A Variational Approach to MR Bias Correction

Ayres Fan, W. Wells, J. Fisher,
M. Cetin, S. Haker, A. Willsky

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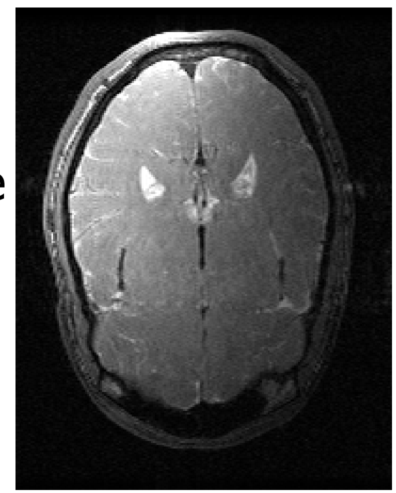
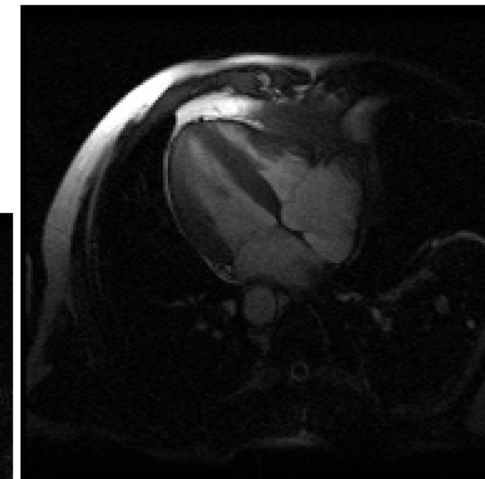
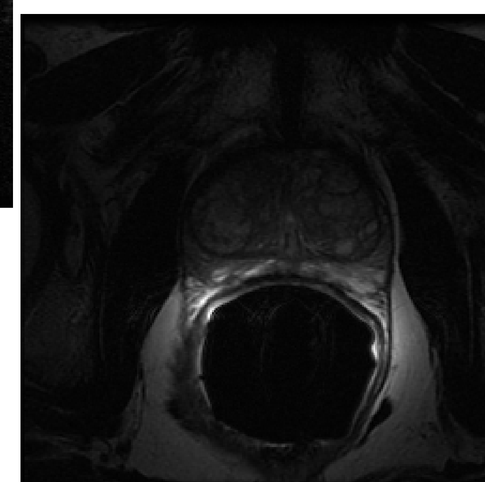
Problem Statement

- The bias field is a systematic intensity inhomogeneity that corrupts magnetic resonance (MR) images.
- Correcting for the bias field makes both human analysis (e.g., tumor detection, cartilage damage assessment) and computer analysis easier (e.g., segmentation, registration).
- General assumptions:
  - The bias field is slowly varying in space.
  - The bias field is tissue independent.
  - Tissue intensities are piecewise constant.







Measurement Model

- Brey and Narayana (1988) proposed capturing images from both the body coil and the surface coil. The measurement model is then:

$$\begin{aligned} I_B(x) &= \varphi(x) + n_B(x) \\ I_S(x) &= \beta(x)\varphi(x) + n_S(x) \end{aligned}$$

- I_B is homogeneous but noisy. I_S has high SNR in the region of interest, but a potentially severe bias artifact.
- Note that gain in SNR from using a surface coil does not come from reduction of noise, but from increased signal gain from the bias field.

Energy Functional

- We construct an augmented energy function that encourages smoothness in **b** and piecewise smoothness in **f**:

$$E(\boldsymbol{b}, \boldsymbol{f}) = \lambda_B ||\boldsymbol{y}_{\mathsf{B}} - \boldsymbol{f}||^2 + \lambda_S ||\boldsymbol{y}_{\mathsf{S}} - \boldsymbol{b} \circ \boldsymbol{f}||^2 + \alpha ||\boldsymbol{D}\boldsymbol{b}||^2 + \gamma ||\boldsymbol{L}\boldsymbol{f}||_p^p$$

  - We generally choose p <= 1 to help preserve edges
  - **D** and **L** are matrices chosen to implement differential operators
  - λ_B, λ_S, α, and γ are all positive constants
  - The λ's can be seen to be related to the inverse noise variances of the observed images.

Optimizing Energy Function

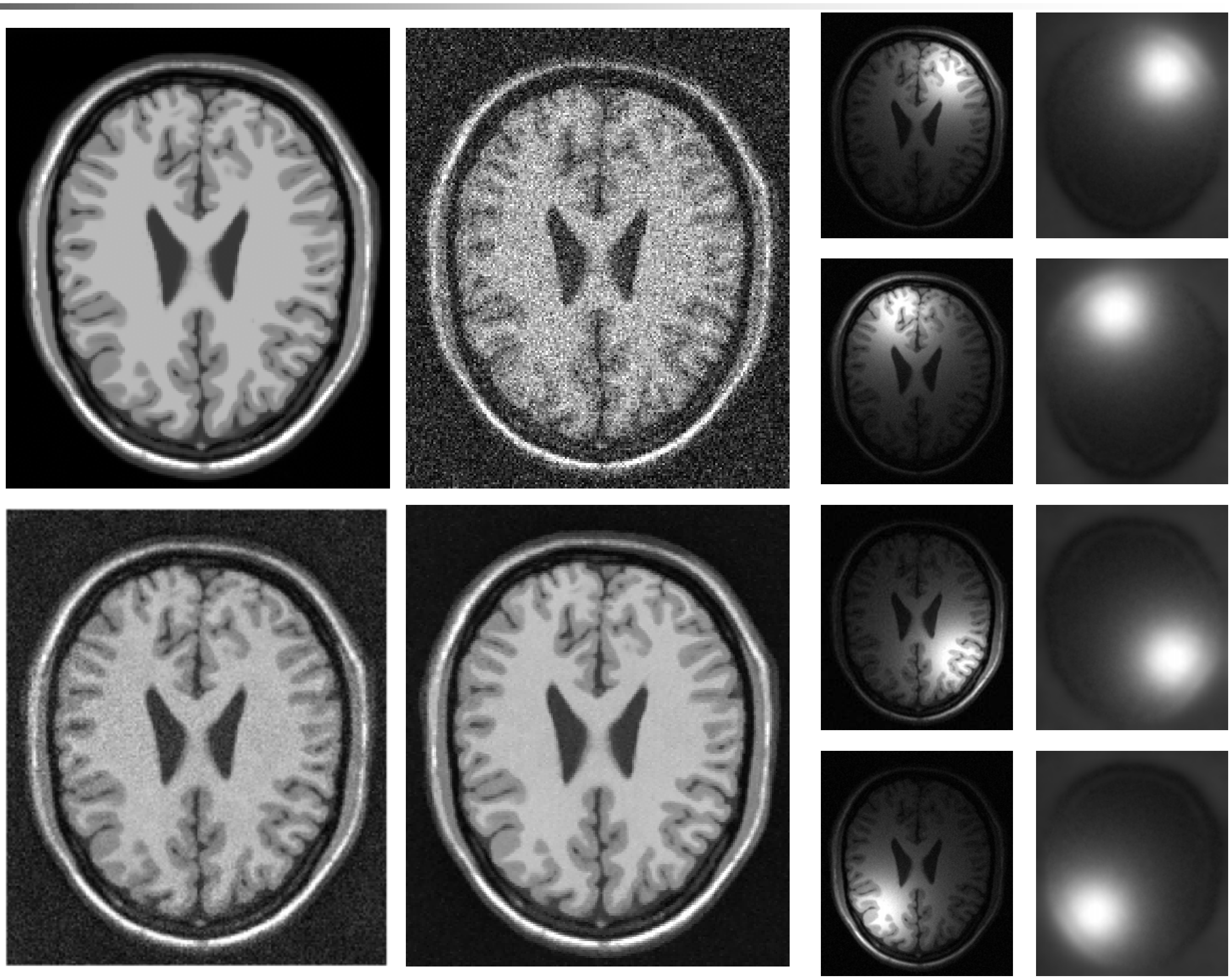
- Overall problem is non-convex.
- Use coordinate descent to alternately optimize **b** and **f**
  - Minimizing the energy simultaneously with respect to **b** and **f** is difficult. But given b, f is relatively easy to obtain, and vice versa.
  - A stationary point found using coordinate descent is also a stationary point of the overall energy functional.
- b-step
  - Minimize $\lambda_S \|\boldsymbol{y}_\mathrm{S} - \boldsymbol{F}\boldsymbol{b}\|^2 + \alpha \|\boldsymbol{D}\boldsymbol{b}\|^2$

    F is a diagonal matrix with **f** along the diagonal
- f-step
  - Minimize $\lambda_B \|\boldsymbol{y}_\mathrm{B} - \boldsymbol{f}\|^2 + \lambda_S \|\boldsymbol{y}_\mathrm{S} - \boldsymbol{B}\boldsymbol{f}\|^2 + \gamma \|\boldsymbol{L}\boldsymbol{f}\|_p^p$

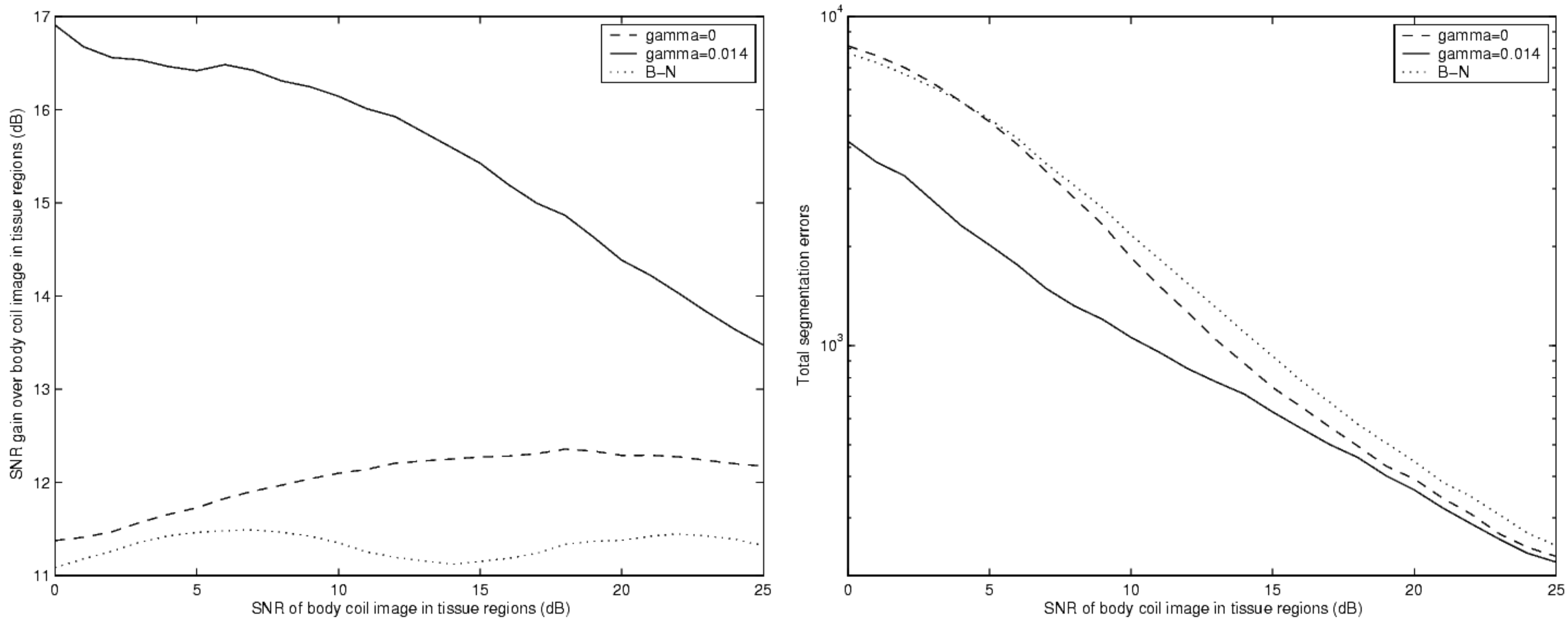
    B is a diagonal matrix with **b** along the diagonal

MNI Example

(left to right) True image (top), B-N estimate (bottom), body coil image (top), our f estimate (bottom), surface coil images, and bias field estimates.



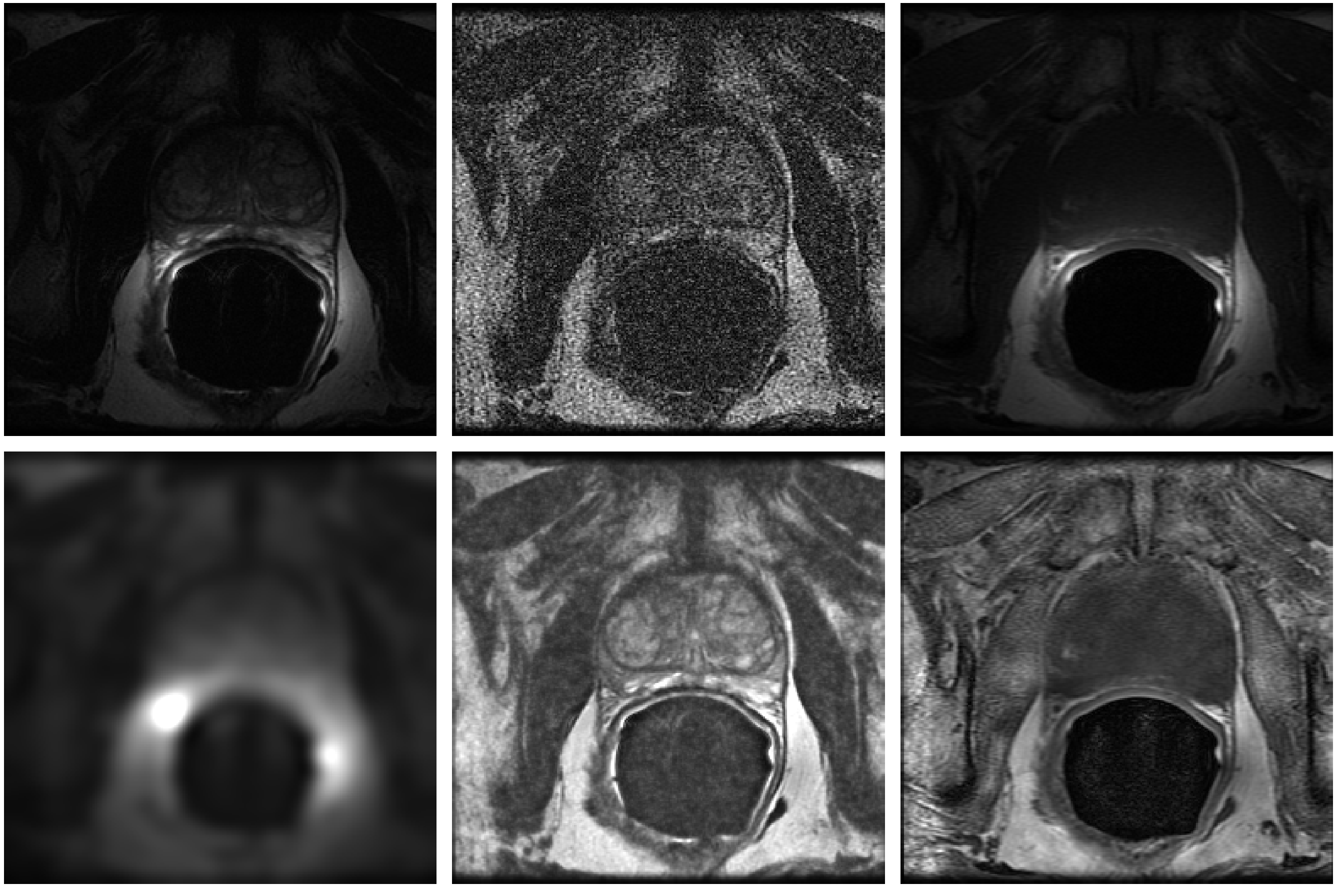
MNI Scaling



(a) (b)

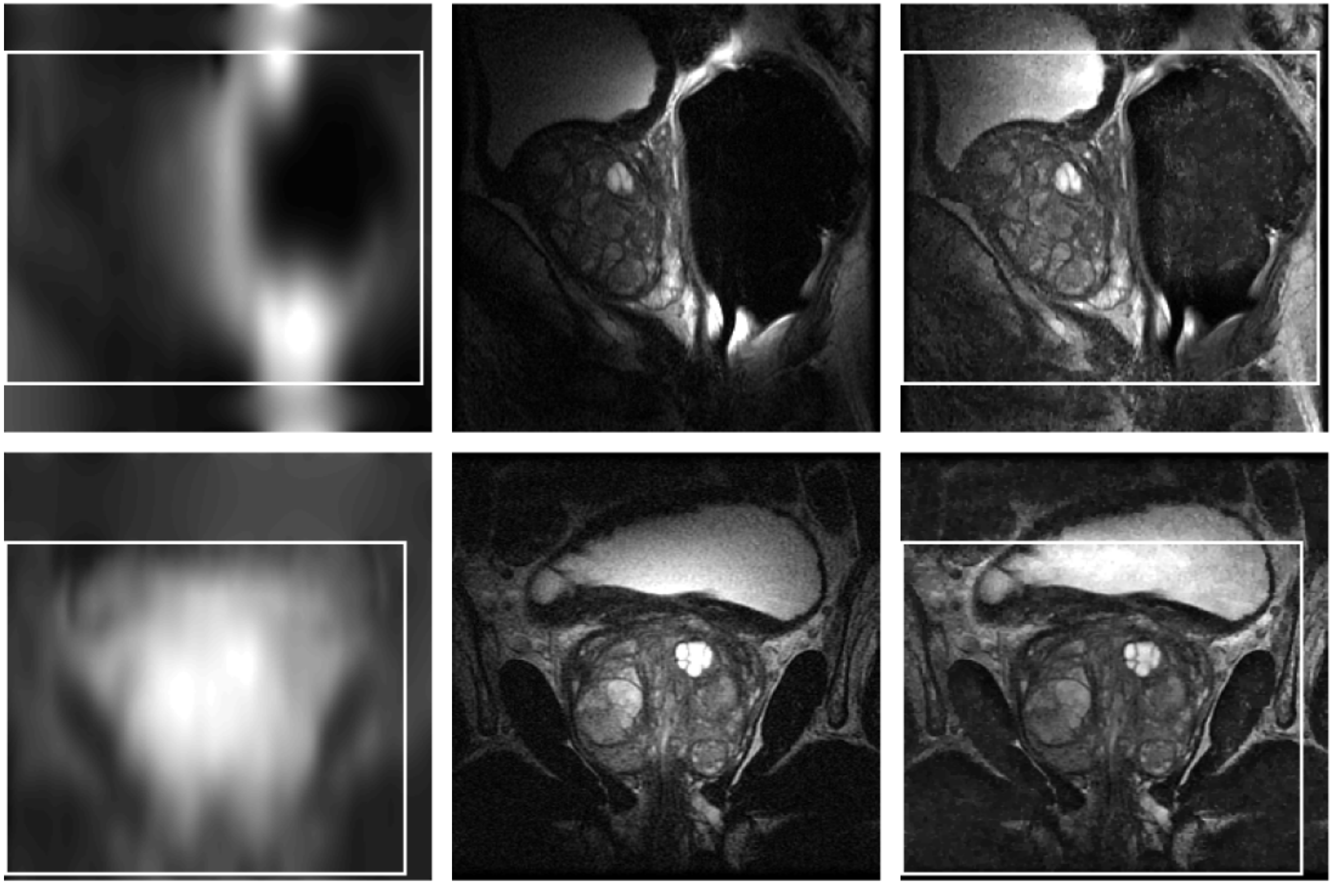
Performance of (a) SNR gain and (b) segmentation errors as a function of image acquisition SNR

Prostate Results



Top: T2W surface coil image, T2W body coil image, T1W surface coil image.
Bottom: Estimated bias field, true T2W image estimate, true T1W image estimate.

Coronal and Sagittal Correction



Sagittal (top), coronal (bottom). Bias field (left), surface coil image (middle), true image (right).