Decsision Making in Intellingent Systems:

Partially observable Markov decision processes

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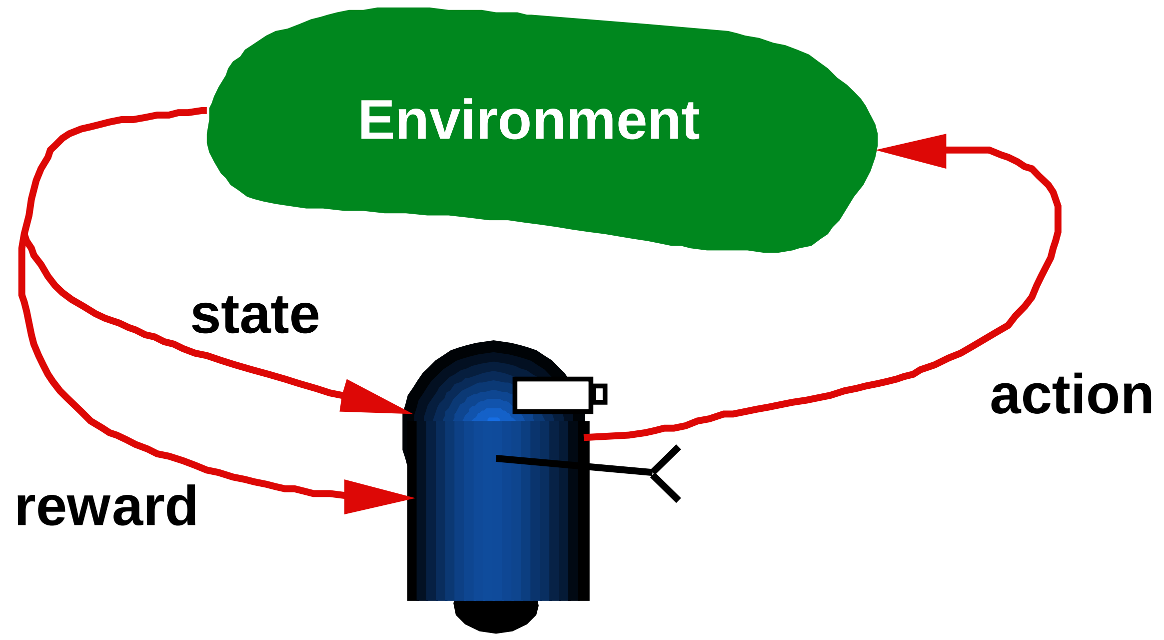
Regular MDPs

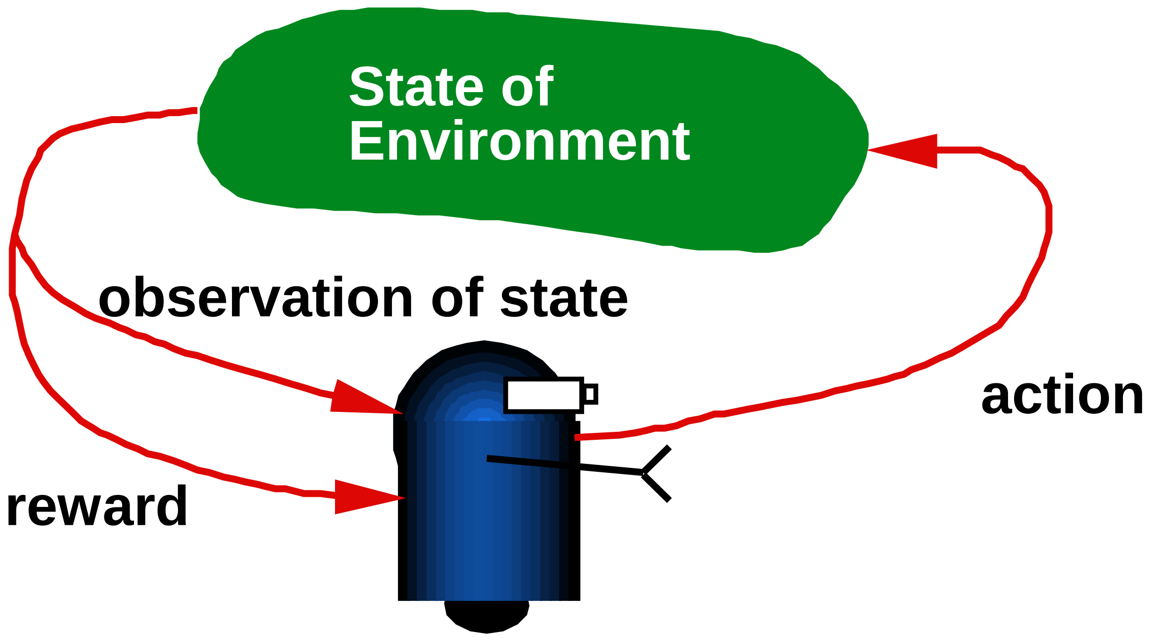
- Up to now…



State = state of environment!

Regular MDPs

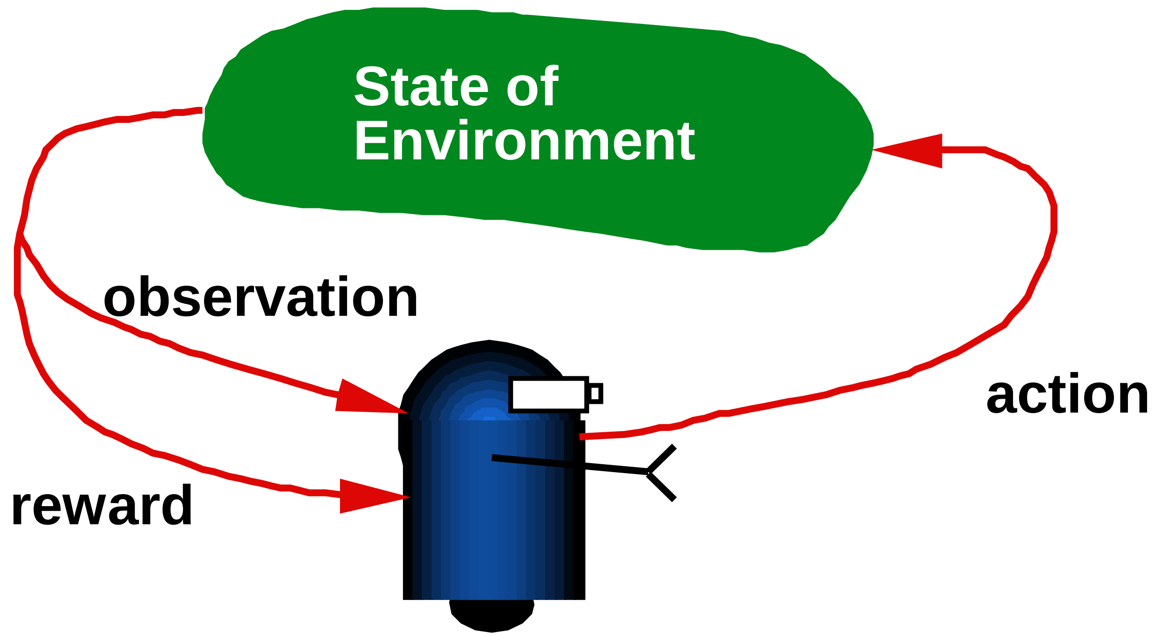
- Up to now...



Agents observe the state (of the environment)
-> Fully Observable MDP

Partially observable MDPs

- Now: Partially observable environment
  - agent can't observe the full state.



  - ...but observation gives hint about the true state.

RL vs Planning

- In this course: focus on reinforcement learning(RL).
- RL = learning the model + planning
  - Planning is `using the model'
  - explicit: `model-based RL'
  - implicit: Q-learning etc.
- In this lecture: only planning!
  - We assume we have a perfect model of the (partially observable) world.

Partially observable MDPs

POMDP

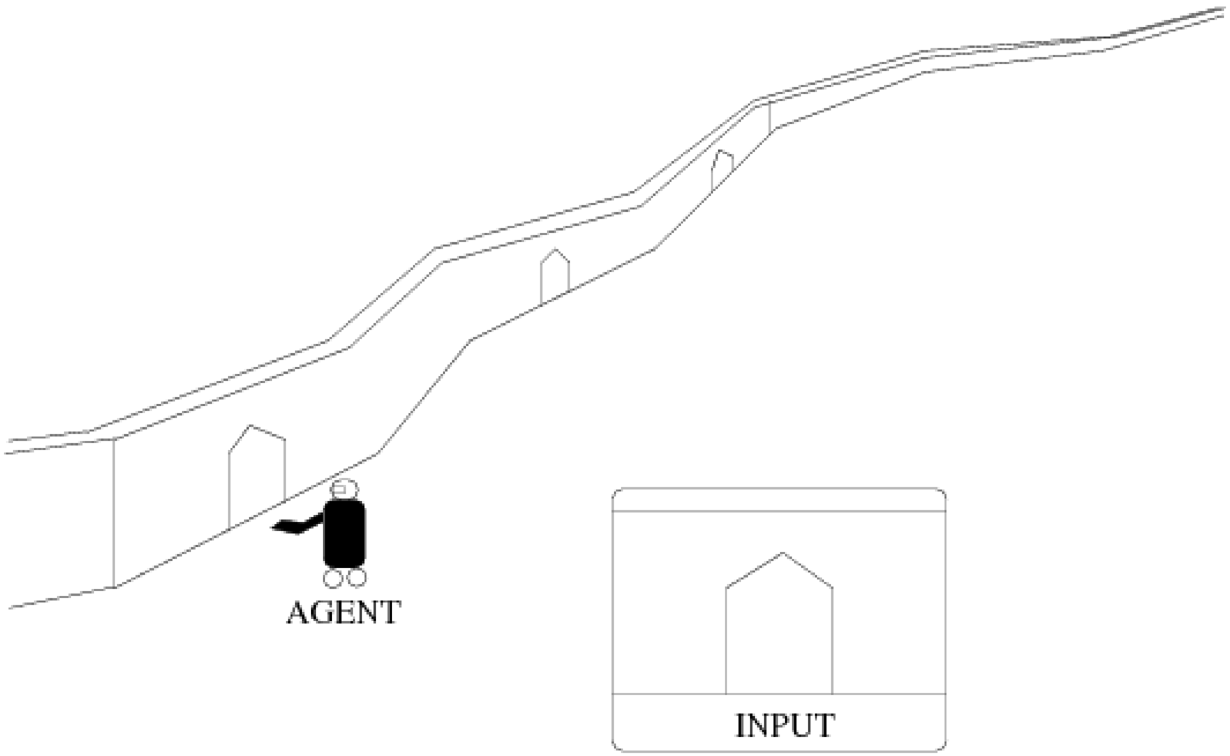
MDP

- States – s_1 ... s_n
- Transitions – P(s' | s,a)
- Rewards - R(s,a)

- Observations – o_1...o_m
- Observation probs – P(o | a, s')

POMDP: an example

- Where am I?

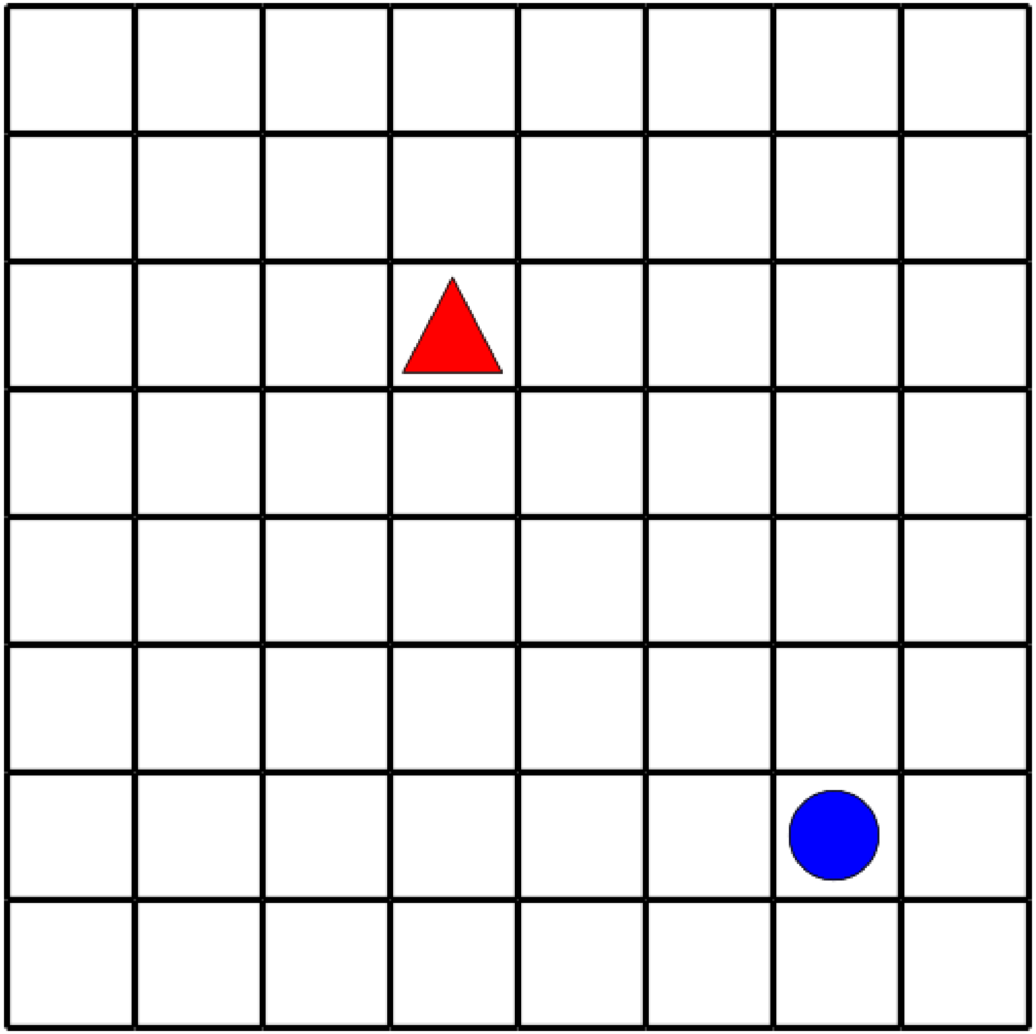


Partial observability

- When is an agent's environment partially observable?
  - Real world: almost always.
- Types of partial observability
  - Noise
    - Sensors have measurement errors.
    - Sensor (or other part of the agent) can fail.
  - Perceptual aliasing
    - When multiple situations can't be discriminated. I.e., multiple states give the same observation.
      - e.g. what is behind a wall?

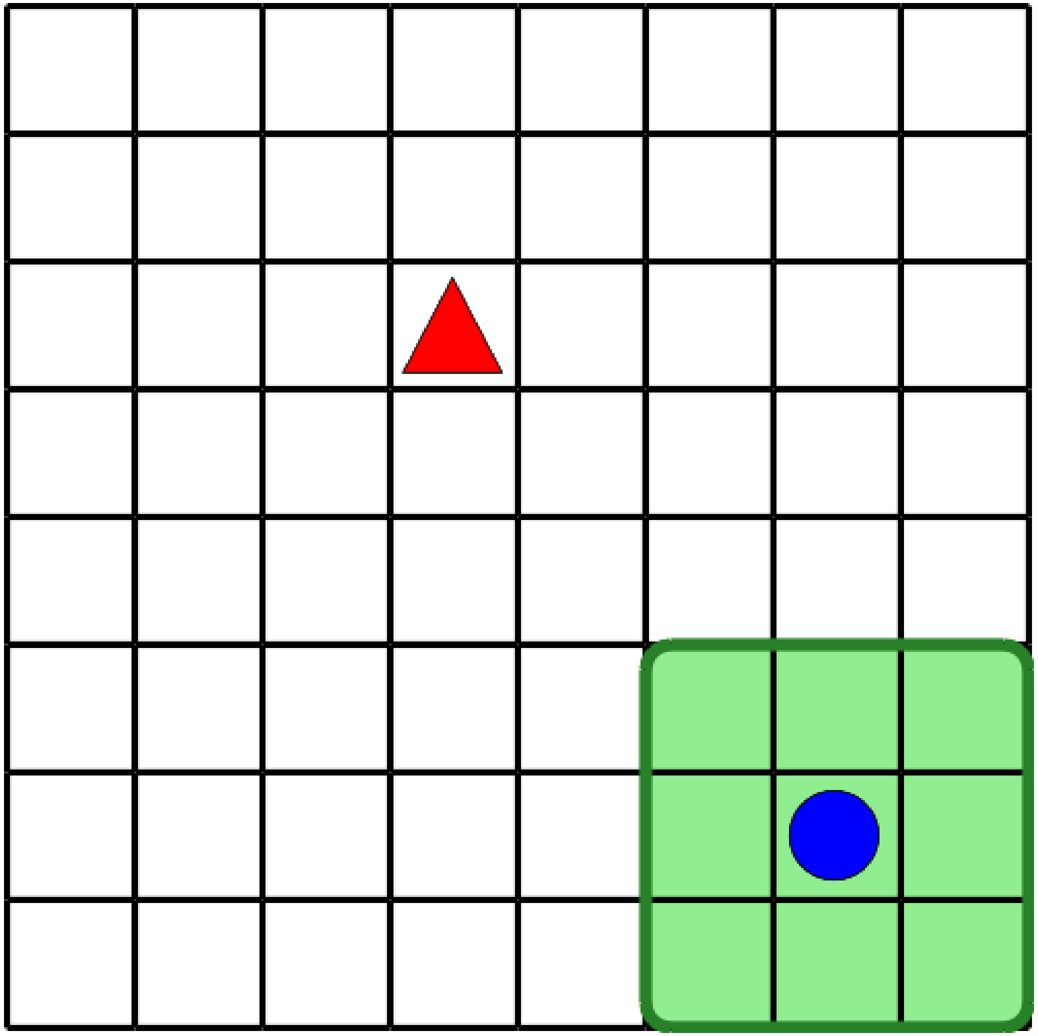
Example: predator-prey

- Fully observable
- o=s=(-3,4)



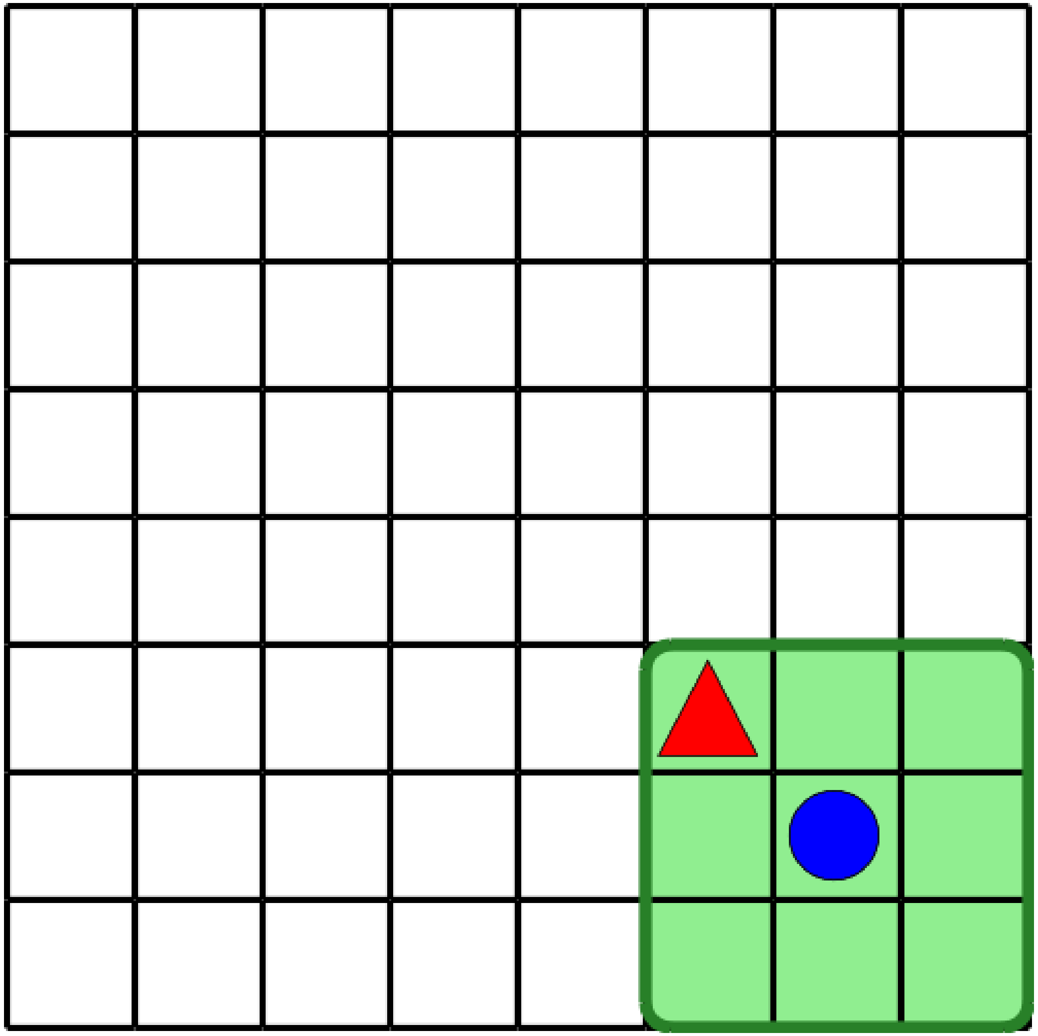
Example: predator-prey

- Partially observable – perceptual aliasing
- o=Null



Example: predator-prey

- Partially observable – (noise?)
- o=(-1,1)



Policies under partial observability?

- Now given that the agent only gets some observations, what policy should he follow?
  - How does such a policy look?

Policies under partial observability?

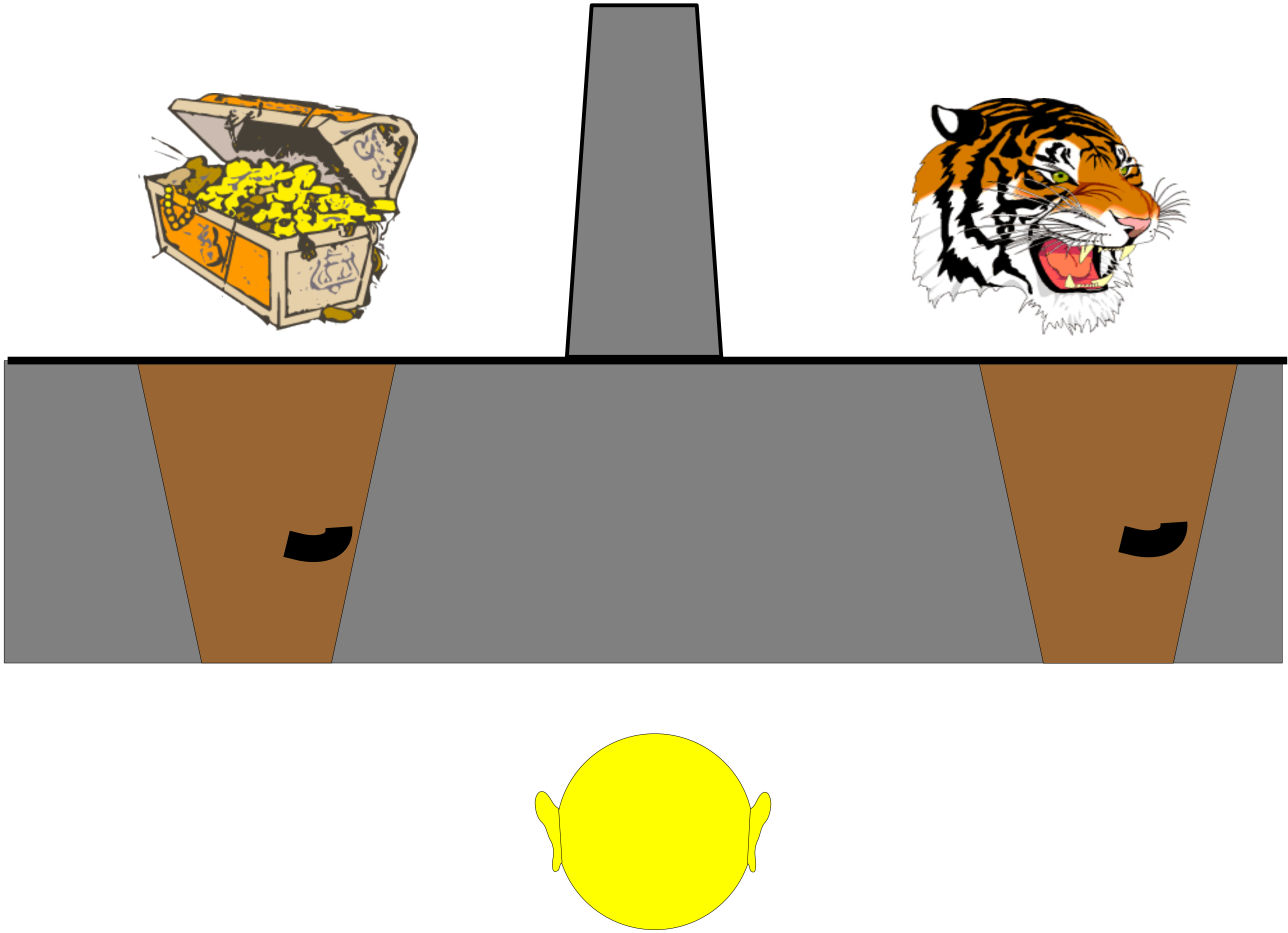
- Now given that the agent only gets some observations, what policy should he follow?
  - How does such a policy look?

- No more Markovian signal (i.e. the state) directly available to the agent...
  - ➔ In general: should use all information!
  - ➔ The full history of observations.

- We will do something smarter in a moment...

A full POMDP: the Tiger problem

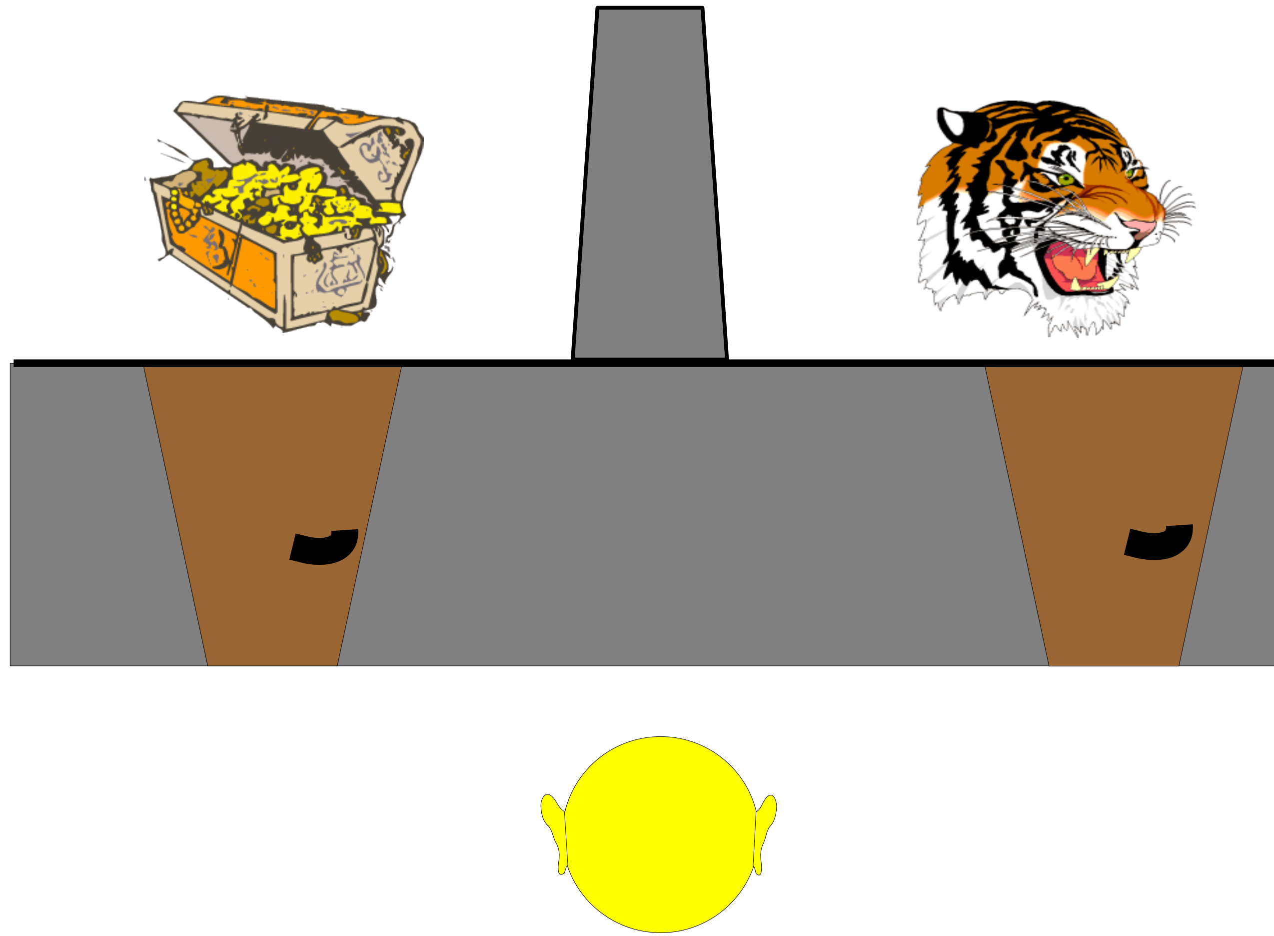
- **States**: left / right (50% prob.)
- **Actions**: Open left, open right, listen
- **Observation**: Hear left, Hear right
- **Transitions**: static, but opening resets.



- **Rewards**:
  - correct door +10,
  - wrong door -100
  - listen -1
- **Observations** are correct 85% of the time.
  - P(HearLeft | Listen, State=left) = 0.85
  - P(HearRight | Listen, State=left) = 0.15

The Tiger problem

- When do you open...?



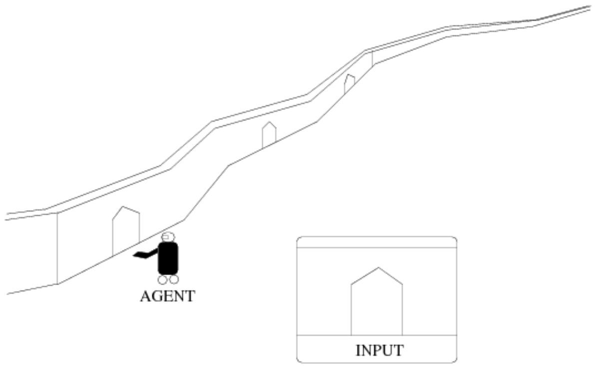
- At the beginning?
- After HL ?
- After HL, HL ?
- After HL, HL, HL ?

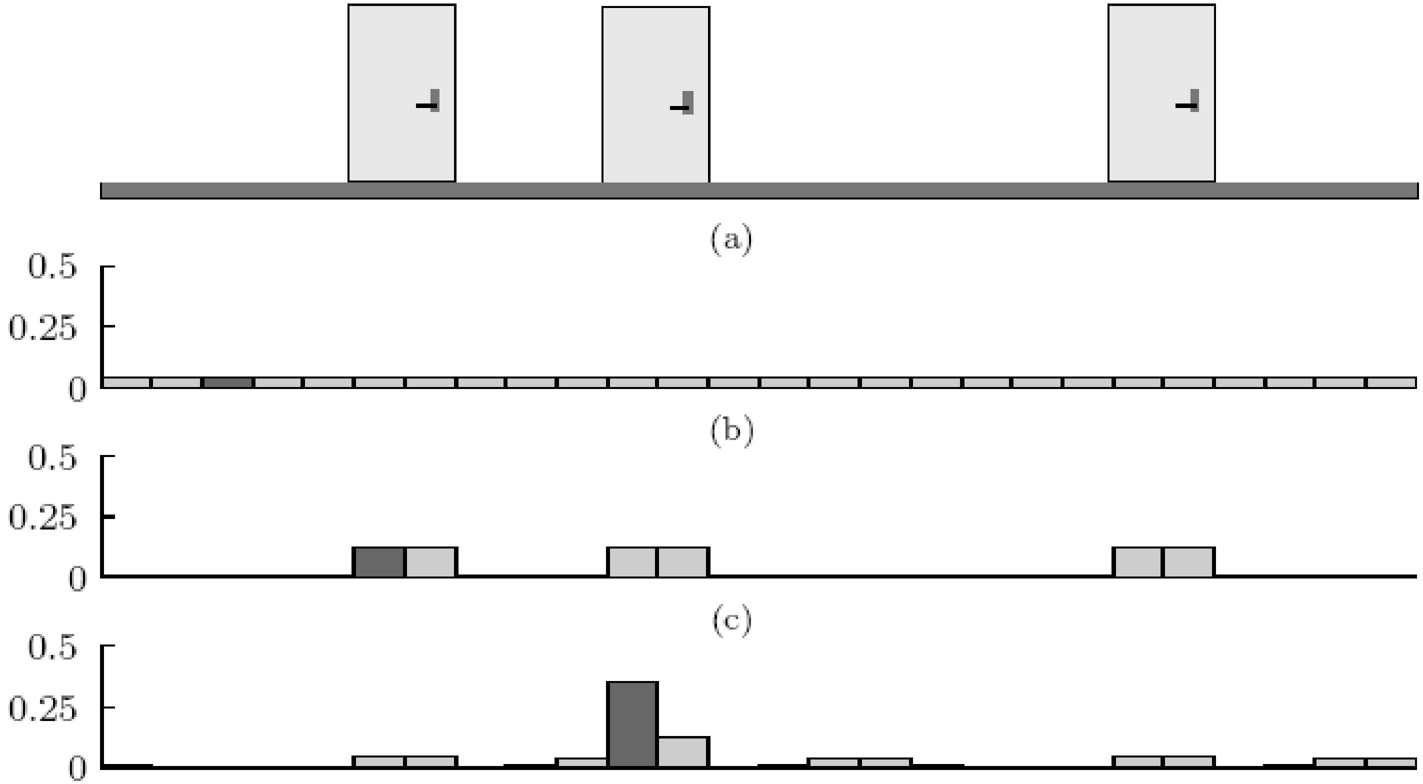
Beliefs

- As promised: there is something smarter than trying all possible policies.
  - mappings from obs. histories -> actions is approx. $A^{(O^t)}$
- Maintain the probability of all states.
  - Use that to make your decisions.
    - Did you estimate the probability of the states for the tiger problem?
  - The probability distribution over states at some time step, is called the belief b.
    - For all s: b(s) = Pr(s)
    - Sufficient statistic for the history.

Beliefs: an example

- For the hallway problem





Calculating the belief

- A POMDP is often specified with an initial belief.
  - So we want to keep track of the probs. of the states.
  - I.e., given b, a and o, we want to find the new belief b'_{ao}.
  - Process is called belief update.

DO not forget: the term `belief' can be misleading.

Not: `something that one agent can belief, but some other agent would not'
But: The actual probability of the states, given the history.

Belief update – prerequisites

- b'_{ao} can be calculated from b and T, O...

  (resp. the transition, observation model)

- ...using Bayes' rule.

Bayes rule:

$$P(A|B) = P\frac{(A,B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

Belief update

- substituting relevant vars in Bayes' rule.

$$P(s'|o)=\frac{P(o|s')P(s')}{P(o)}$$

- adding same arguments to `given'

$$P(s'|b,a,o)=\frac{P(o|b,a,s')P(s'|b,a)}{P(o|b,a)}$$

- expanding P(s'|b,a) gives the belief update:

$$b'_{ao}(s')=\frac{P(o|a,s')\sum_{s}P(s'|s,a)b(s)}{P(o|b,a)}$$

with $P(o|b,a)=\sum_{s'}P(o|a,s')\sum_{s}P(s'|s,a)b(s)$

POMDPs: making decisions

- Now we know how to maintain a belief over states...
  ->but what decisions should we make?
- We treat 3 methods
  - Approximate
    - most likely state (MLS)
    - Q_{MDP}
  - Exact, given the initial belief
    - Solving the `belief MDP'

Most likely state

- Take the action that would seem best in... ...the most likely state s_{ml}.
  - I.e., state with highest probability.
  - $b = (0.1\ \ 0.3\ \ 0.5\ \ 0.1)^T$ -> state 3
- But what is the best action in s_{ml} ?
  - Solve the `underlying MDP'.
    - pretend there are no observations.
    - Solve the MDP.
    - Result: the MDP policy π_{MDP}
  - Perform action $\pi_{MDP}(s_{ml})$.

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Q-MDP

- Also uses solution of the `underlying MDP'
  - but now uses the found Q values, not the policy.

- Find the MDP Q(s,a)-values
  - E.g., using value iteration.

- Given the current belief b, for each action compute

$$Q(b,a) = \sum_s Q(s,a)\, b(s)$$

- select the action with highest Q-value

$$a_{Qmdp} = \arg\max_a Q(b,a)$$

Solve the beliefs MDP

- For a finite (and not too large) horizon...
- and given an initial belief...
- → we can compute all possible beliefs.
  - `belief tree'
- Propagate back the expected reward

$$V(b) = max_a \left(R(b,a) + \sum_o P(o|b,a) V(b_{ao}) \right)$$

with

$$R(b,a) = \sum_s R(s,a) b(s)$$

  - The optimal action a* is the one that maximizes the above expression.

Pros and cons

- Exact (`belief MDP').
  - Gives **the** optimal policy.
  - Only applicable to fairly small problems.
    - Few actions and observations.
    - Small horizon.

- Approximate (MLS, Q-MDP)
  - Scales to larger problems.
    - Solving the underlying MDP is the hardest.
    - Also selecting the final action can be done on-line.
  - Not optimal:
    - Too positive.
    - Information gaining actions are undervaluated.

Solving for ANY initial belief

- In some cases no initial belief b^0 available.
  - Perform planning for all possible initial beliefs.

- This is possible because of special property of the POMDP value function:
- Piecewise-linear and convex (PWLC)

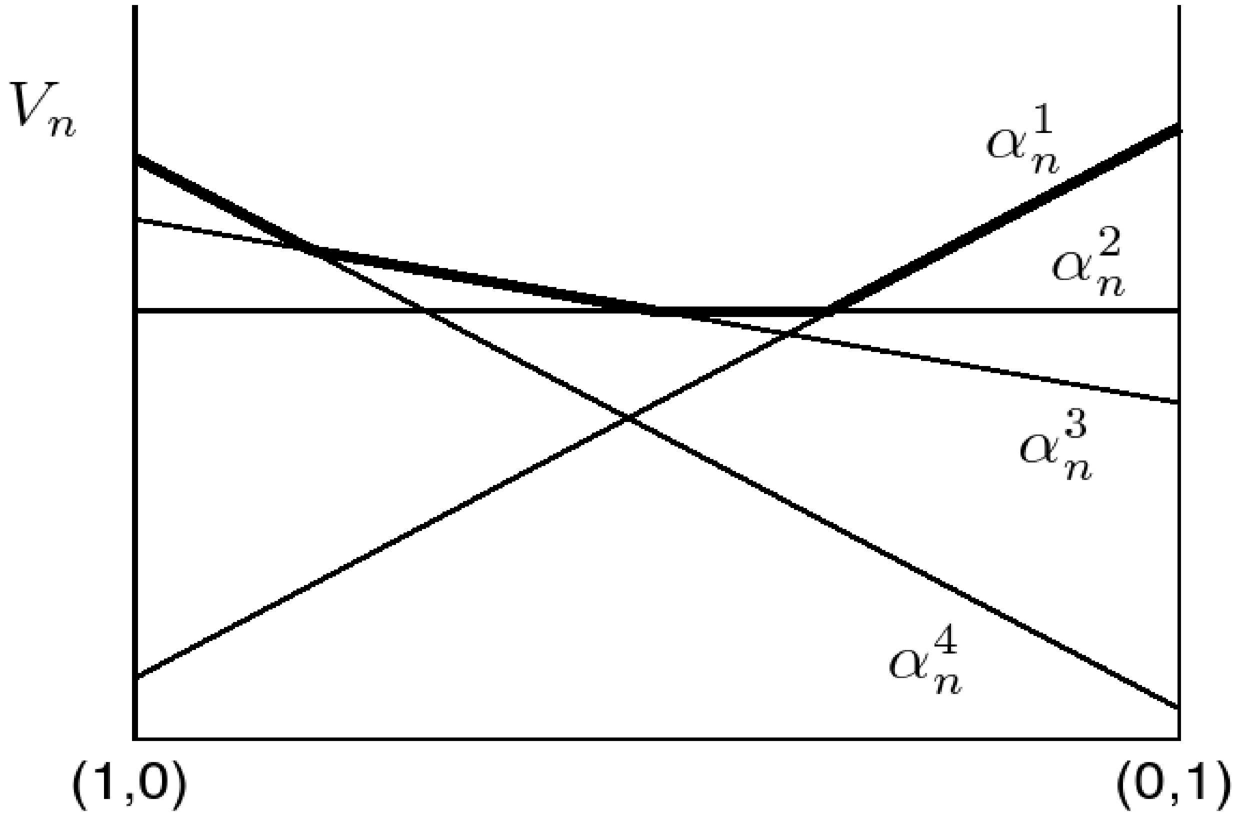
- Like VI for MDPs: use a backup operator H
  - $V_{k+1}=HV_k$
  - inf. horizon: $V^*=HV^*$

$$V(b)=max_a\left(R(b,a)+\gamma\sum_o P(o|b,a)V(b_{ao})\right)$$

PWLC-property

- V_k is PWLC (when k is finite)
  - Can be represented by a set of vectors.

$$V_n(b) = \max_i b \cdot \alpha_n^i$$



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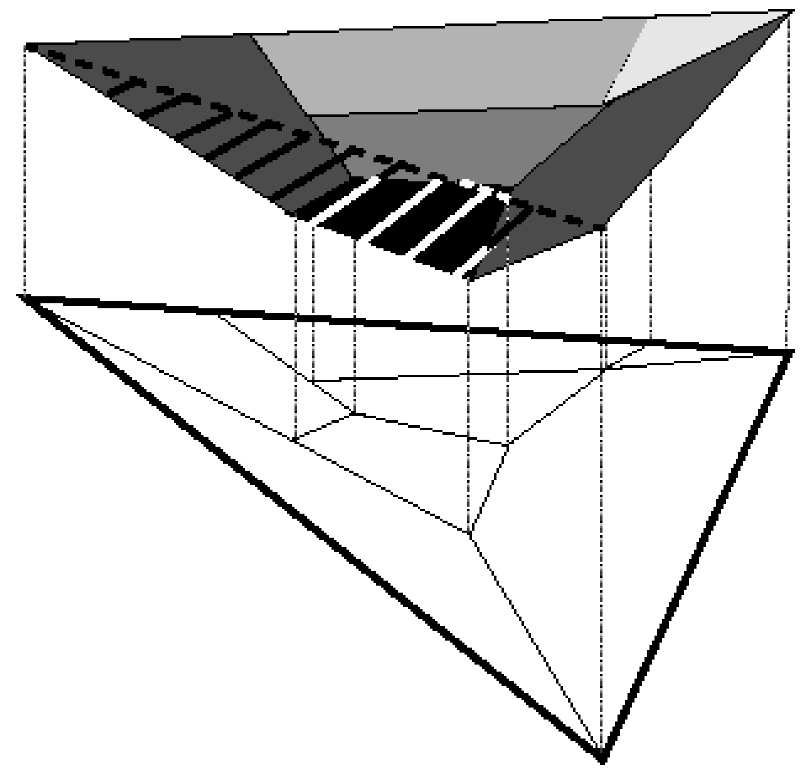
PWLC in 3D

- 3 states

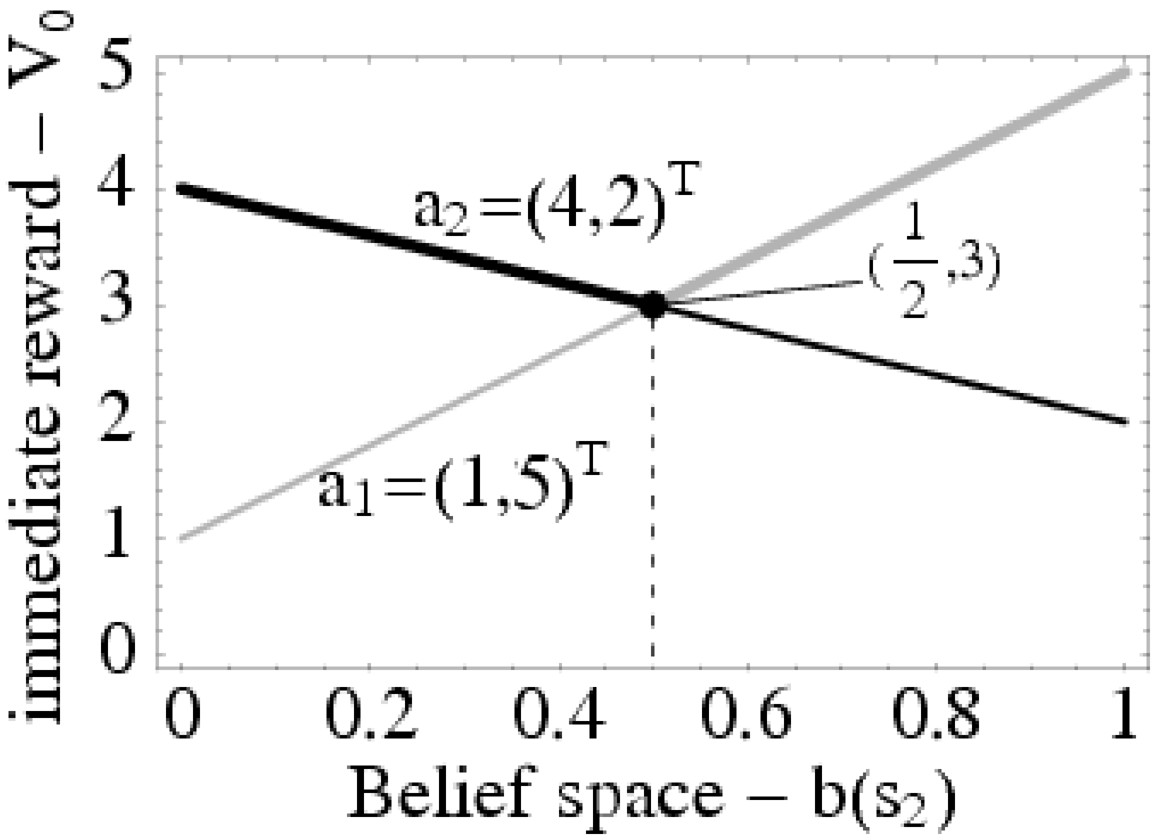


- Generalizes to arbitrary number of states.
  - Although hard to visualize.

A numeric example

- V_0 given by the immediate rewards

$R(s,a)$	a_1	a_2
s_1	1	4
s_2	5	2



Constructing V_{k+1} from V_k

- Basic procedure for a particular belief b
  - for all a
    - $\alpha_{temp} = (0 \ldots\ 0)^T$
    - for all o
      - calculate b_{ao}
      - Select α_{ao} the maximizing vector from V_k at b_{ao}
      - α_{temp} += $P(o|b,a) * \alpha_{ao}$
    - create a new vector: $\alpha_a = R_a + \alpha_{temp}$
  - Select the action that maximizes $\alpha_a . b$

- However, need to do this for all beliefs...
  - Just generate all possible vectors.

Summary

- Planning in a partially observable world.
- In such a setting an agent can maintain a belief over states.
  - using Bayes' Rule
- We considered 3 planning methods for use with an initial belief:
  - Exact: `solving the belief MDP'
  - Approximate: MLS and Q-MDP
- When no initial belief:
  - use PWLC property to generate a value function.
- PWLC property also basis for more advanced algorithms.