PRA1004 Scientific Computing - Lab Assignments

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Week 2

Overview Lab 2

In this lab session, you will learn many things:

- You will implement Newton's method.
- You will learn about indexing in matrices and using a number of convenient functions.
- You will develop a feeling of what can go wrong with *floating point numbers*.
- You will implement your own matrix multiplication function and benchmark it.

As always, you are expected to hand in a report where you answer your questions with a short explanation toghether with the source code you produce in this lab session. Make sure that your source code is in the form of a script (one for each assignment unless indicated otherwise.)

1 Newton's Method

In this assignment you will implement Newton's method to compute the digits of $\sqrt{2}$.

1. Write down the function f for which $\sqrt{2}$ is a root.
2. Write down the derivative f'.
3. Derive (2.7) on page 47 of the book.
4. Implement Newton's method to find the digits of $\sqrt{2}$. Use `format long` to display all 15 significant digits.
5. Compare your implementation to the one in the book. What are the differences? Are there parts of your implementation that you think could be improved?

2 Matrices and Indexing

1. Generate the following sequence with 1 line: [1, 4, 7, 10]
2. Compute the inner product of [1:4] and [5:8] in three different ways.
3. A number of related tasks:
   (a) Generate an invertible random 5x5 matrix called `Ar` using `rand`.
   (b) Save the matrix using `save`. Make sure to submit is with your report.
   (c) Compute `Id`, the result of the product of `Ar` with its inverse. Use `abs` to make `Id` positive
   (d) `eps(1)` is the difference between 1 and the first number larger than 1. Use '>' to show the matrix that indicates which entries are bigger than `eps(1)`, call it `Id2`. How many such entries are there? How many did you expect?
   (e) Compute in one line the number of entries of `Id` that are larger than `eps(1)`. (Hint: use `sum` twice.)
   (f) Use `eye` to generate the expected identity matrix, save it as `Id3` .
   (g) Use `Id2` and `Id3` to generate a matrix `ErrorEntries` that contains 1's only in those places where you did not expect them in `Id2`.
   (h) Use `find` to print the so-called 'linear indices' (that pretends that the columns of a matrix form one long vector) of the error entries.
   (i) Use those indices to replace the error entries in `Id` by 0.
4. Generate a random 70x70 matrix, find the maximum element as well as its row and column index.

3 Floating Point Numbers

1. Compute the 15 digits (in decimal) of the smallest non-normalized floating point number: 4.9407e-324. Put the line you used in the report. (Hint: what is the floating point representation of "0.5" ?)
2. Have a look at the following code

```
i = 0;
fraction = 1.0/49;
while i ~= 1
    i = i + fraction;
end
disp('i = '),
disp(i)
```

   (a) What is wrong with it?

(b) How would you fix this code?

(c) What is the output of your corrected script?

3. Count the number of integers between 1 and 1000 for which the product with their inverse does not equal 1. (tip: use a for loop)

4. Compute

```
1e-4 + 1 - 1
1e-20 + 1 - 1
```

(a) What is the difference ?

(b) Why does this difference occur?

(c) The above formulas have form '1e-x+1-1'. What is the largest 'x' that does not suffer from this problem? Can you explain that?

4 Computational Cost

1. Create a function called `C = MatMult(A,B)` that implement matrix multiplication using a triple for loop.

(a) use `size` to check whether AB is a valid matrix product.

(b) If the dimensions are incompatible throw an error "cannot multiply, sizes incorrect" using `error`.

(c) For easy debugging, use the following test matrices:

```
A = reshape(1:6, 2, 3)
B = reshape(10:60, 3, 2)
```

2. Create a plot showing the run time of MatMult on random matrices of size 10, 20, ..., 100.

3. Also create a plot showing the runtime of Matlab's multiply operator on random matrices of size 100, 200, ..., 1000.

4. What reasons can you give to explain the difference?