cage coils also sometimes used)
- The signal observed at the receiver is then:

$$I(x) = b(x)f(x) + n(x)$$

b(x) is the magnetic field induced by the receiving coil.

f(x) is the ideal MR signal

- The severity of the bias field is determined by the spatial homogeneity of the surface coil reception profile

Previous Work

- Earliest work in mid-80’s: Axel et al (1984) with homomorphic unsharp filtering, Axel et al (1987) with phantom correction
- More modern approaches include Dawant et al (1993) with spline fitting, Haselgrove-Prammer (1986) with embedded coil markers
- Various simultaneous bias correction and segmentation approaches such as Meyer et al (1995) and Wells et al (1996) using Expectation-Maximization

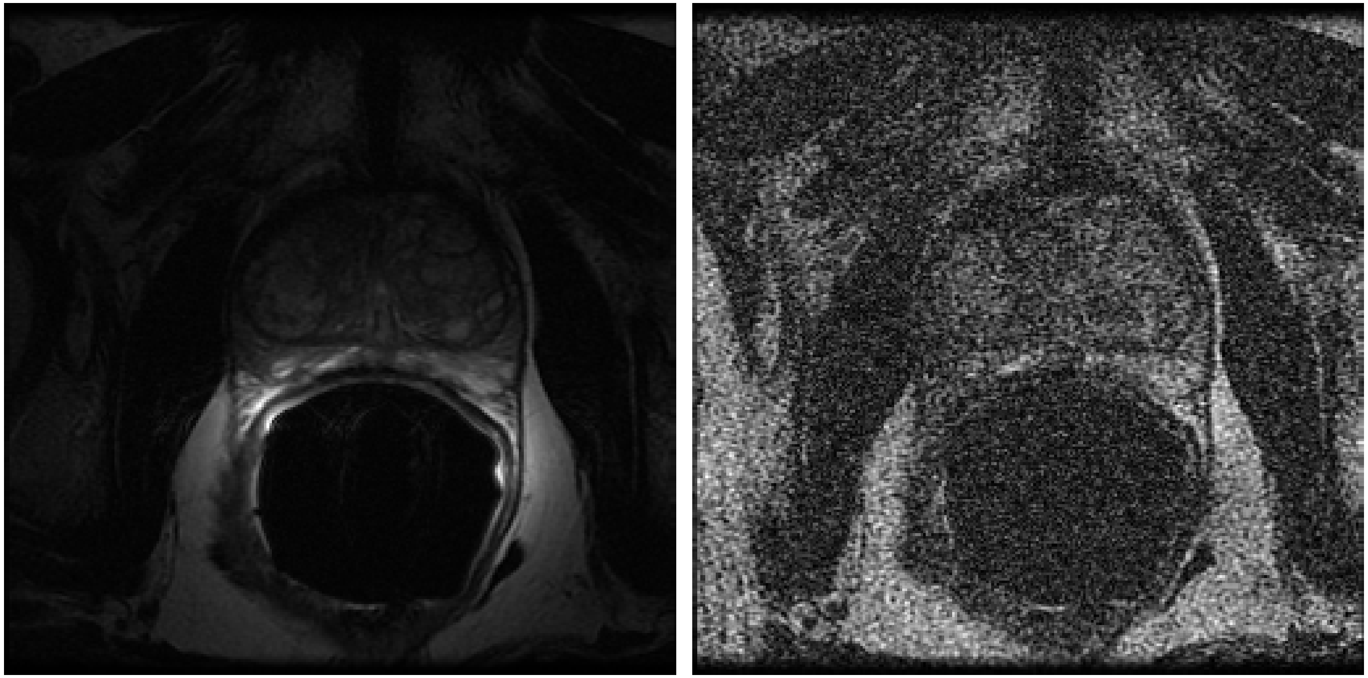
Measurement Model

- Brey and Narayana (1988) proposed capturing images from both the body coil and the surface coil. One image is noisy, other has large bias field.
- The measurement model is then:

$$\begin{aligned} I_B(x) &= kf(x) + n_B(x) \\ I_S(x) &= b(x)f(x) + n_S(x) \end{aligned}$$

- I_B is homogeneous but noisy. I_S has high SNR in the region of interest, but severe bias artifact

Example Data



Surface coil (left) and body coil (right) prostate images

Previous Work (2)

- Brey-Narayana filter the two observation images to denoise them and divide the results to estimate the bias field:

$$\begin{aligned} \hat{b} &= (h_1 * I_S)/(h_2 * I_B) \\ \hat{f} &= I_S/\hat{b} \end{aligned}$$

- Other body coil/surface coil approaches include Lai-Fang (1998) who fit splines using both images, Pruessmann et al (2001) who fit local polynomials

ML Formulation

- We model the noise as Gaussian and IID. This is approximately true in high SNR regions. Generally low SNR regions correspond to air regions which we do not care about
- Stack the 2D or 3D images into vectors. We can then write the log likelihood as (ignoring constant terms):

$$l(I; b, f) = -\frac{1}{2\sigma_B^2}||I_B - kf||^2 - \frac{1}{2\sigma_S^2}||I_S - b \circ f||^2$$

Regularization

- Maximizing that function results in useless estimates:

$$\hat{b} = I_S / I_B$$
$$\hat{f} = I_B$$

- We construct augmented energy function that encourages smoothness in b and piecewise smoothness in f:

$$E(b, f) = ||I_B - kf||^2 + \lambda ||I_S - b \circ f||^2 + \alpha ||Db||^2 + \gamma ||Lf||_p^p$$

- Generally choose p < 2 to help preserve edges

Optimizing Energy Function

- D and L in previous equation are high-pass linear operators. Thus we penalize high frequency components
- Difficult to minimize with respect to b and f simultaneously. But given b, f is relatively easy to obtain, and vice versa. So use coordinate descent to alternately optimize b and f
- Also use multigrid to increase convergence speed. This entails finding low frequency components on a coarser grid and propagating the results to the finest grid

Solving for b

- With f fixed, the energy is quadratic in terms of b. The quadratic matrix is positive definite which means that the energy function is convex
- Setting the gradient to zero leads to a linear equation for the solution (F is a diagonal matrix formed from f):

$$\left(F^2/\sigma_S^2 + \alpha D^T D\right) b = f \circ I_S$$

- We can solve by inverting or using a suboptimal iterative scheme. We use conjugate gradient with a tridiagonal preconditioner

Solution for f (γ=0)

- With no regularization on f, we can minimize the function pointwise:

$$\hat{f}_n = \frac{kI_B(n) + \lambda b_n I_S(n)}{b_n^2 + k^2}$$

- This equation can be interpreted as a noise-weighted convex combination between I_B/k and I_S/b. Essentially the same fusion equation found by Roemer et al (1990)

Half-Quadratic Optimization

- To solve l_p regularization problems, use half-quadratic optimization (Geman-Reynolds 92). Begin with an initial guess. Form a quadratic approximation at that point:

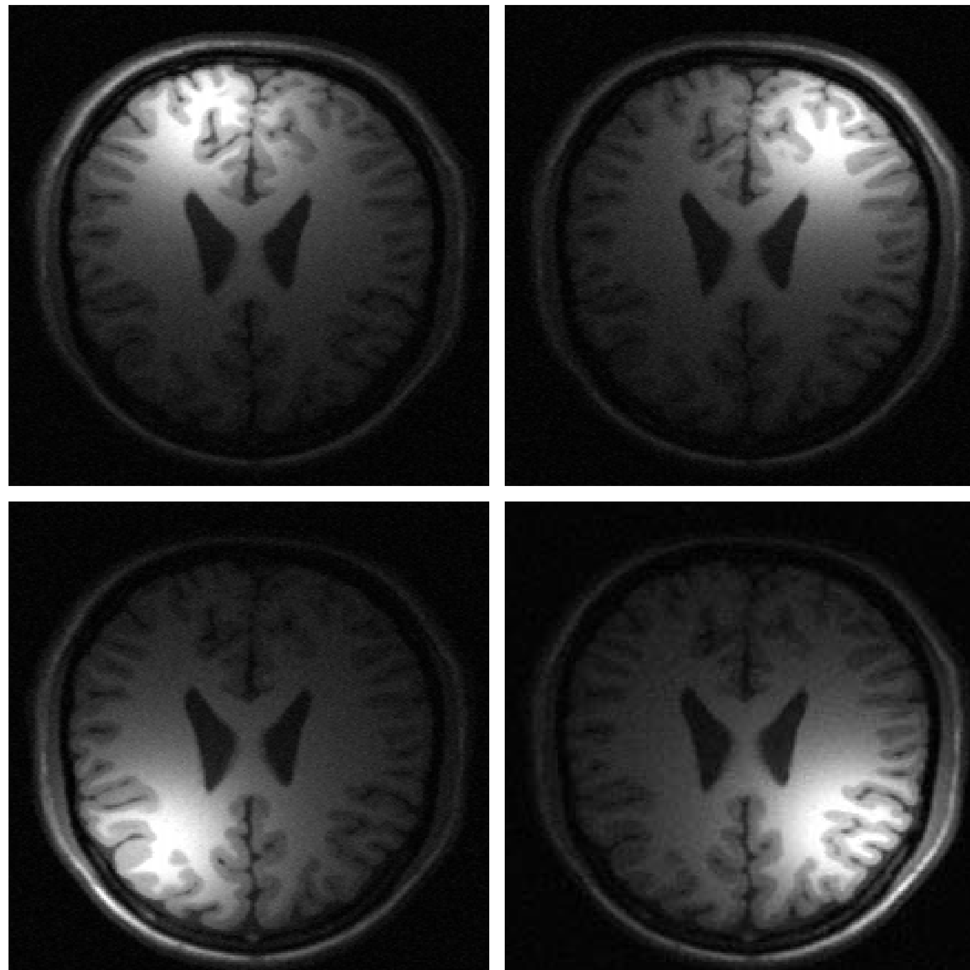
$$\left(\lambda_B I + \lambda_S B^2 + \gamma D^T W^{(i)} D\right) \widehat{f}^{(i)} = \lambda_B y_B + \lambda_S B y_S$$
$$W^{(i)}[n,n] = \tfrac{p}{2}\left(((D\widehat{f}^{(i-1)})[n])^2 + \xi\right)^{p/2-1}$$

- Solve this problem. Continue iterating in this way until convergence

Multiple Coils, Pulse Sequences

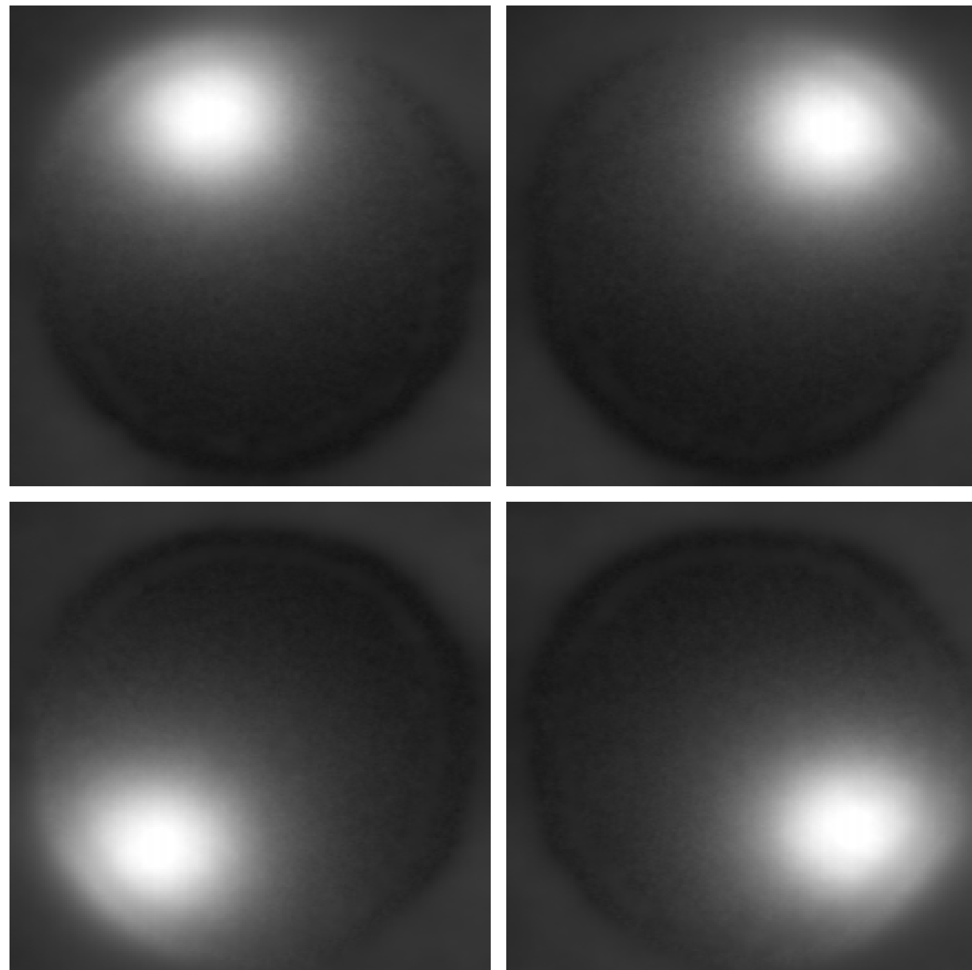
- Common to use multiple surface coils to simultaneously receive the MR signal. This allows for better spatial coverage. Generally combine into a composite image using sum-of-squares.
- We can generalize our energy functional to find the bias field estimate for each coil and one composite true image estimate.
- When acquiring multiple pulse sequences, only need one body coil image. Bias field largely unchanged (though some minor effects may crop up due to, e.g., magnetic susceptibility of the tissue).

MNI Example



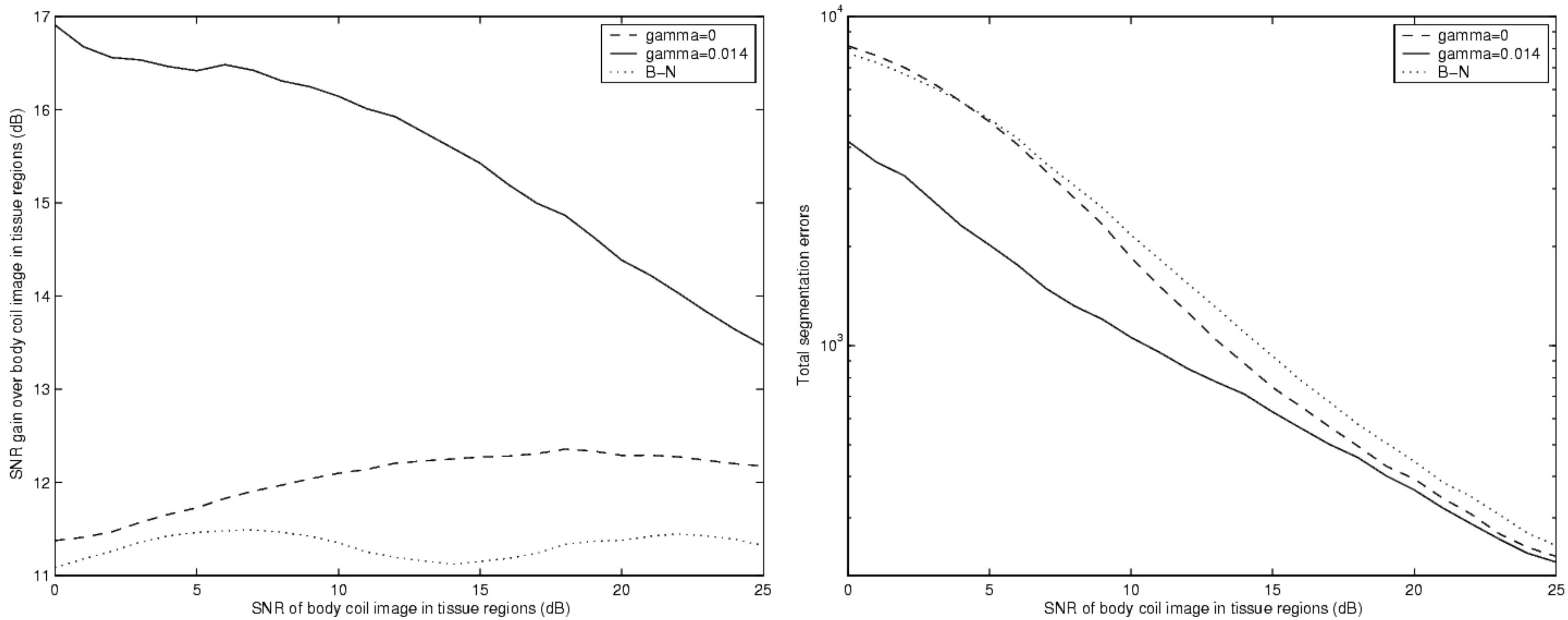
Body coil image (left) and surface coil images (right)

MNI Results



Estimated image (left) and bias fields (right)

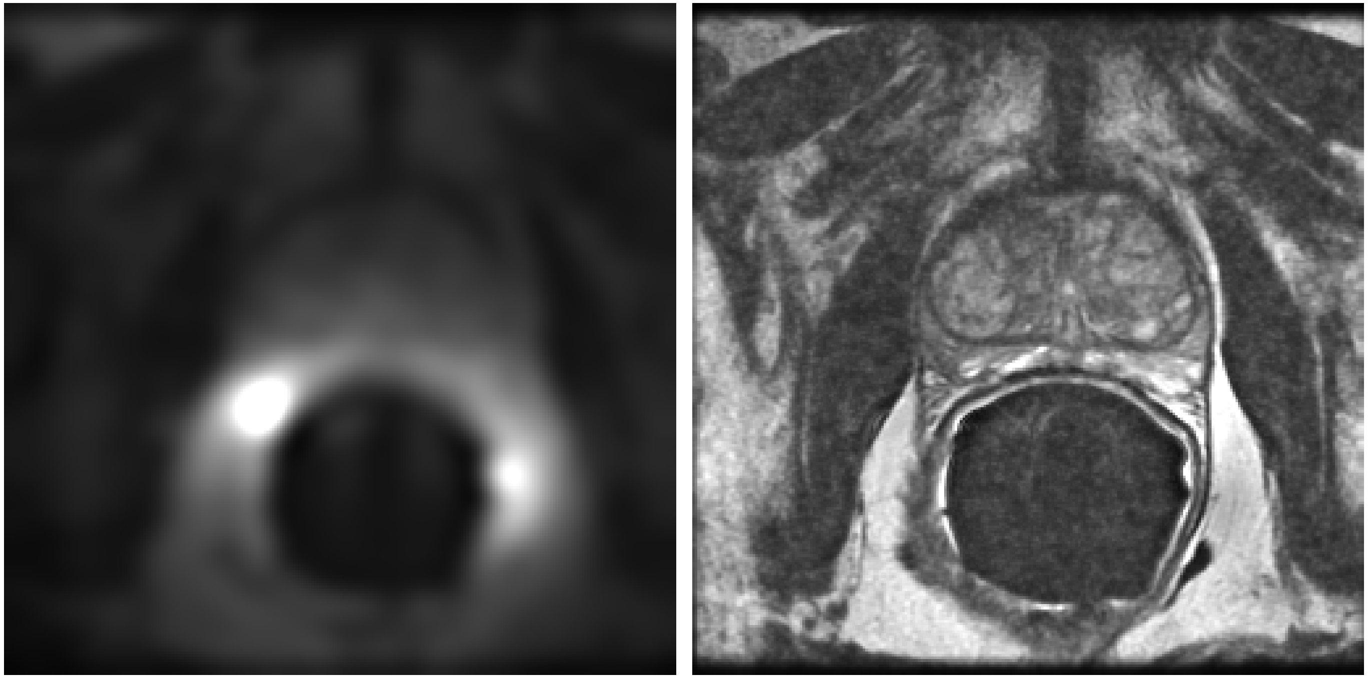
MNI Scaling



(a) (b)

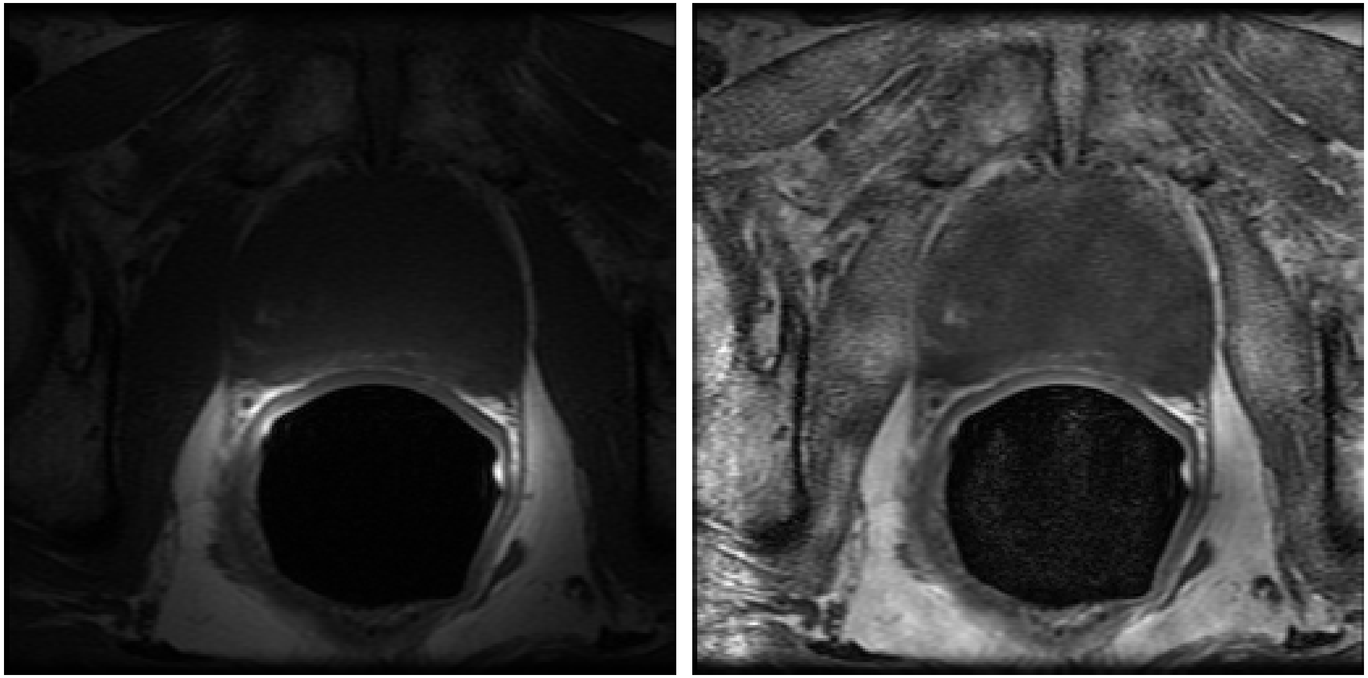
Performance of (a) SNR gain and (b) segmentation errors as a function of image acquisition SNR

Prostate Results



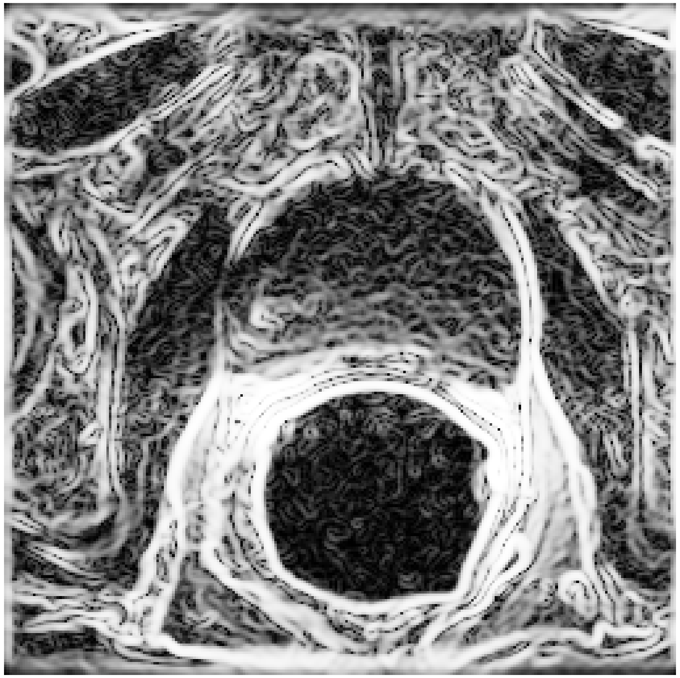
Estimated bias field (left) and true MR image (right), T2-weighted image

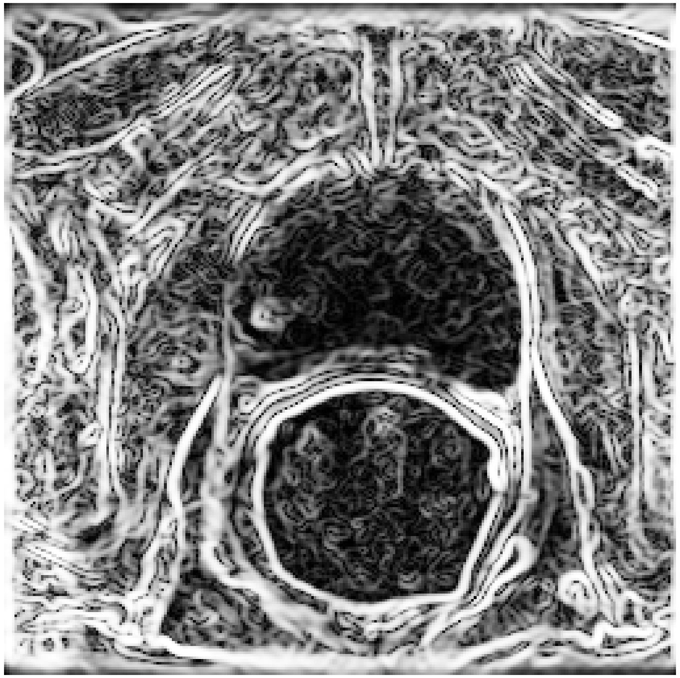
Applying correction to T1



T1 surface coil image (left), result from applying bias field estimate from T2 image (right)

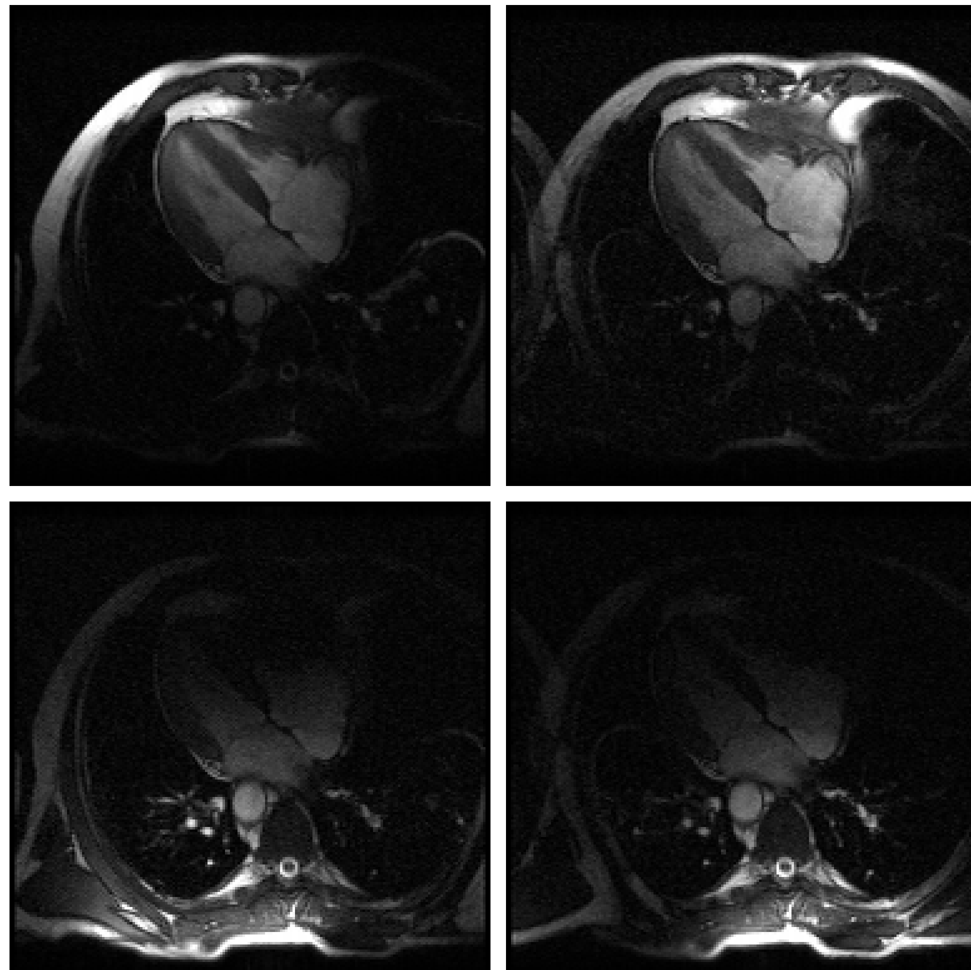
Image Gradients





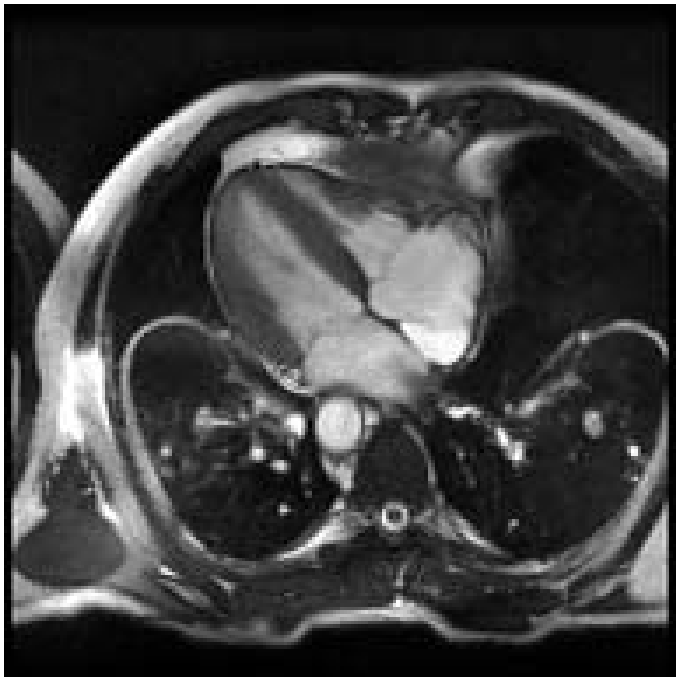
Original surface coil T1-weighted image (left), corrected image (right). Smaller gradients signify greater homogeneity

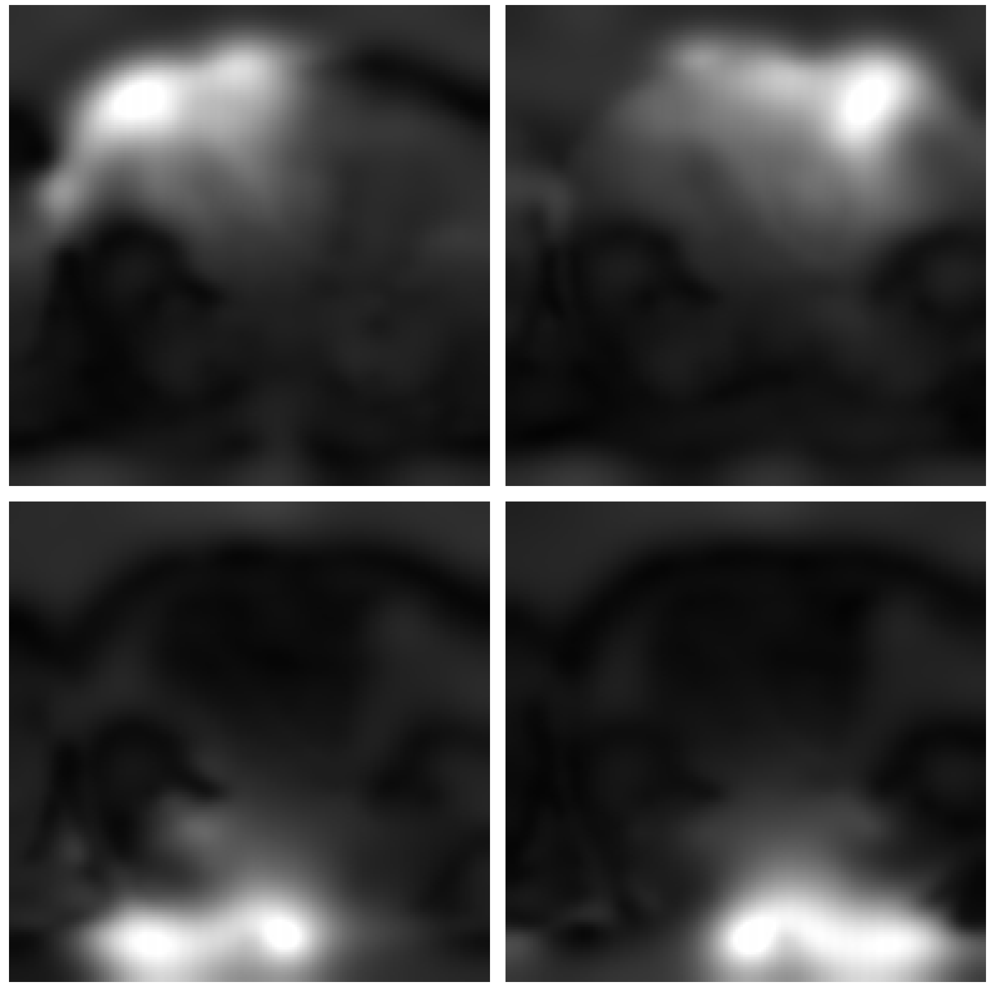
Heart Example



Body coil image (left) and
surface coil images (right)

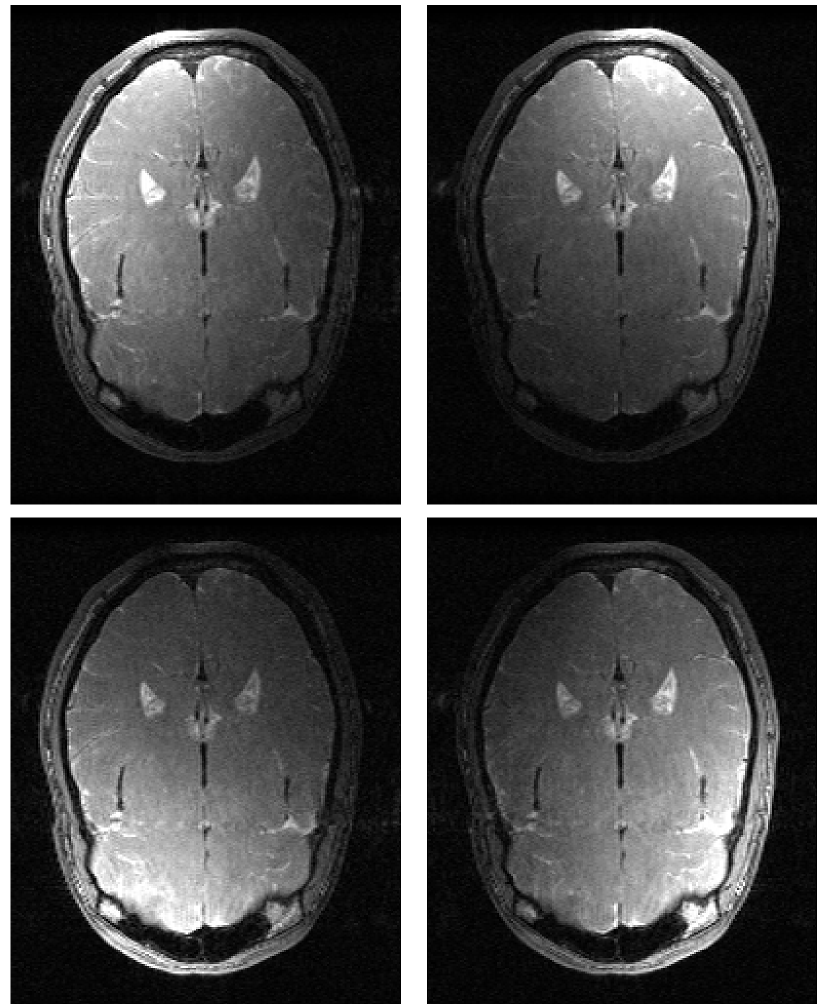
Heart Results





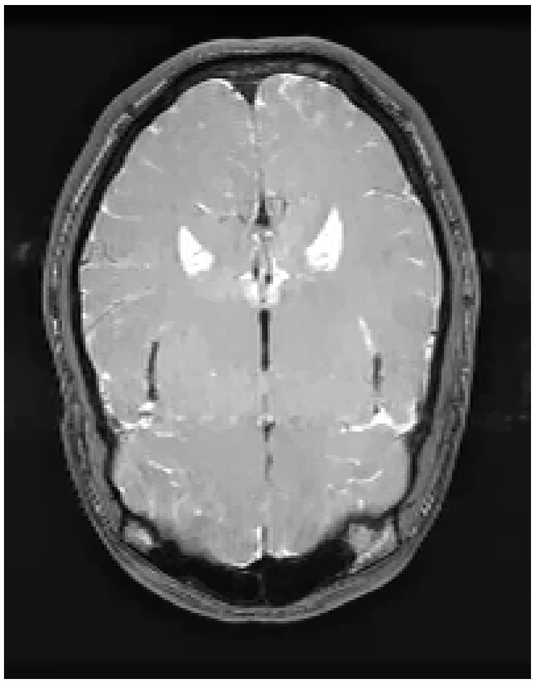
Estimated image (left) and bias fields (right)

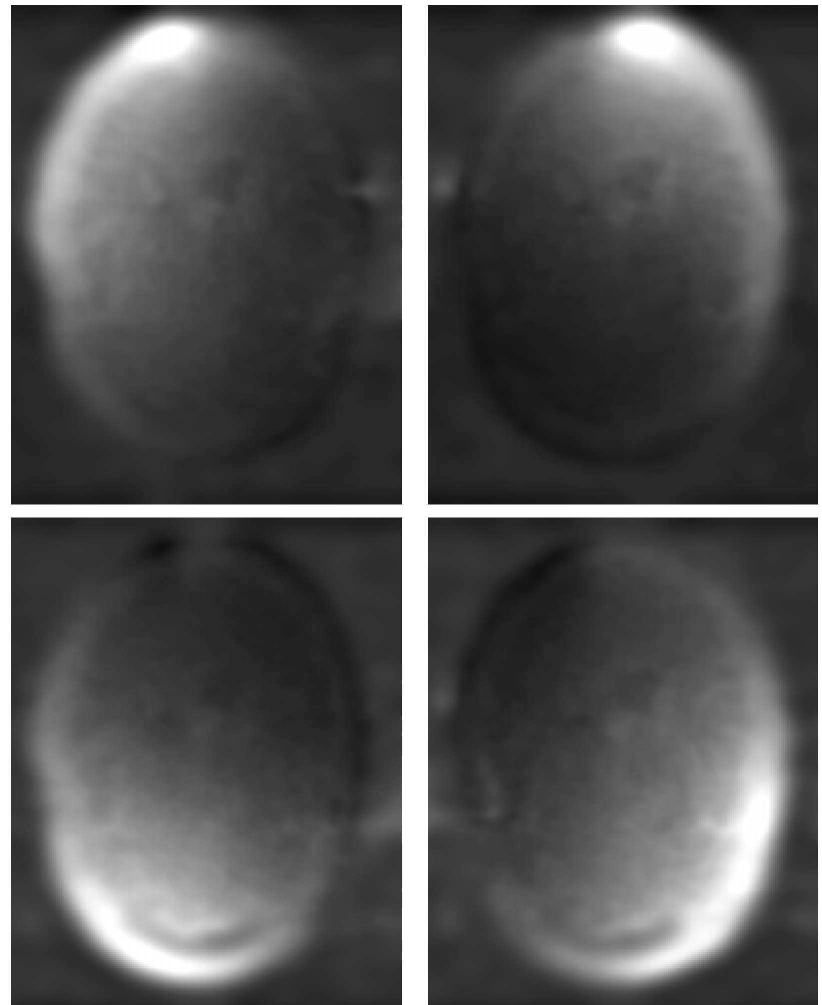
Brain Example



Body coil image (left) and surface coil images (right), gradient recalled echo (GRE)

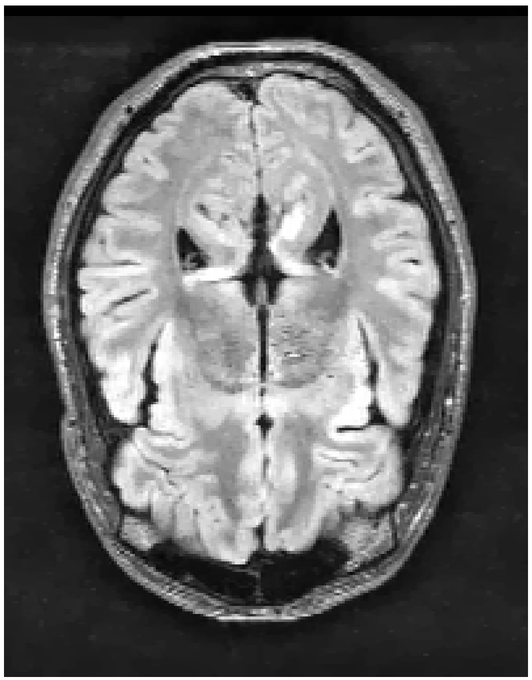
Brain Results

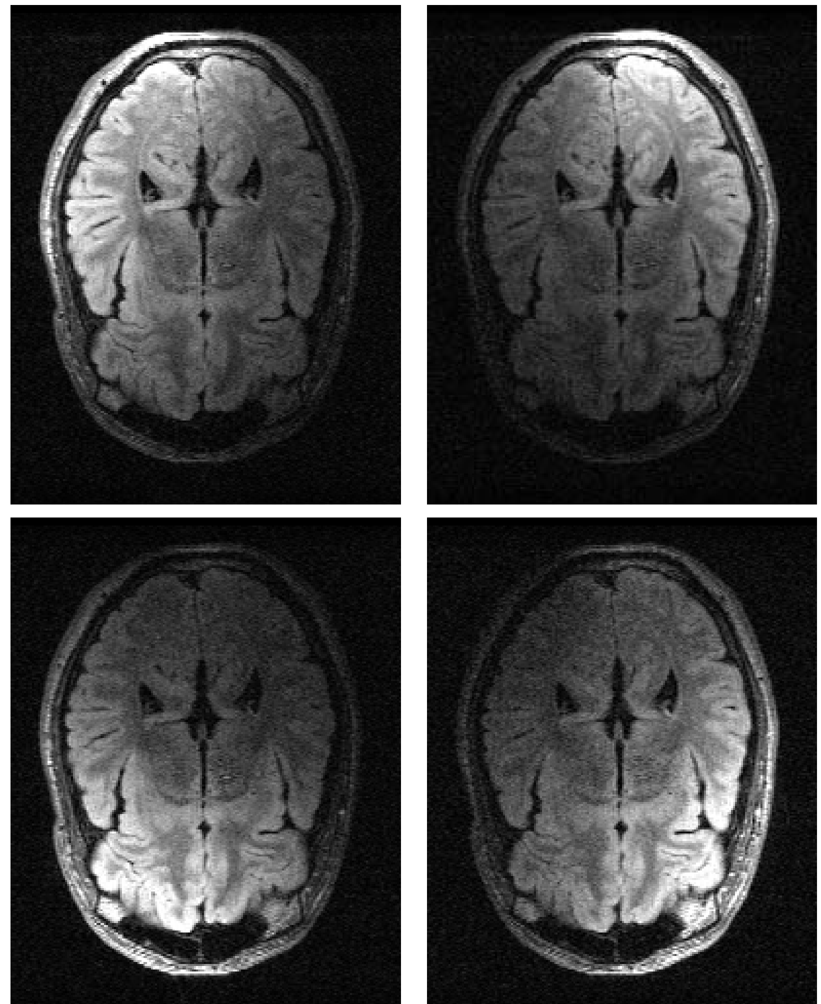




Estimated image (left) and bias fields (right)

Brain FLAIR Results





Estimated true image (left),
surface coil images (right),
FLAIR pulse sequence

Contributions

- Non-parametric variational formulation of image fusion problem with statistical estimation flavor
- Demonstrably superior results on synthetic examples
- Simultaneous bias correction and denoising
- Seamless handling of multiple surface coils and multiple pulse sequences
- Efficient solver using coordinate descent, preconditioned CG, multigrid