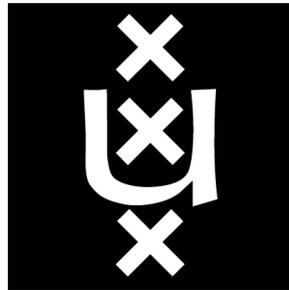


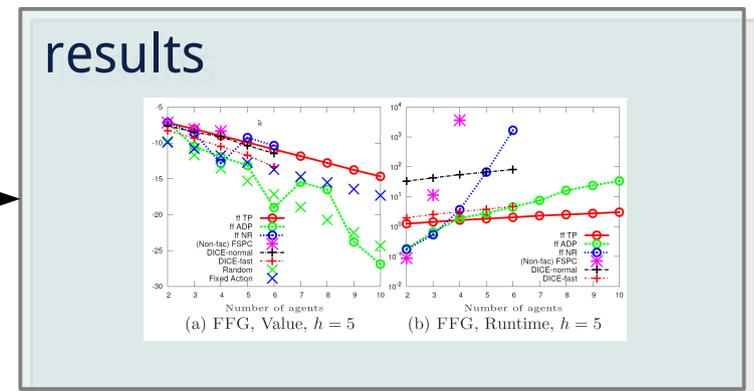
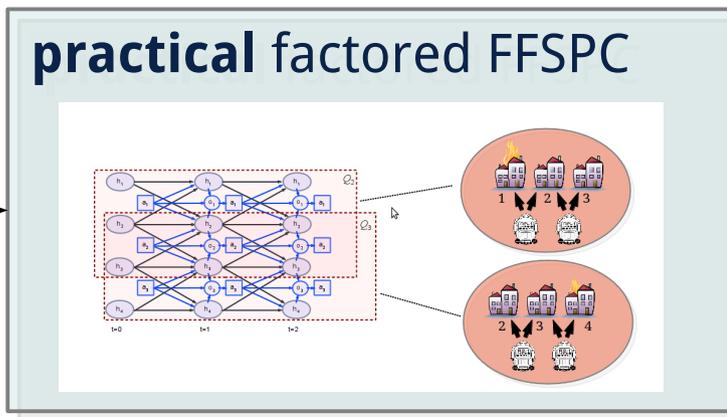
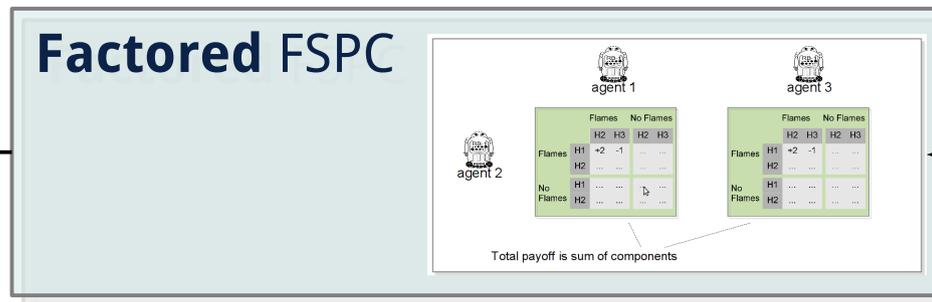
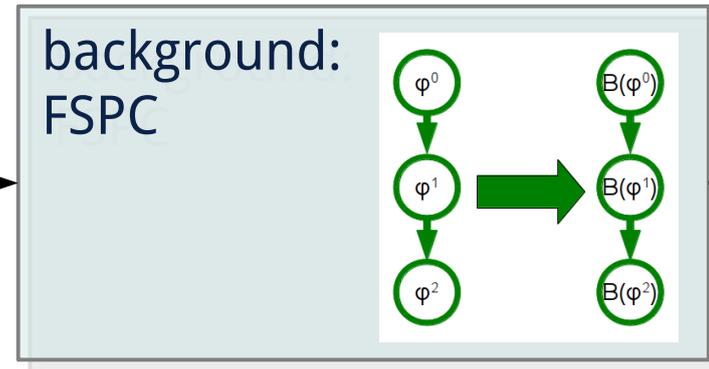
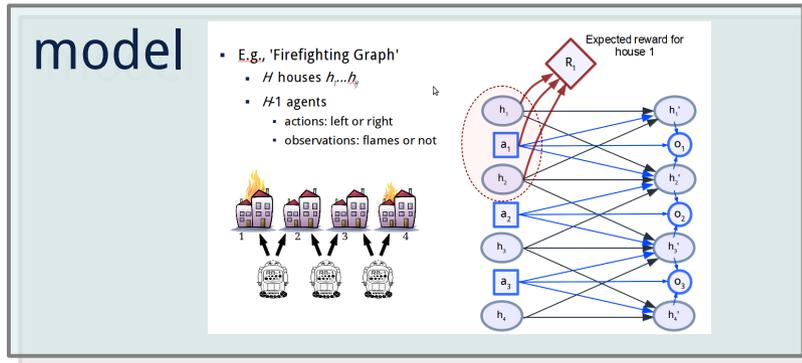
# Approximate Solutions for Factored Dec-POMDPs with Many Agents

**Frans A. Oliehoek**, Shimon Whiteson, & Matthijs T.J. Spaan



AAMAS, Wednesday May 8, 2013

# Visual Roadmap

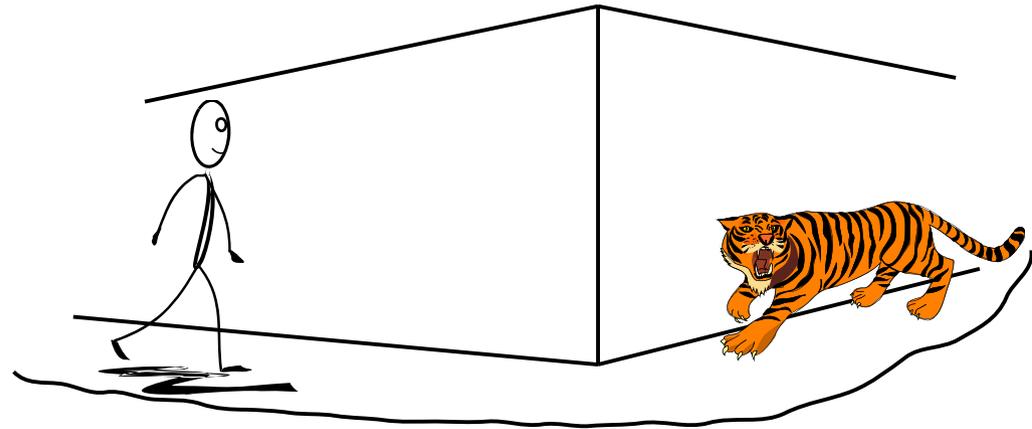


# Multiagent decision making under Uncertainty

- Outcome Uncertainty



- Partial Observability

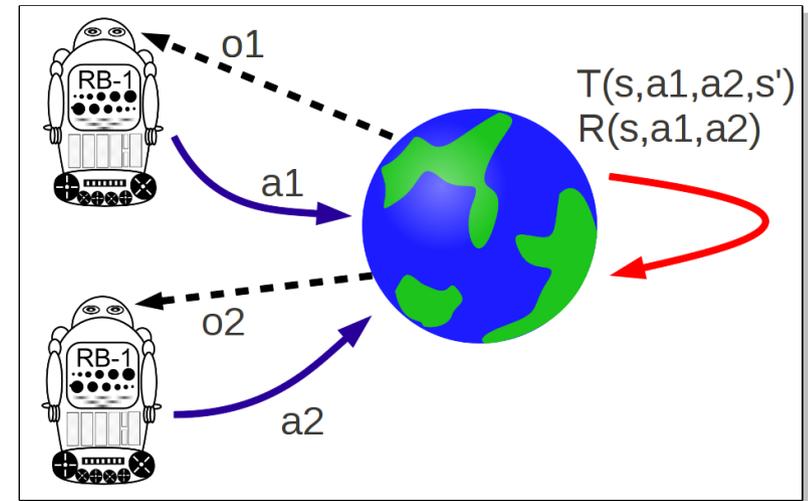


- Multiagent Systems: uncertainty about others

# Formal Model

- A Dec-POMDP

- $\langle S, A, P_T, O, P_O, R, h \rangle$
- $n$  agents
- $S$  – set of states
- $A$  – set of **joint** actions
- $P_T$  – transition function
- $O$  – set of **joint** observations
- $P_O$  – observation function
- $R$  – reward function
- $h$  – horizon (finite)



$$a = \langle a_1, a_2, \dots, a_n \rangle$$

$$P(s' | s, a)$$

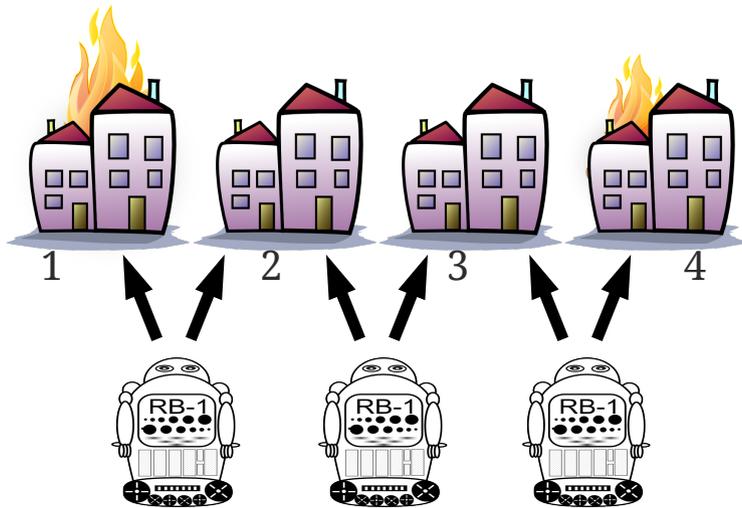
$$o = \langle o_1, o_2, \dots, o_n \rangle$$

$$P(o | a, s')$$

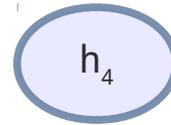
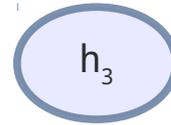
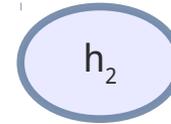
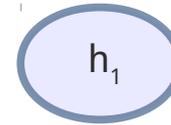
$$R(s, a) = E_{s'} R(s, a, s')$$

# Factored Dec-POMDPs

- E.g., 'Firefighting Graph'
  - $H$  houses  $h_1 \dots h_H$
  - $H-1$  agents
    - actions: left or right
    - observations: flames or not



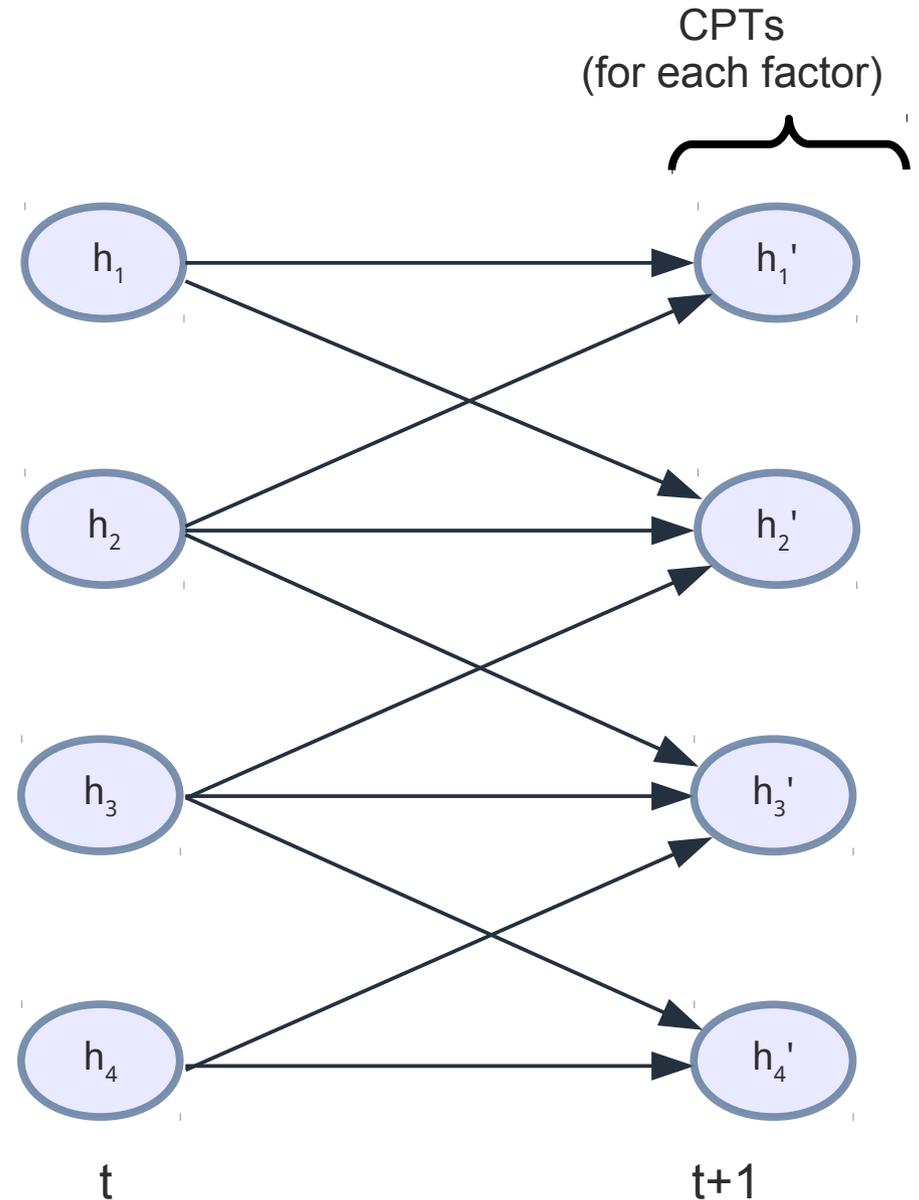
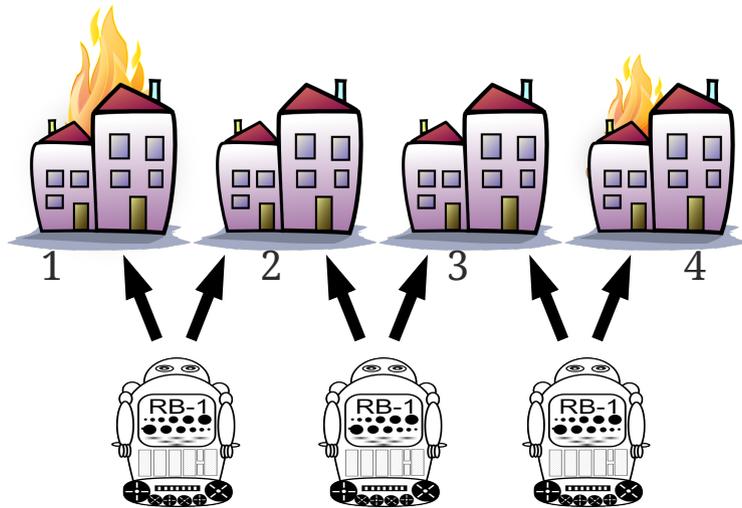
state variables  
or 'factors'



$t$

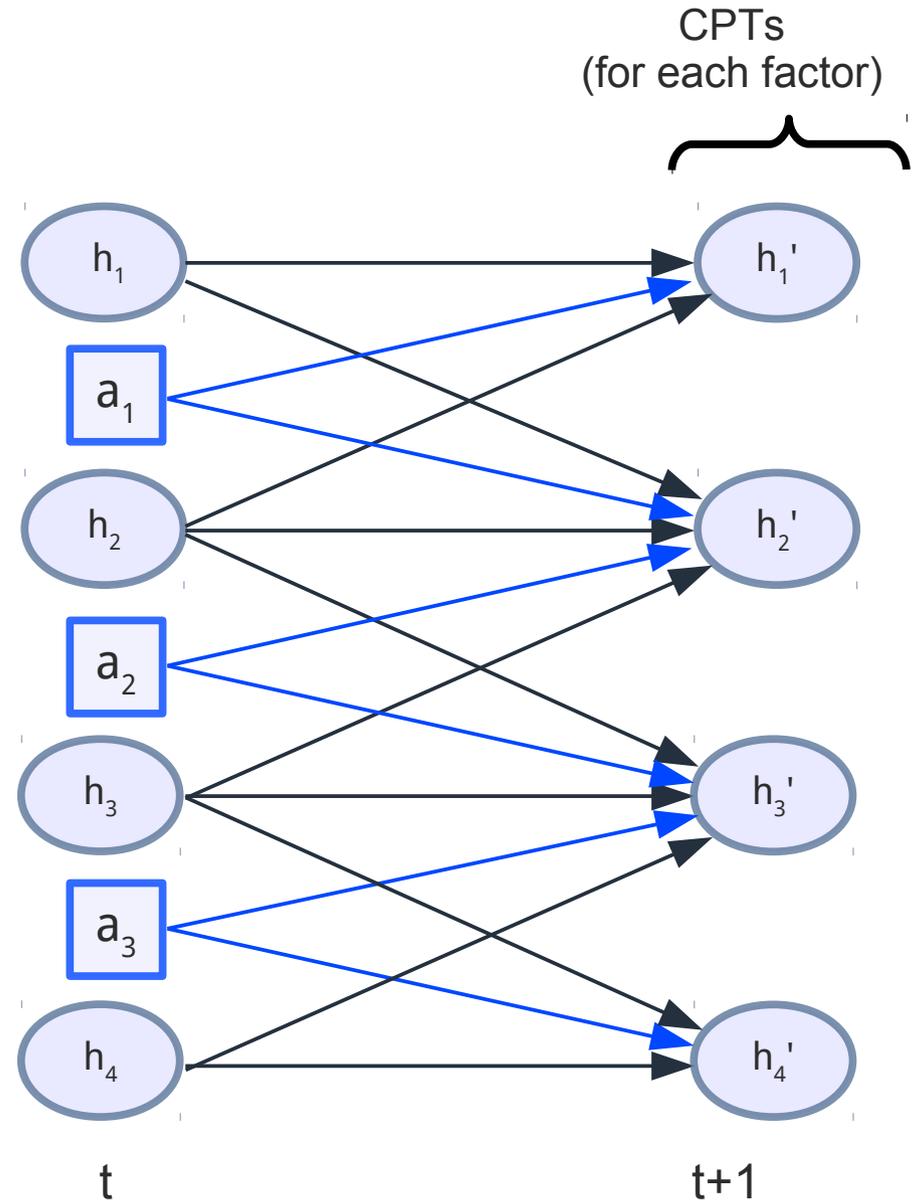
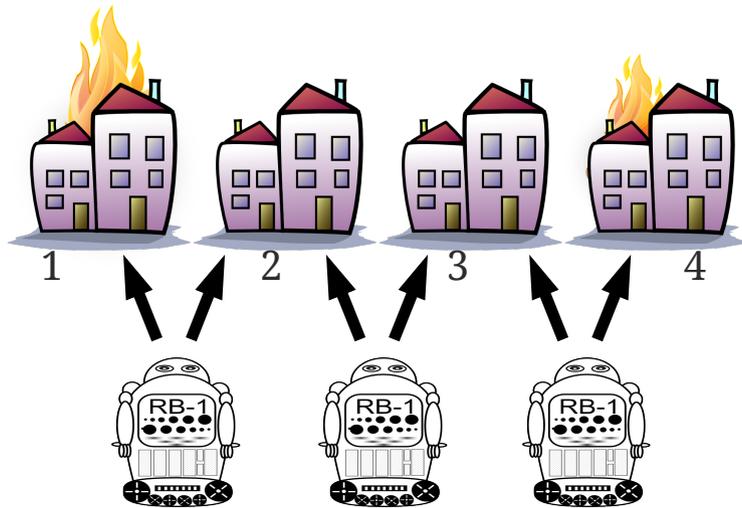
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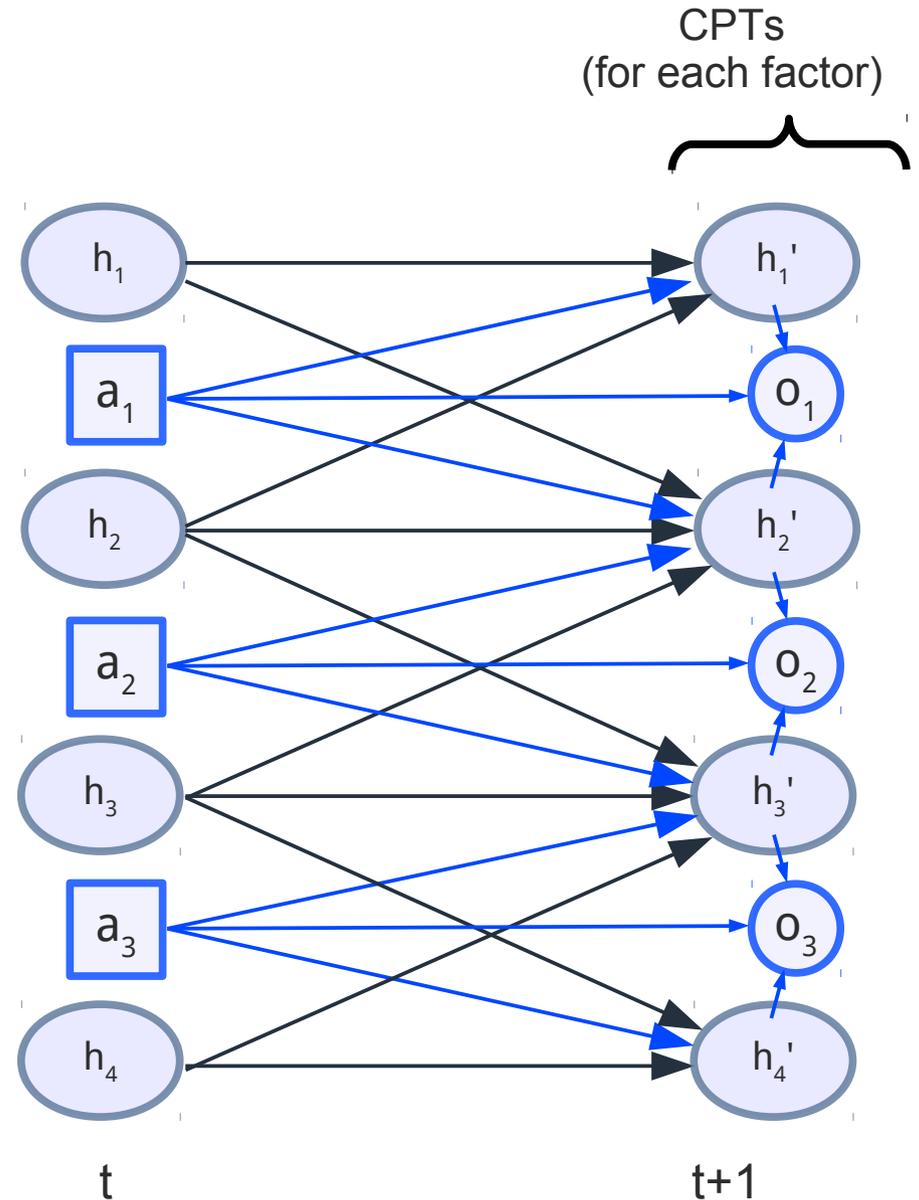
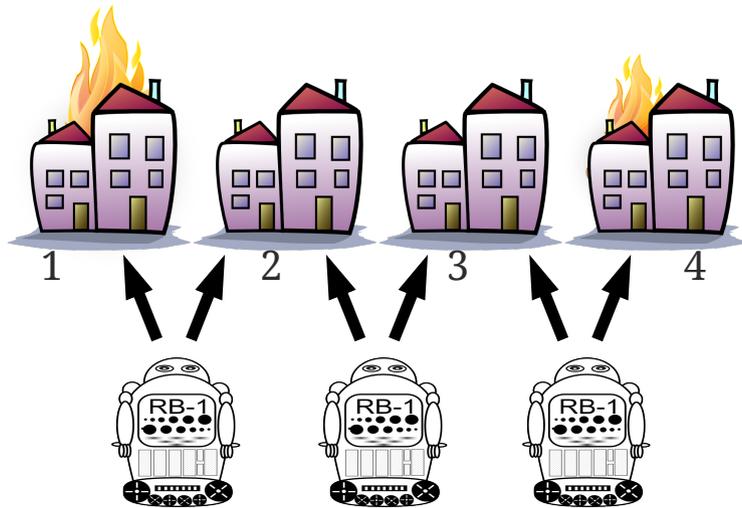
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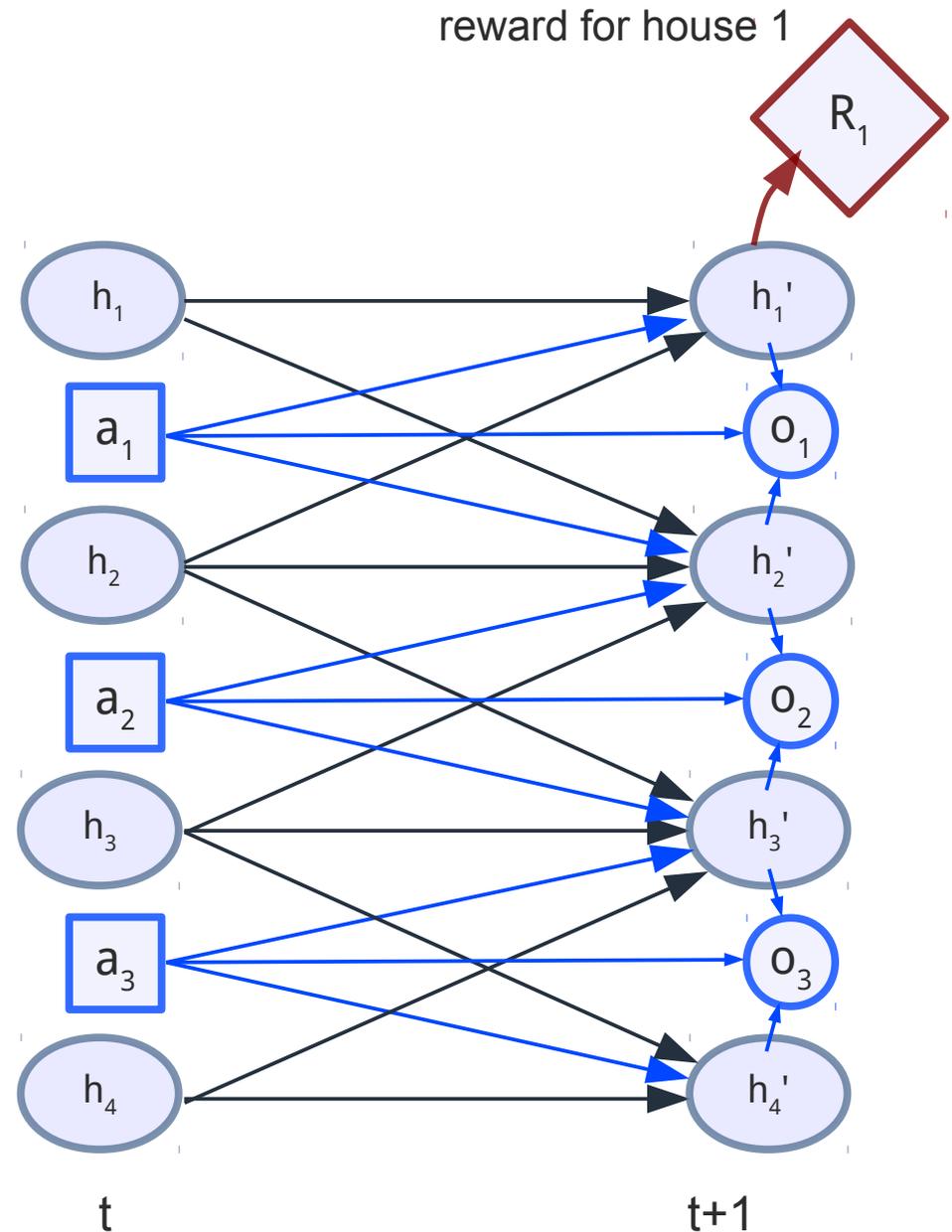
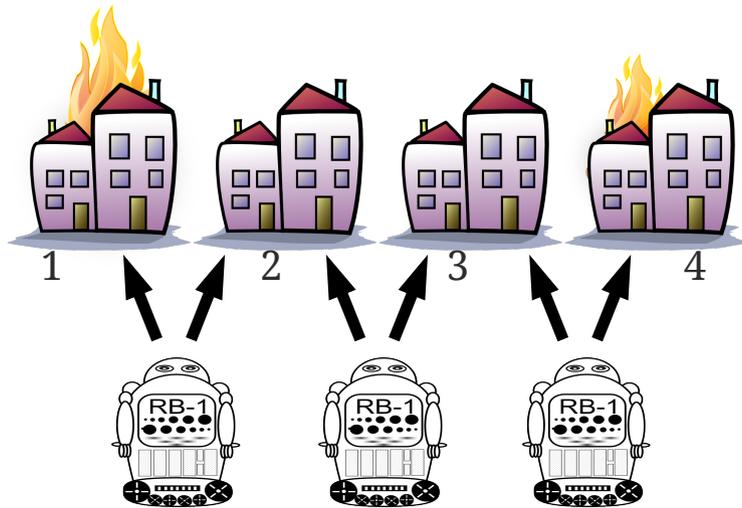
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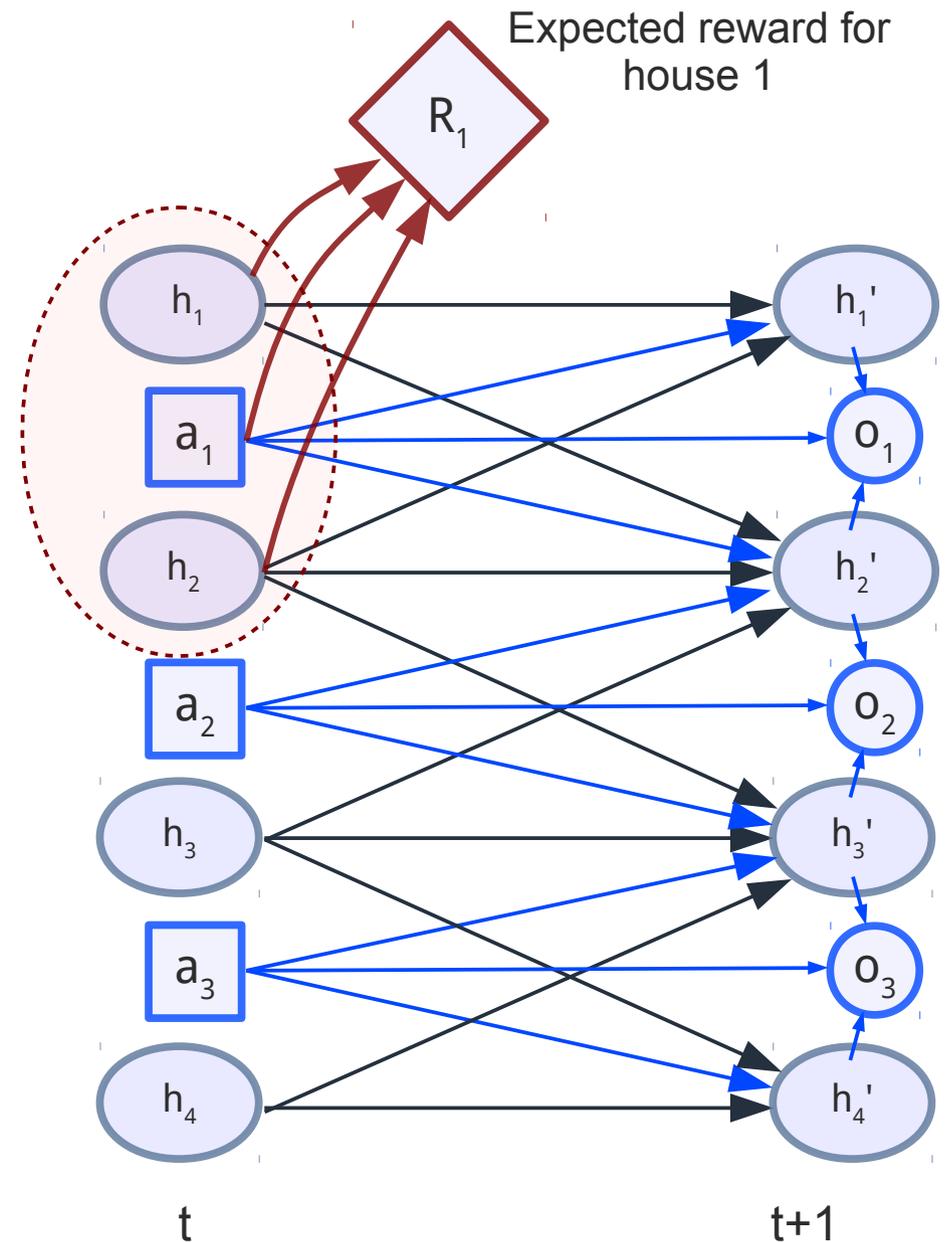
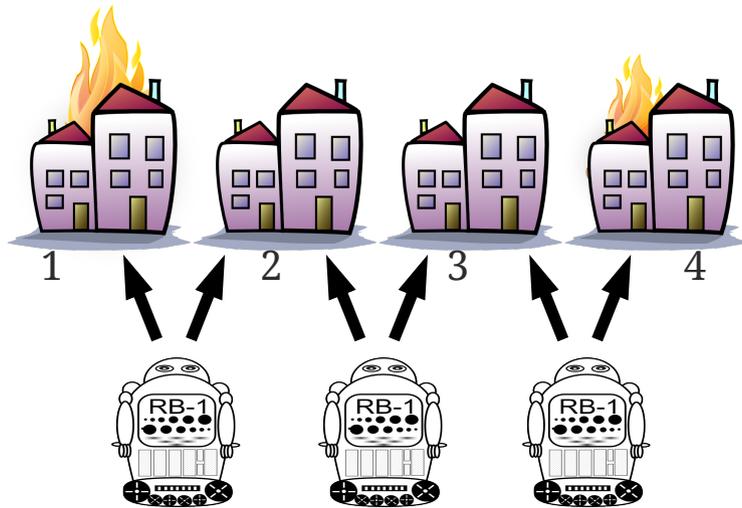
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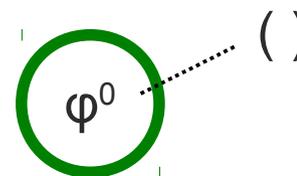


# Background: FSPC – 1

- Goal: scalability w.r.t. #agents
- Forward-sweep policy computation (FSPC)
  - for each  $t=0,\dots,h-1$
  - compute best joint decision rule  $\delta^t$
  - given past joint policy  $\varphi^t$

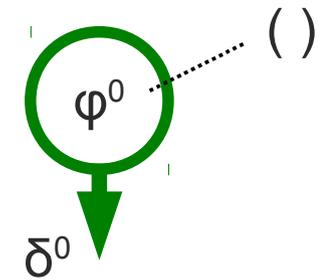
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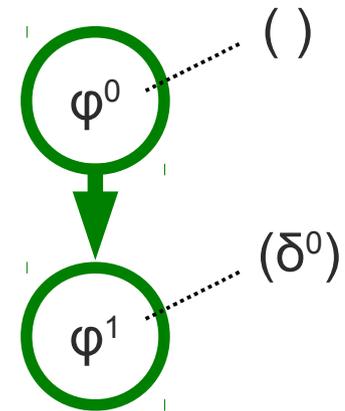
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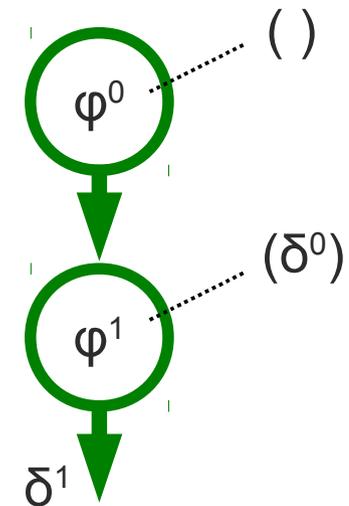
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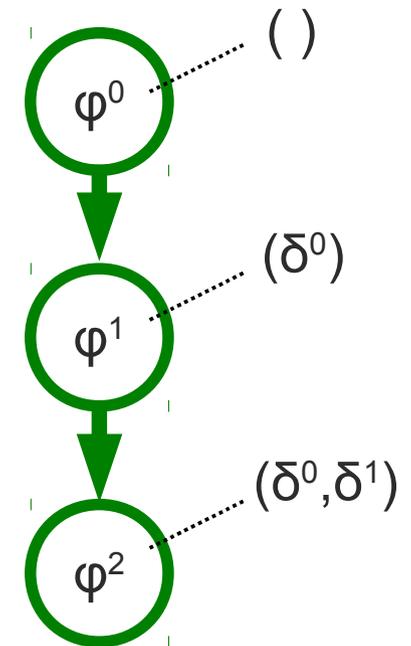
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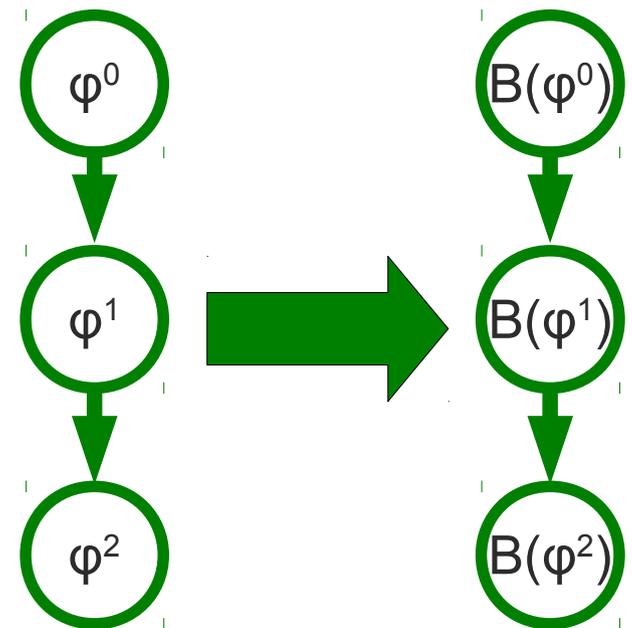
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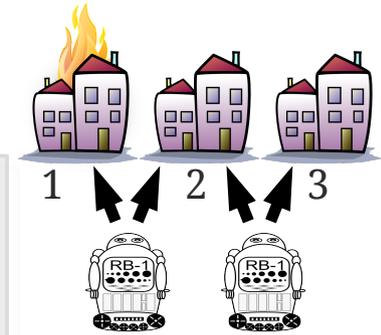
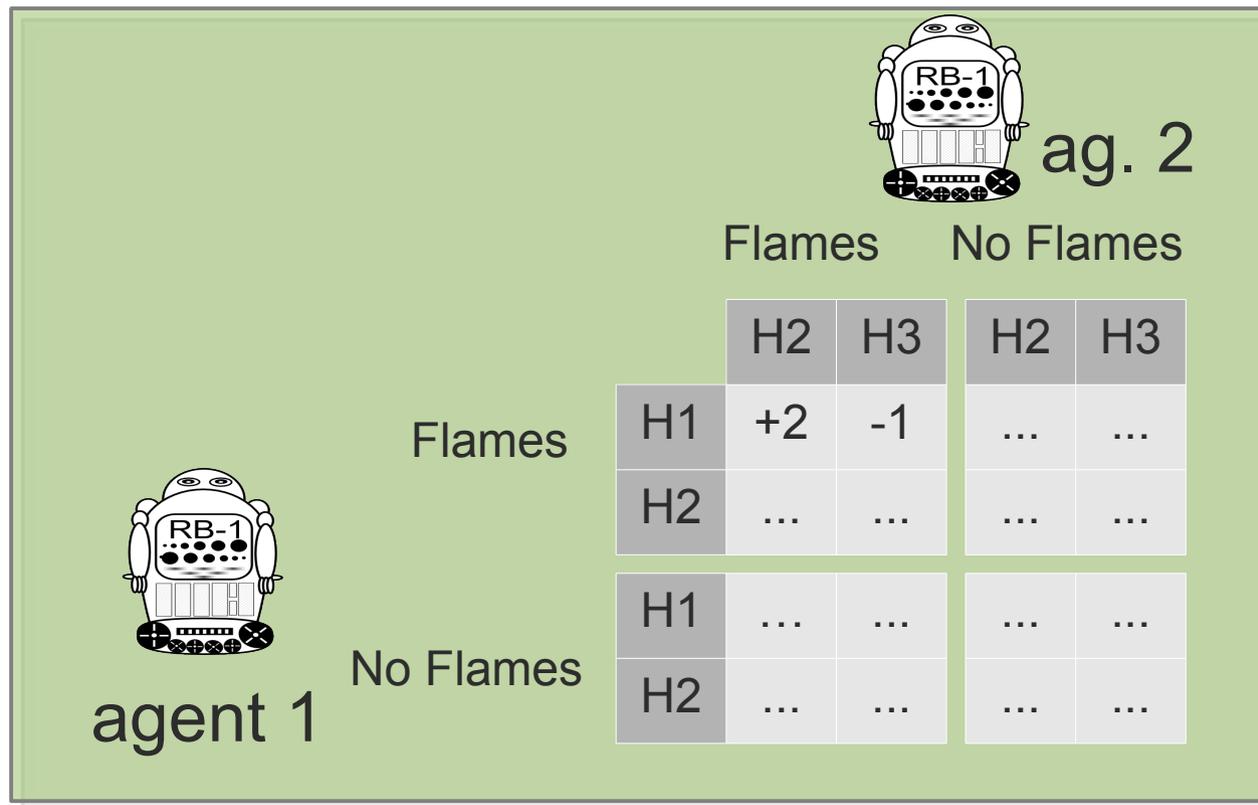
# Background: FSPC – 2

- Problem at stage  $t$ 
  - select  $\delta = \langle \delta_1, \dots, \delta_n \rangle$
  - $\delta_i$ : histories  $\rightarrow$  actions  $\delta_i^t(\vec{\theta}_i^t) = a_i^t$

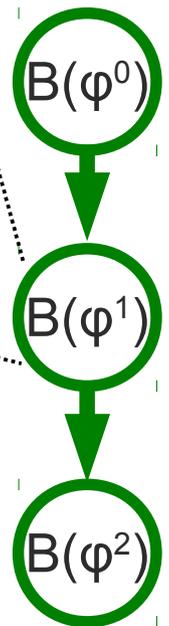
- As **collaborative Bayesian games (CBGs)**
  - agents, actions
  - types  $\theta_i \leftrightarrow$  histories
  - probabilities:  $P(\theta)$
  - payoffs:  $Q(\theta, a)$



# Background: CBGs



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# This paper: Factored FSPC

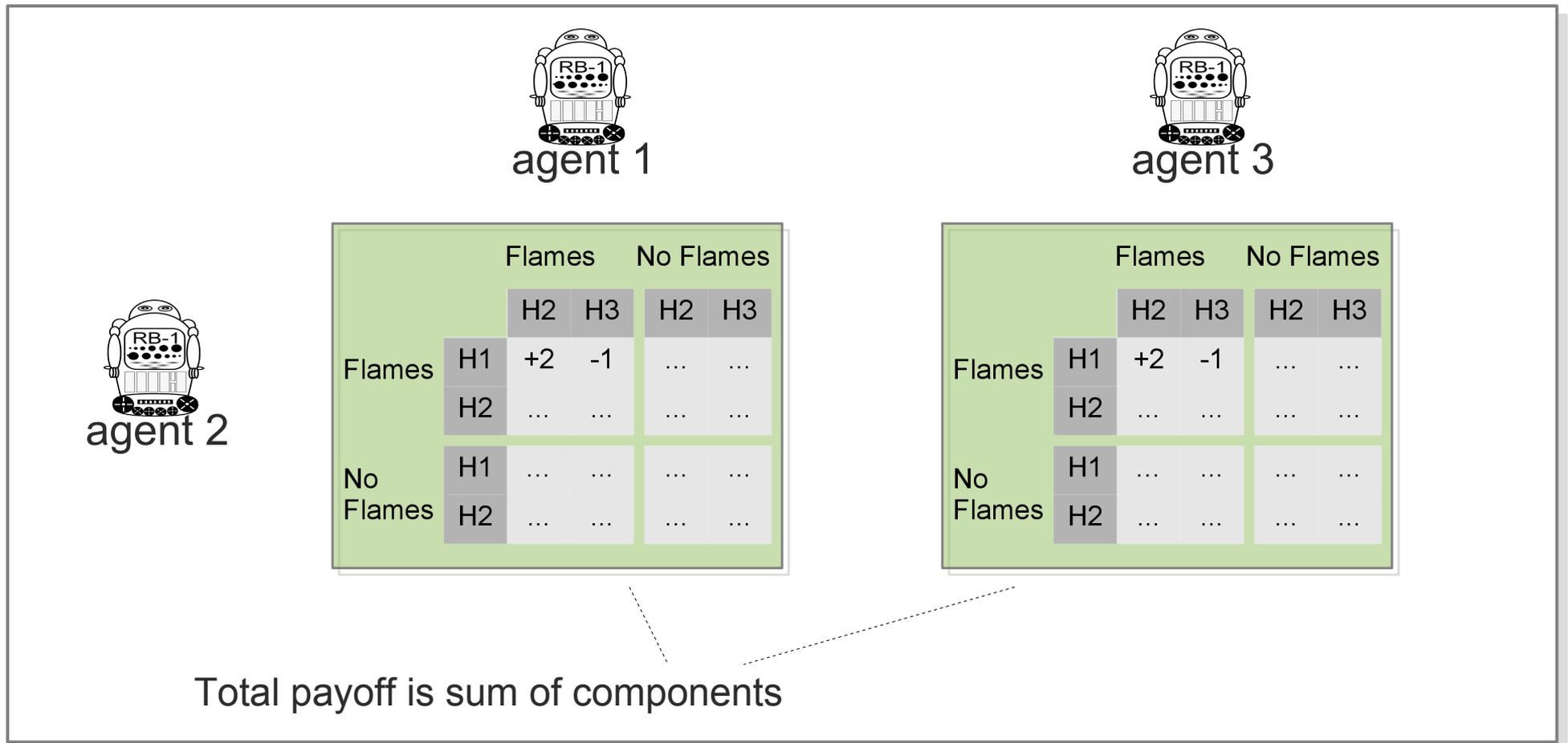
**Factored FSPC** – Basic idea:  
exploit independence between agents in FSPC

- if value function factored  $Q(x, \vec{\theta}, a) = \sum_e Q_e(x_e, \vec{\theta}_e, a_e)$

→ replace CBGs with

Collaborative **graphical** Bayesian games (CGBGs)

# This paper: Factored FSPC



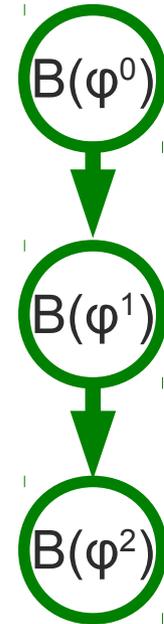
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# Factored FSPC for Many Agents

Improving scalability w.r.t. the number of agents...

- 1) Approximate structure of  $Q^*$ 
  - **Predetermined scope structure**
- 2) Compute CGBG payoff functions
  - **Transfer Planning (TP)**
- 3) Inference techniques to construct CGBGs
  - Extension of factored frontier [Murphy&Weiss UAI 2001]
- 4) Efficient solutions of CGBGs
  - Max-plus to ATI-FG [OWS UAI 2012]



# Factored FSPC for Many Agents

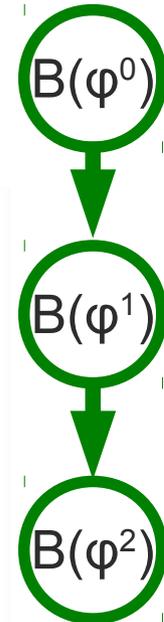
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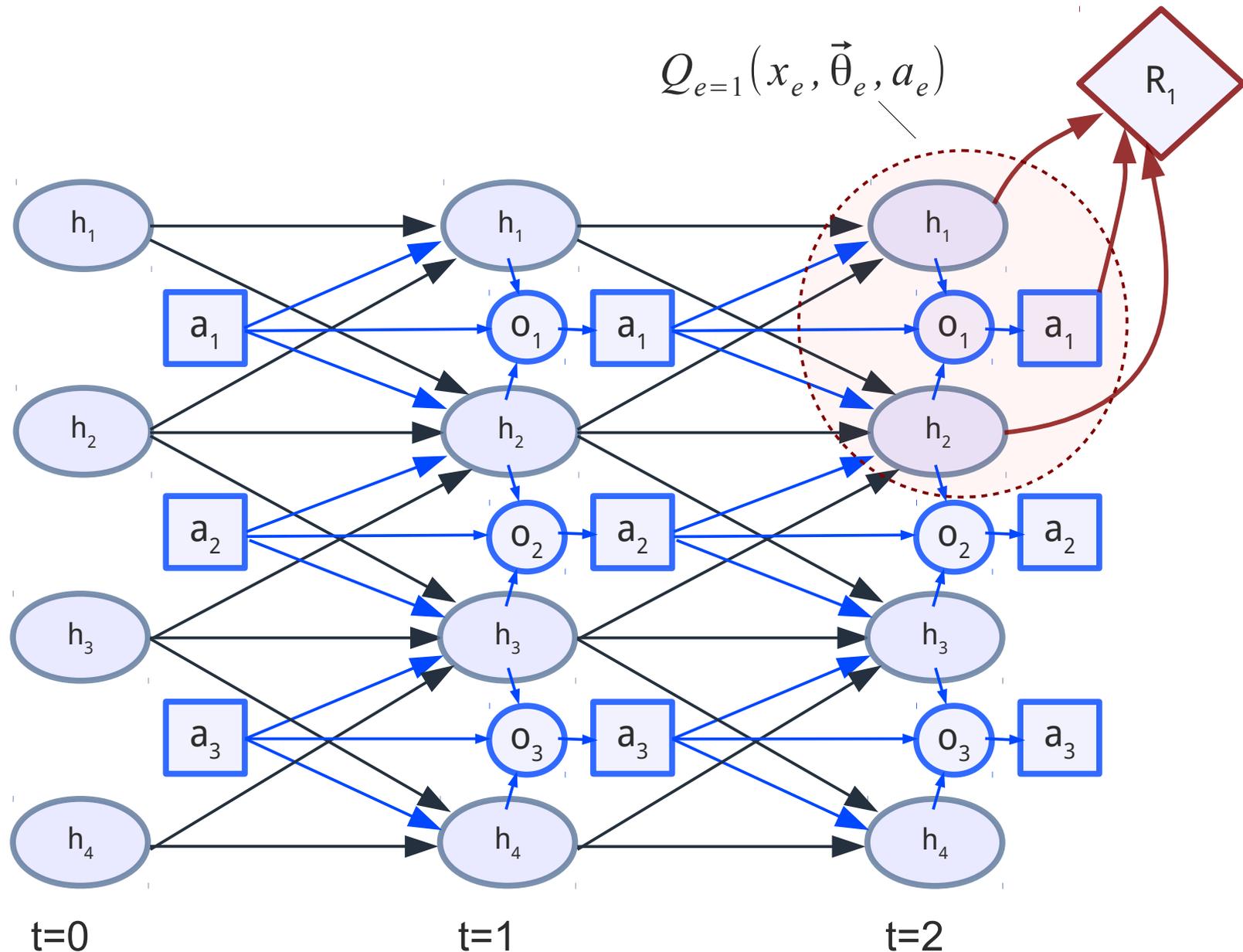
Rest of  
this talk

- 3) Inference techniques to construct CGBGs
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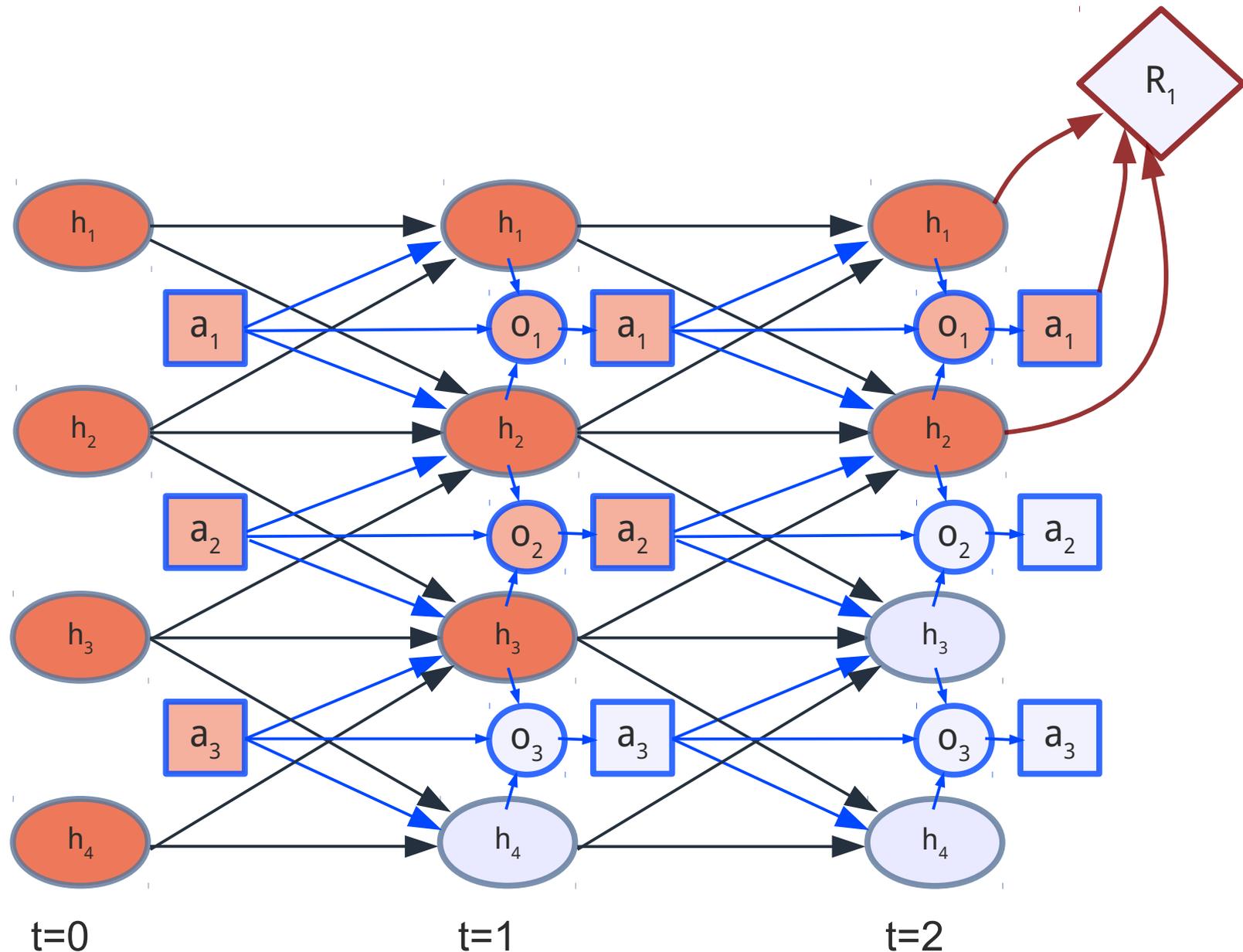
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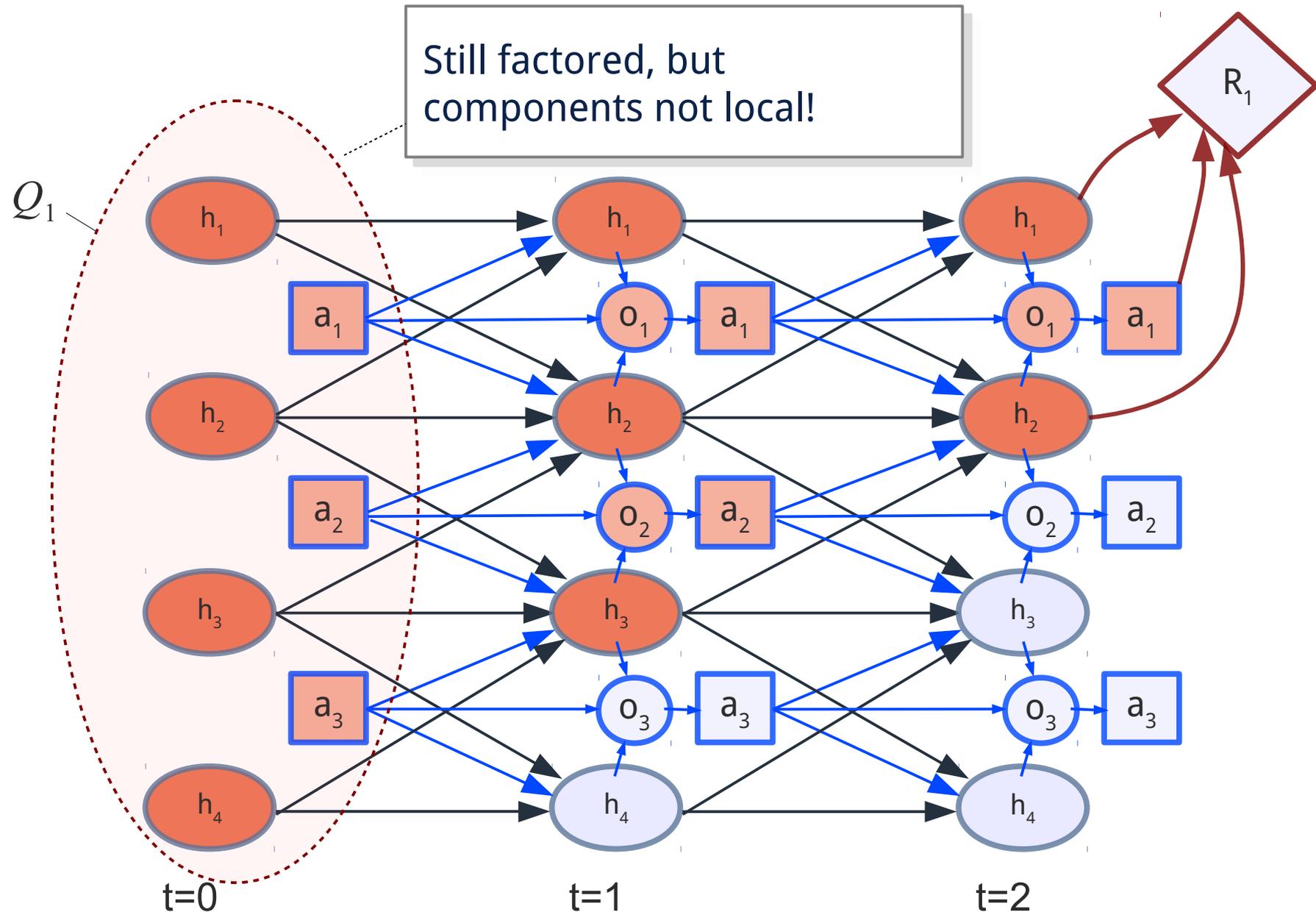
# Accumulation of Reward over Time



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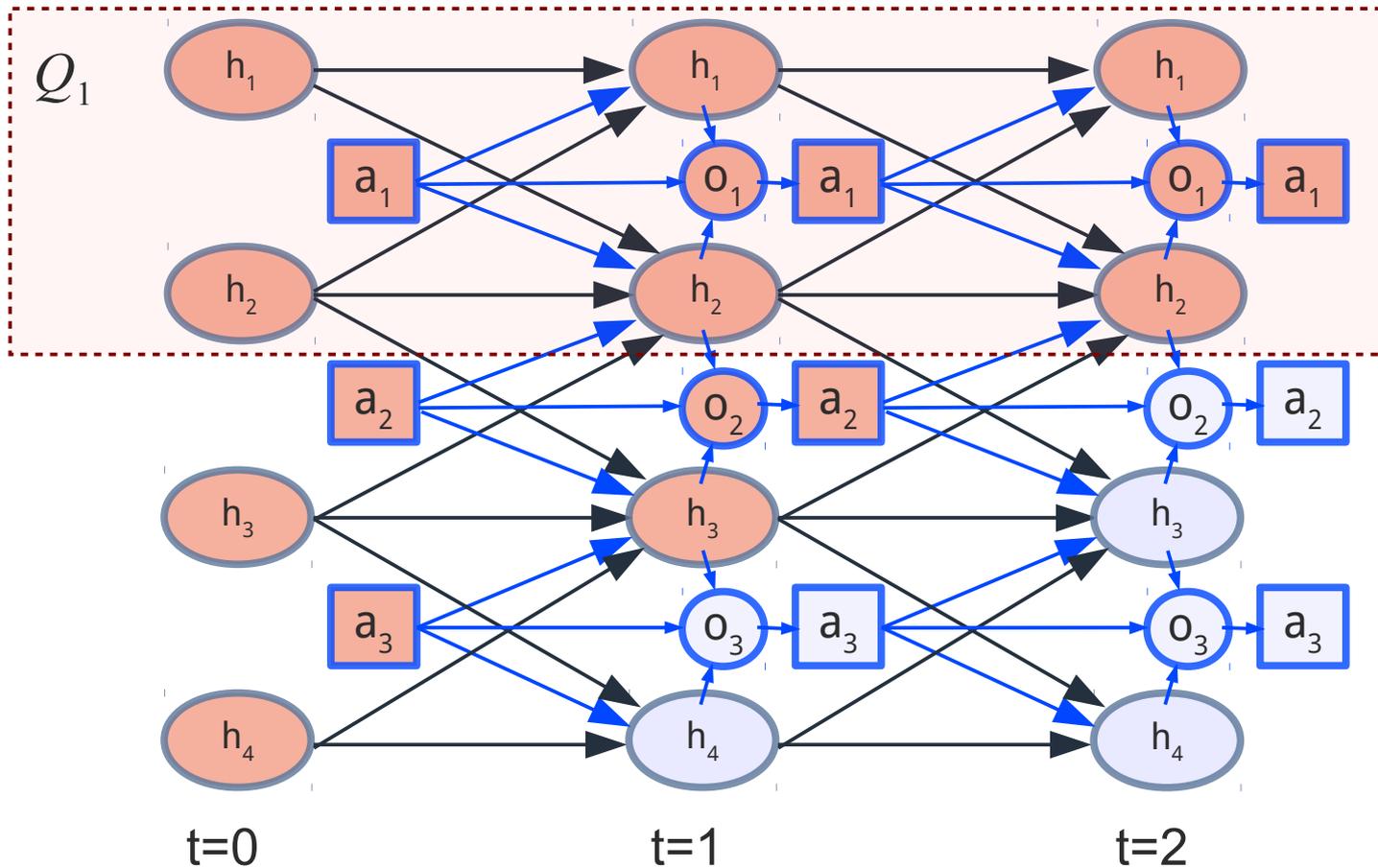


# Accumulation of Reward over Time



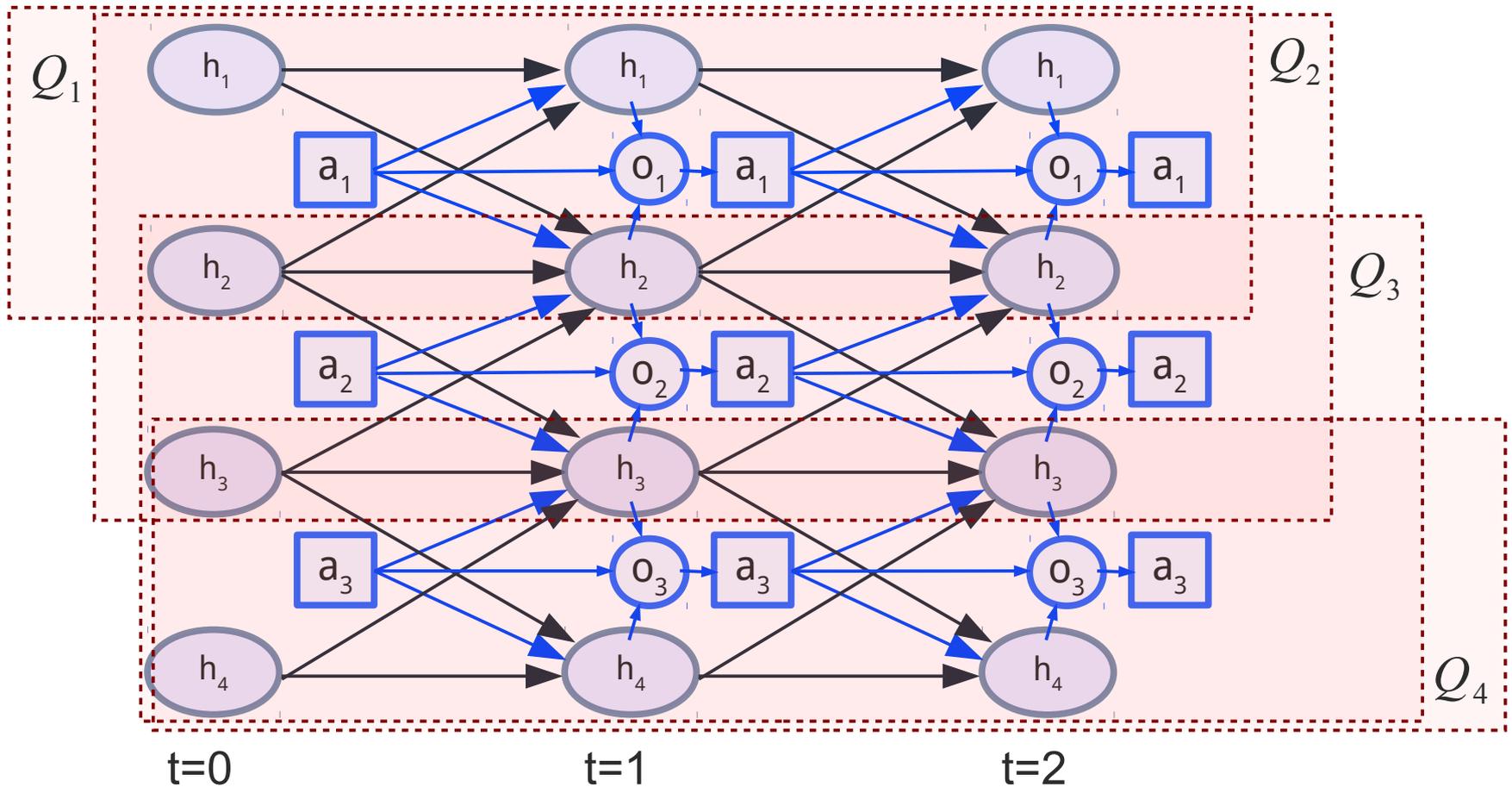
# 1) Predetermined Scope Structure

- Use smaller scopes



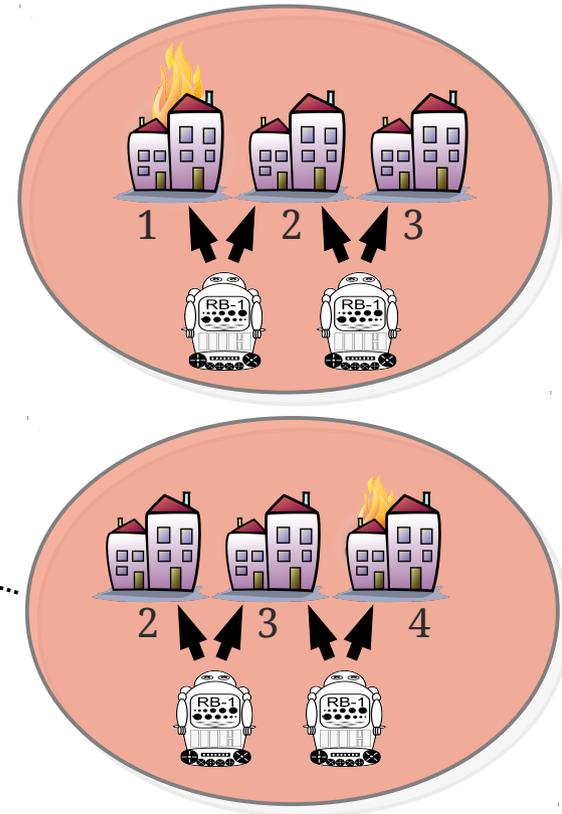
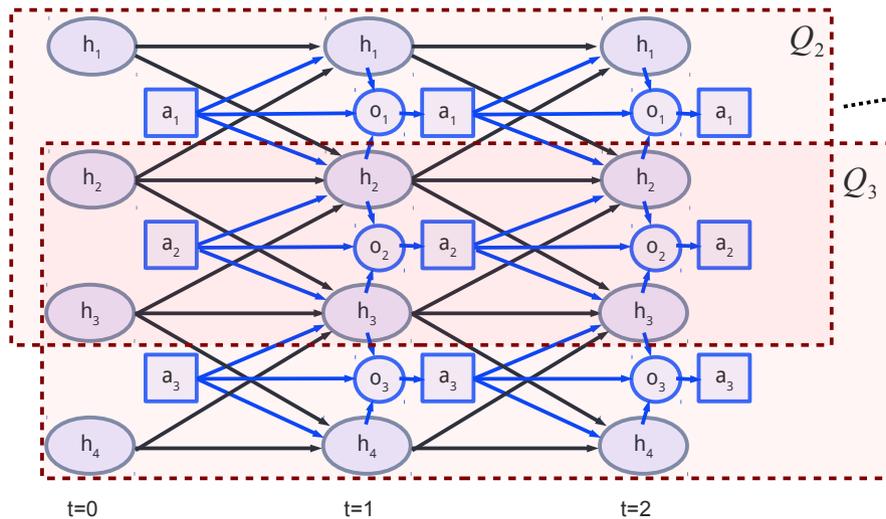
# 1) Predetermined Scope Structure

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# 2) Transfer Planning

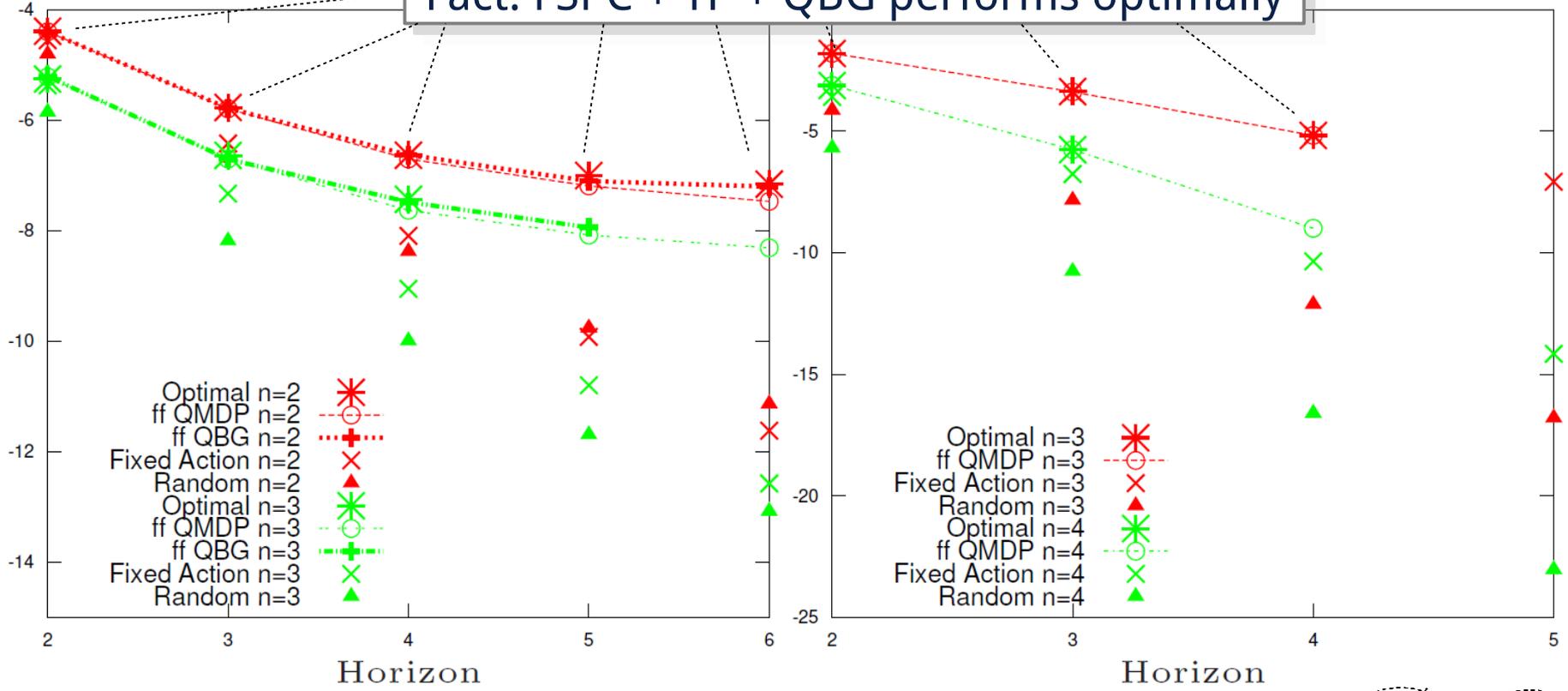
- Define a **source problem** for each component
  - involving fewer agents



- Solve those exactly or approximately (QMDP, etc.)

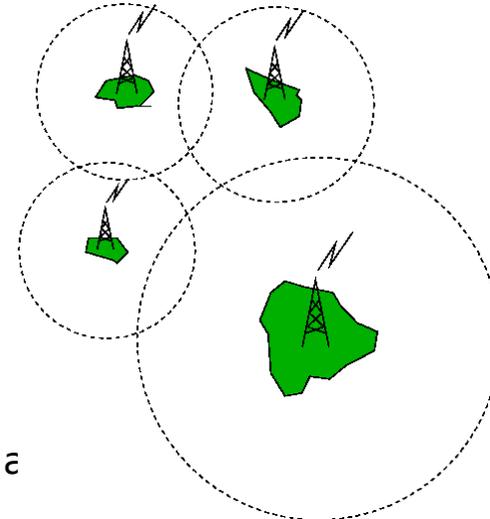
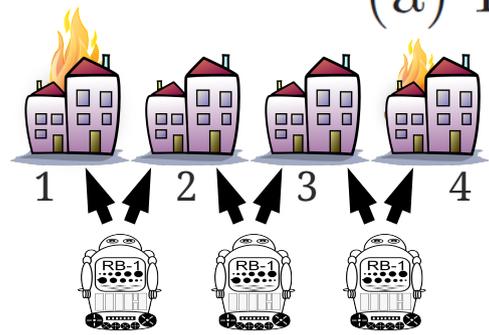
# Results – Compared to Optimal

Fact. FSPC + TP + QBG performs optimally

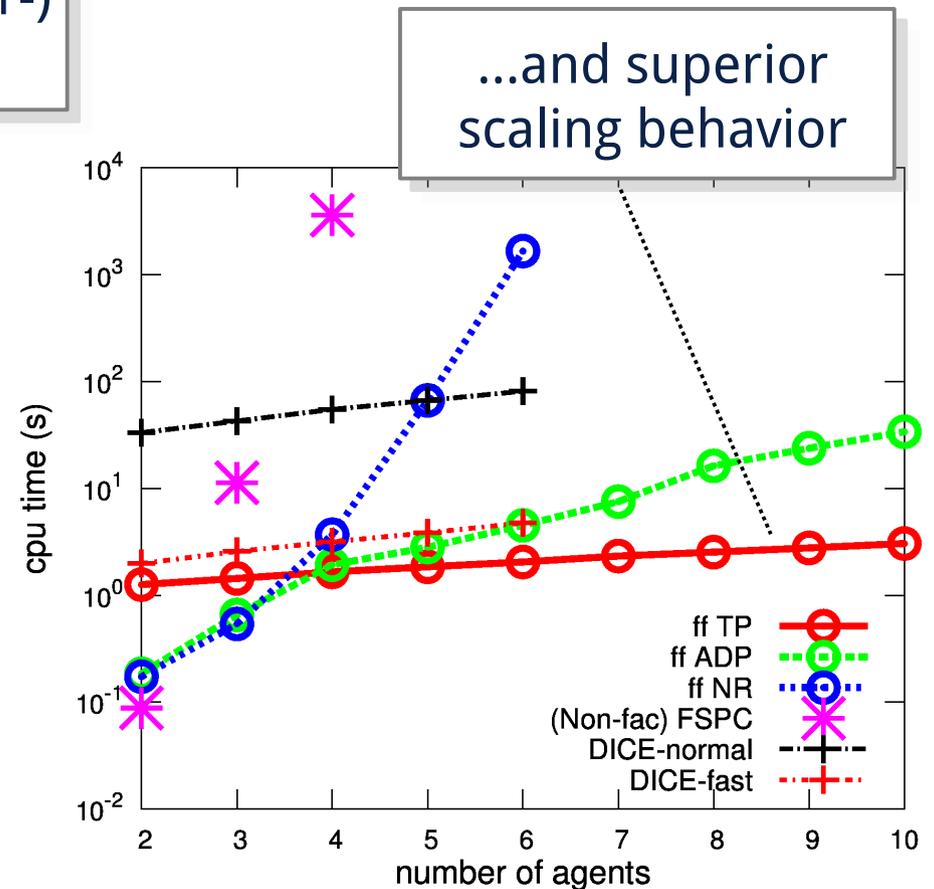
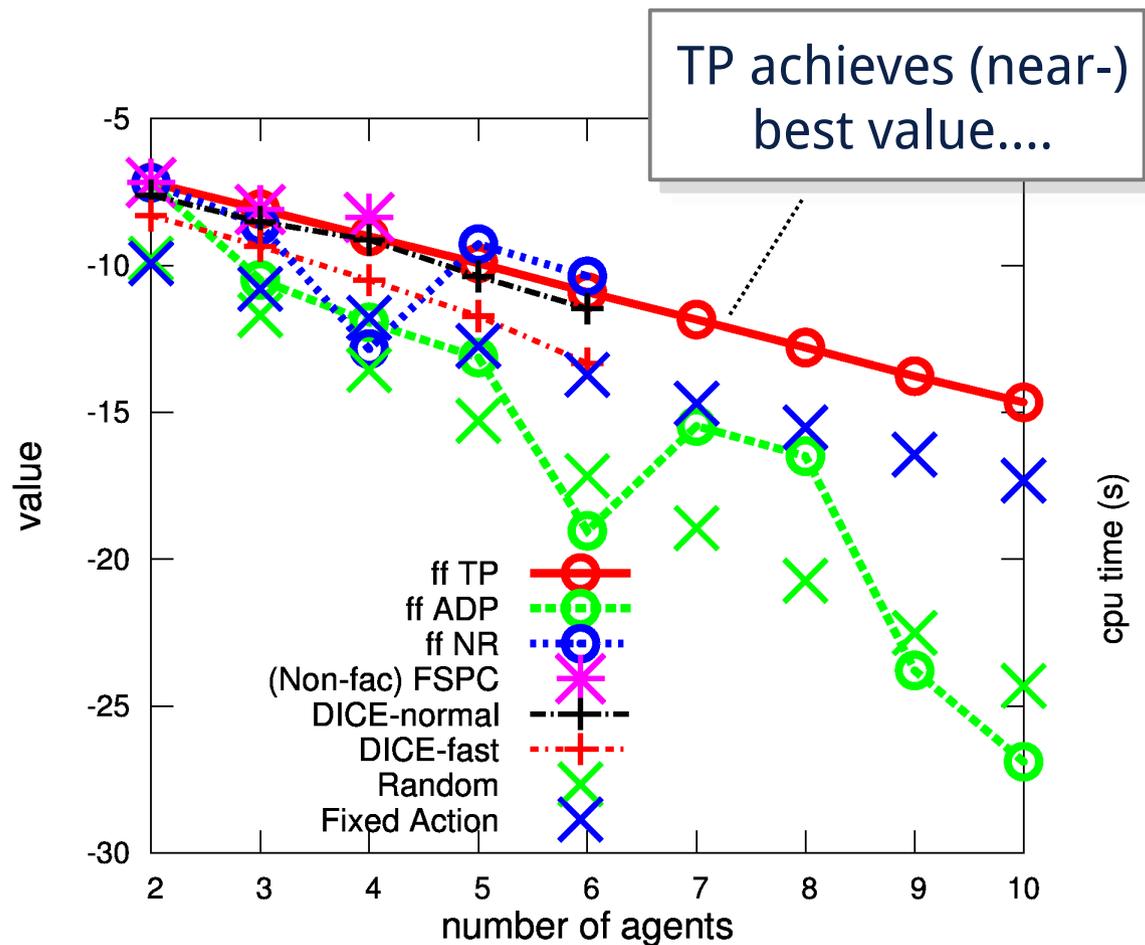


(a) FFG.

(b) ALOHA.

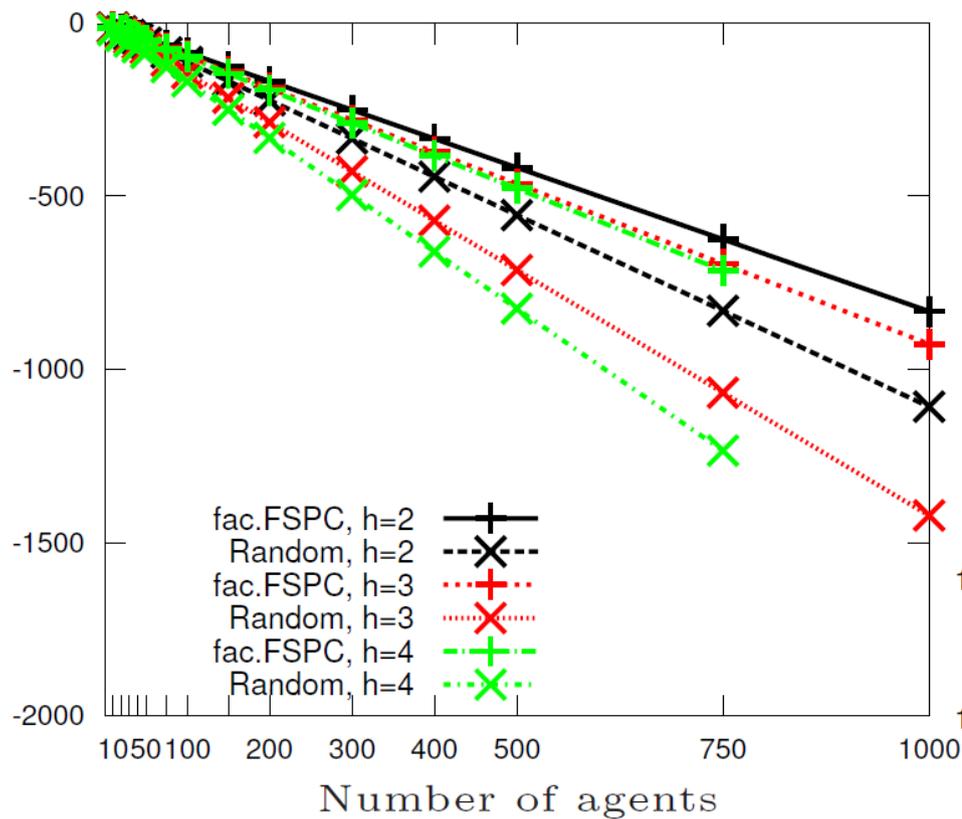


# Results – vs. Approximate

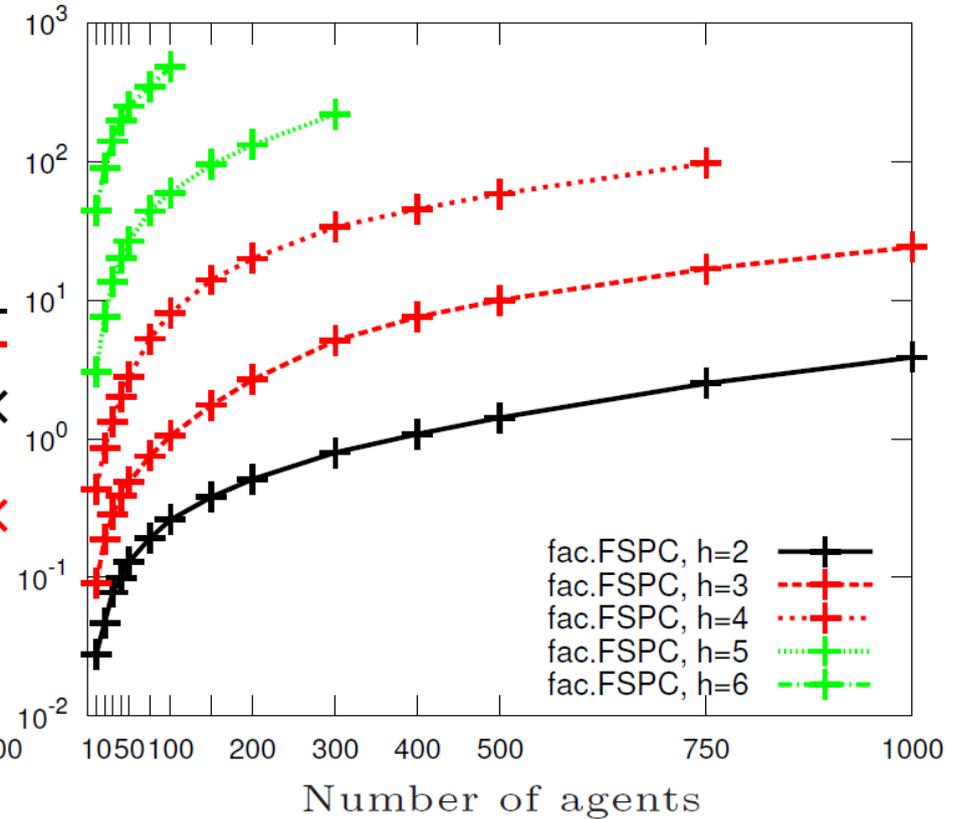


- FFSCP: TP vs ADP, NR
- Non-factored FSPC, DICE [Oliehoek et al. 2008 Informatica]

# Results – Many Agents



(a) Value.  $h = 2 \dots 4$ .



(b) Runtime.  $h = 2 \dots 6$ .

# Conclusions

- Factored FSPC with transfer planning:
  - approximates factored Dec-POMDPs with multiple abstractions involving subsets of agents
- Unprecedented scalability for this class
  - results up to 1000 agents
- Future work:
  - scale to higher horizon
  - understand such abstractions
    - empirically verified near-optimal quality
    - formal understanding influence-based abstraction [AAAI' 12]

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