6.815 Digital and Computational Photography
6.865 Advanced Computational Photography

Fast Bilateral Filtering & Applications

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Many slides by
Sylvain Paris & Jiawen Chen

Demosaicking pset

- **Don’t forget to convert to double**
- **See how to avoid loops on web page**

Recap: HDR imaging

- **Multiple-exposure HDR capture**
  - calibrate response curve
  - combine multiple exposures
- **Bilateral tone mapping**
  - decompose luminance into large scale & detail
    - use bilateral filter
  - reduce contrast of large scale only
    - preserve detail
    - preserve colors

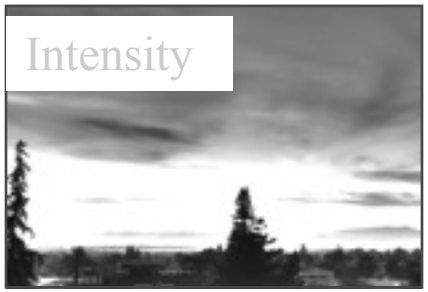
Alternative: exposure fusion

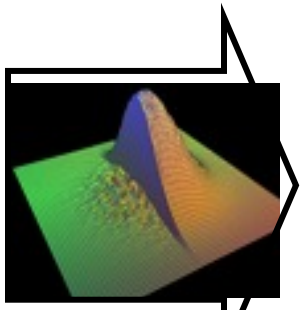
- One single step for both multiple-exposure merging & tone mapping
  - http://research.edm.uhasselt.be/~tmertens/exposure_fusion/

(a) Input images with corresponding weight maps (b) Fused result

Figure 2. Exposure fusion is guided by weight maps for each input image. A high weight means that a pixel should appear in the final image. These weights reflect desired image qualities, such as high contrast and saturation. Image courtesy of Jacques Joffre.

Back to bilateral tone mapping

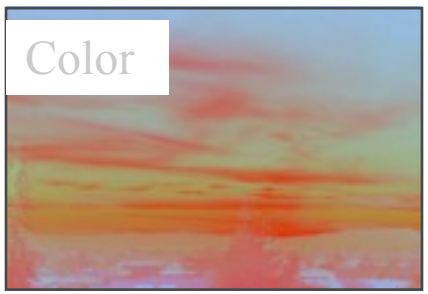


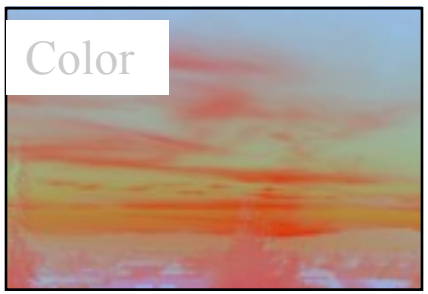


Bilateral Filter

Reduce contrast

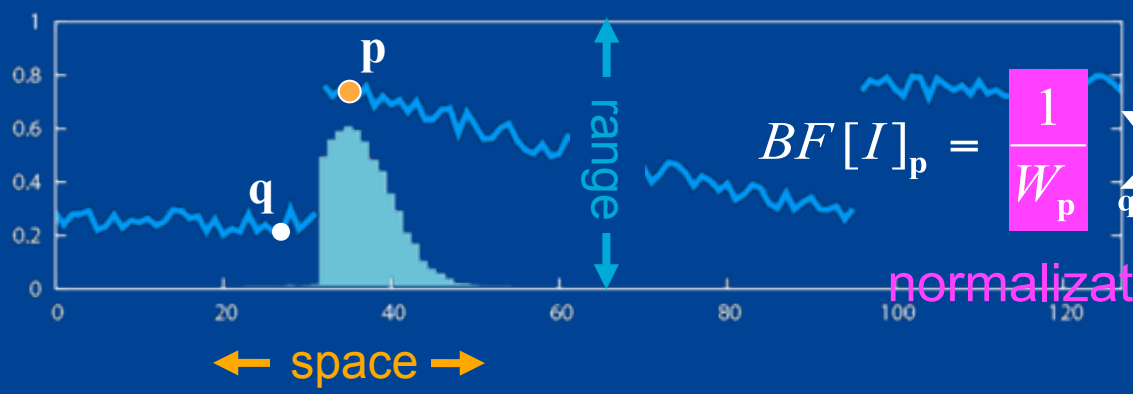
Preserve!





Bilateral Filter: Weighted Average of Pixels

- Depends on spatial distance and intensity difference
  - Pixels across edges have almost no influence



$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|)\, I_{\mathbf{q}}$$

normalization

space

range

Review: Gaussian (Bell curve)

$$G_\sigma(x) = e^{-\frac{x^2}{2\sigma^2}}$$

- σ (standard deviation) determines width
- Can be normalized
  - here, to be 1 at 0
  - or to make area under curve 1 (multiply by $\frac{1}{\sigma\sqrt{2\pi}}$)
- Nice smooth way to have high influence around center and then decrease rapidly beyond

Brute-force Implementation of Bila.

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|) I_{\mathbf{q}}$$

For each pixel **p**

For each pixel **q**

Compute

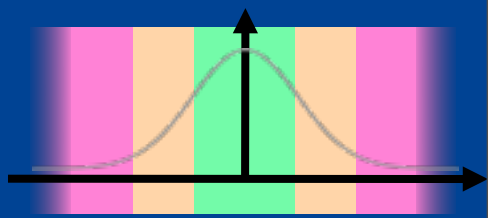
$$G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)\, I_{\mathbf{q}}$$

8 megapixel photo: 64,000,000,000,000 iterations!

V E R Y S L O W !

More than 10 minute per image

Better Brute-force Implementation



Idea: Far away pixels are negligible, truncate the Gaussian

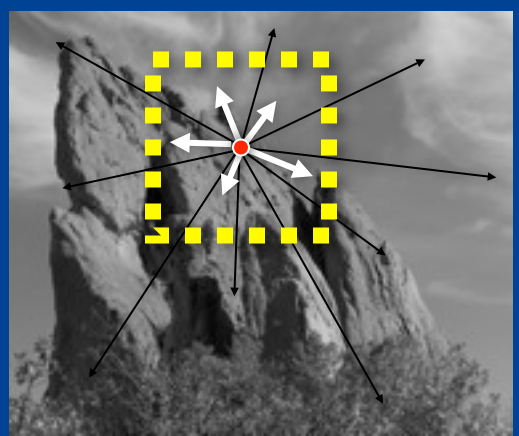
- **usually truncate at 3σ or 4σ**

For each pixel **p**

- For each pixel **q** such that $\| \mathbf{p} - \mathbf{q} \| < constant \times \sigma_s$

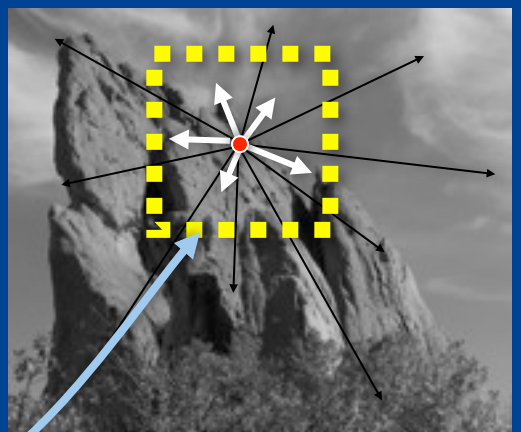
looking at all pixels

looking at neighbors only



Discussion

- Complexity: $\ddot{I}(|S| \times \sigma_s^2)$



neighborhood area

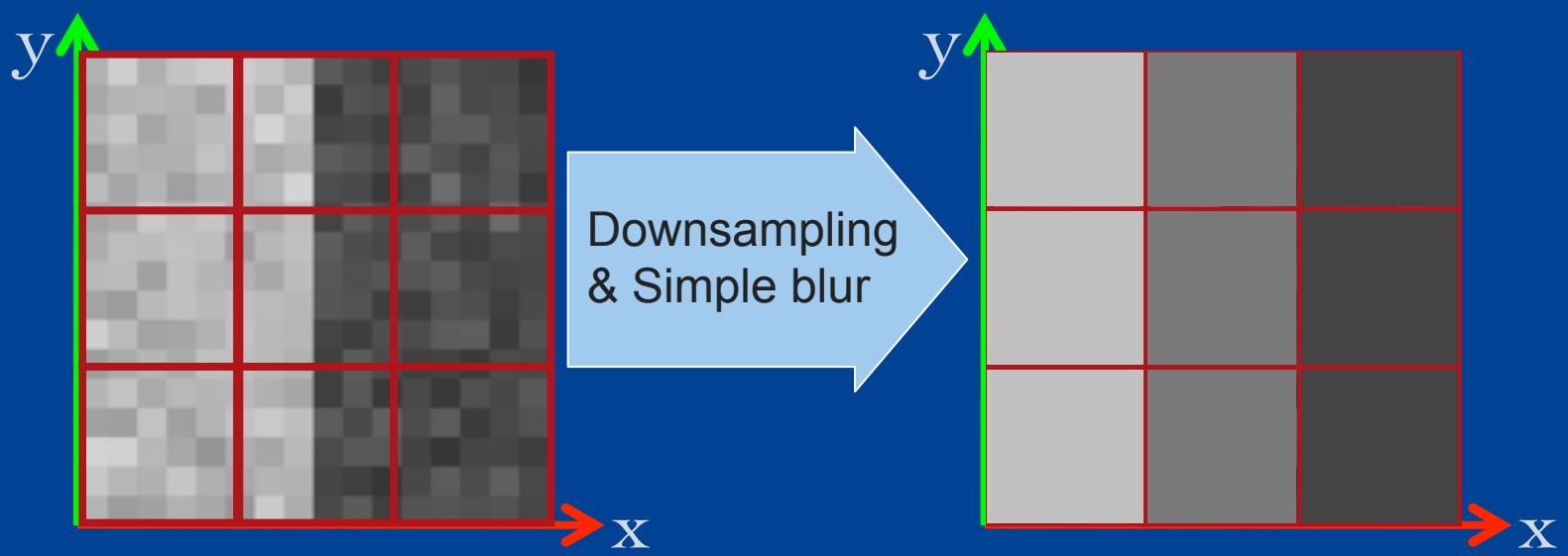
- Fast for small kernels: σ_s ~ 1 or 2 pixels
- BUT: slow for larger kernels

Questions?

Bilateral Grid: basic motivation

[Paris and Durand 06, Chen et al. 07]

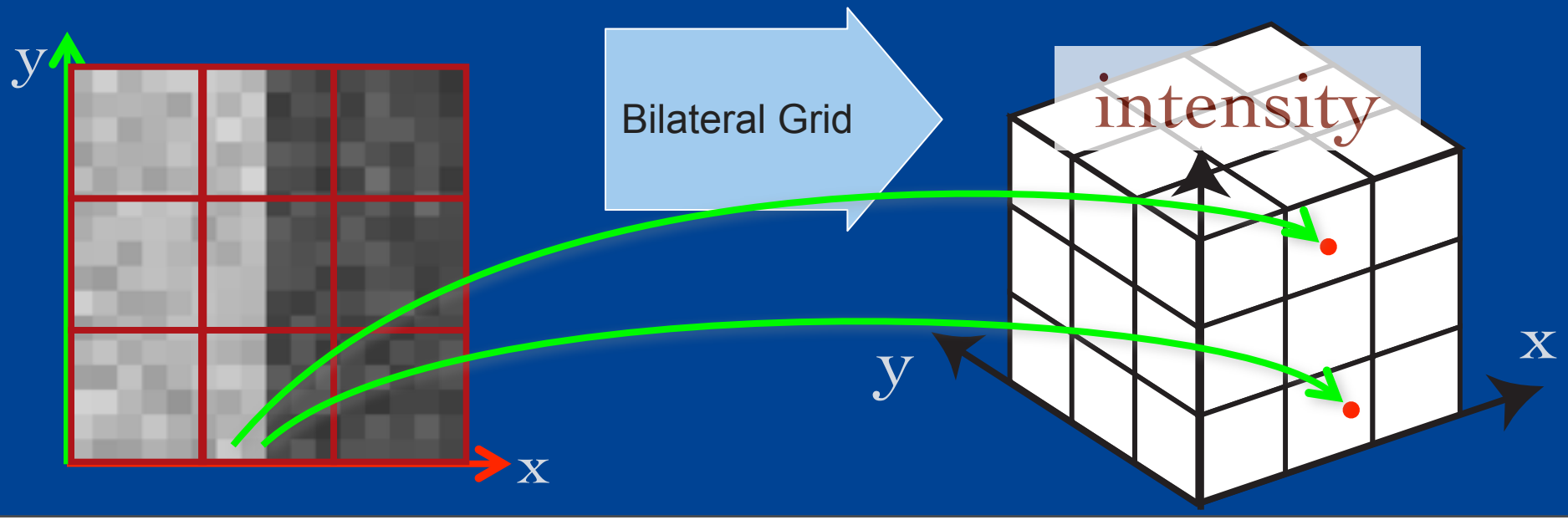
- When we smooth, we reduce complexity of image
  => we should be able to do it at a lower resolution



Bilateral Grid: basic motivation

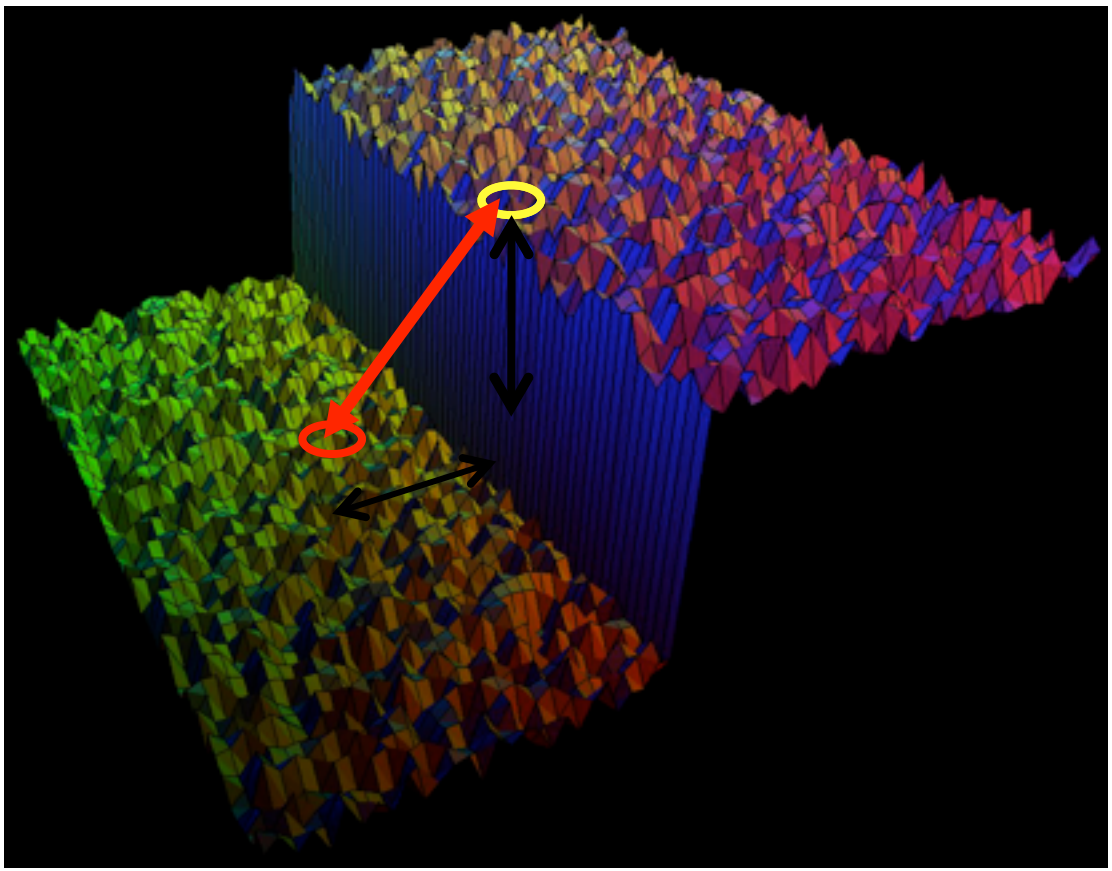
[Paris and Durand 06, Chen et al. 07]

- When we smooth, we reduce complexity of image => we should be able to do it at a lower resolution
- However, the bilateral filter preserves sharp edges and a low resolution image does not
- Idea: add a 3rd dimension to the image so that intensity difference are handled well



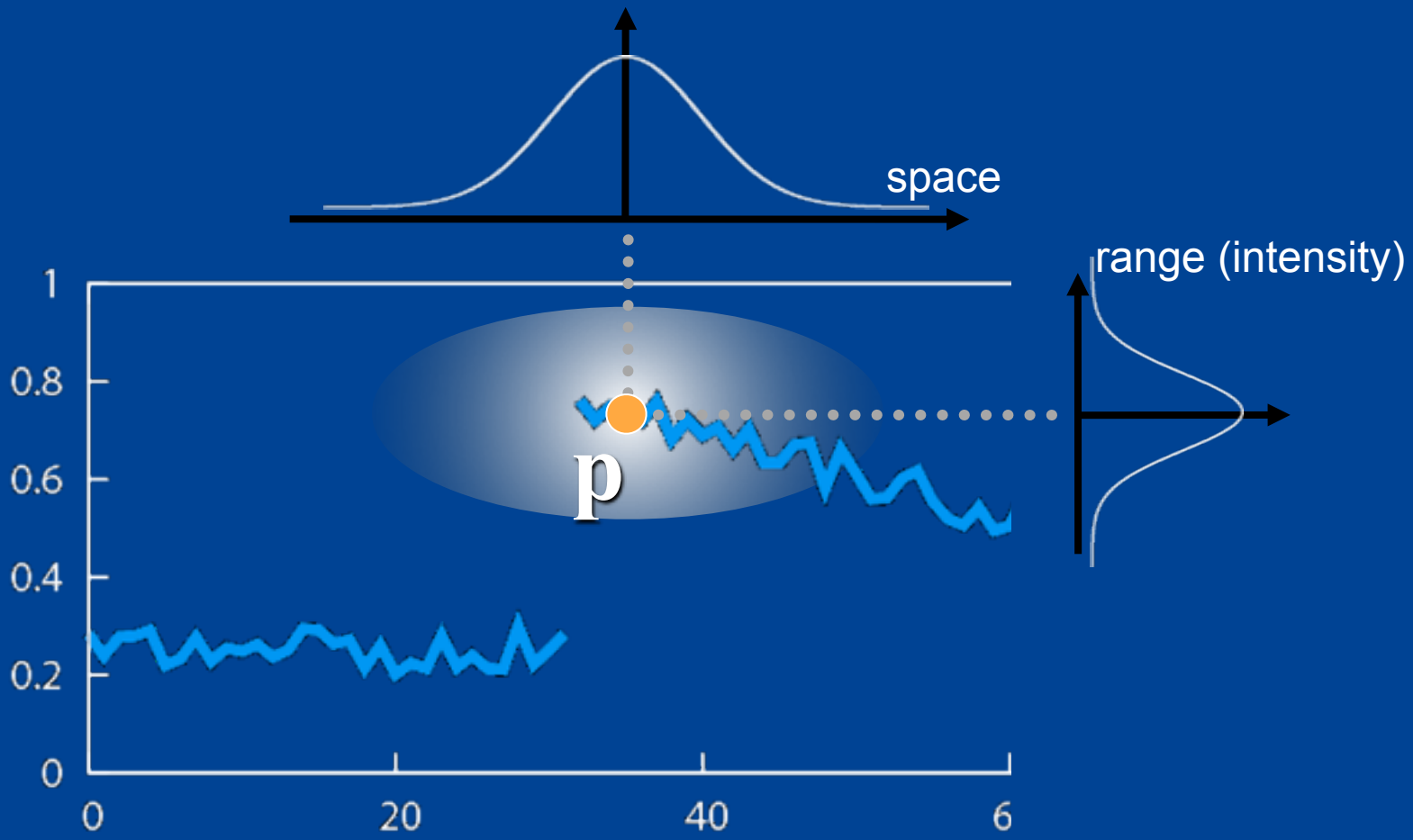
Recall other view

- The bilateral filter uses the 3D distance
- With the bilateral grid, this becomes a 3D blur



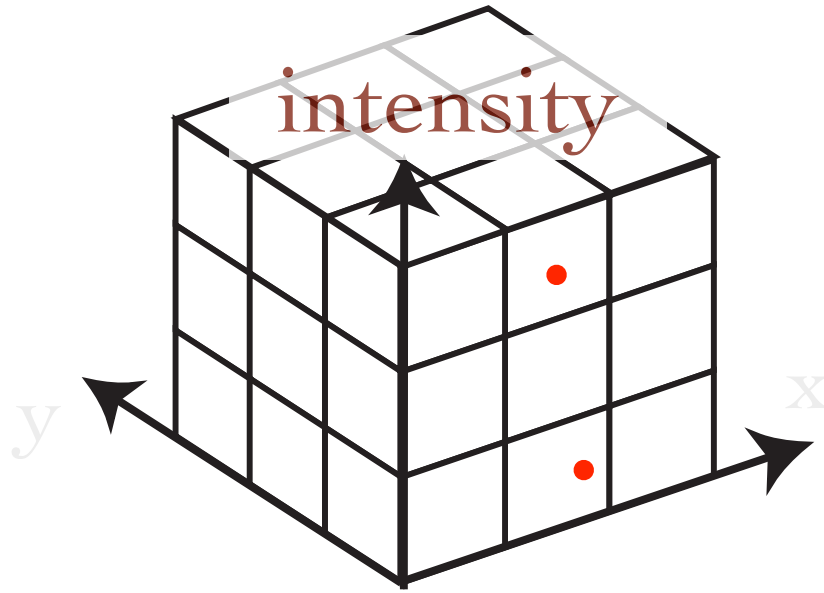
Bilateral Grid [Paris and Durand 06, Chen et al. 07]

- Idea 2.0: The product of spatial and intensity Gaussian defines a 3D Gaussian in x, y, I

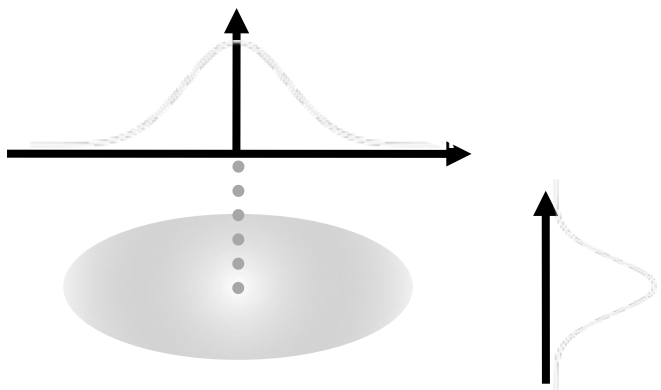


Fast bilateral filter idea

- **Represent image in low-resolution 3D grid**



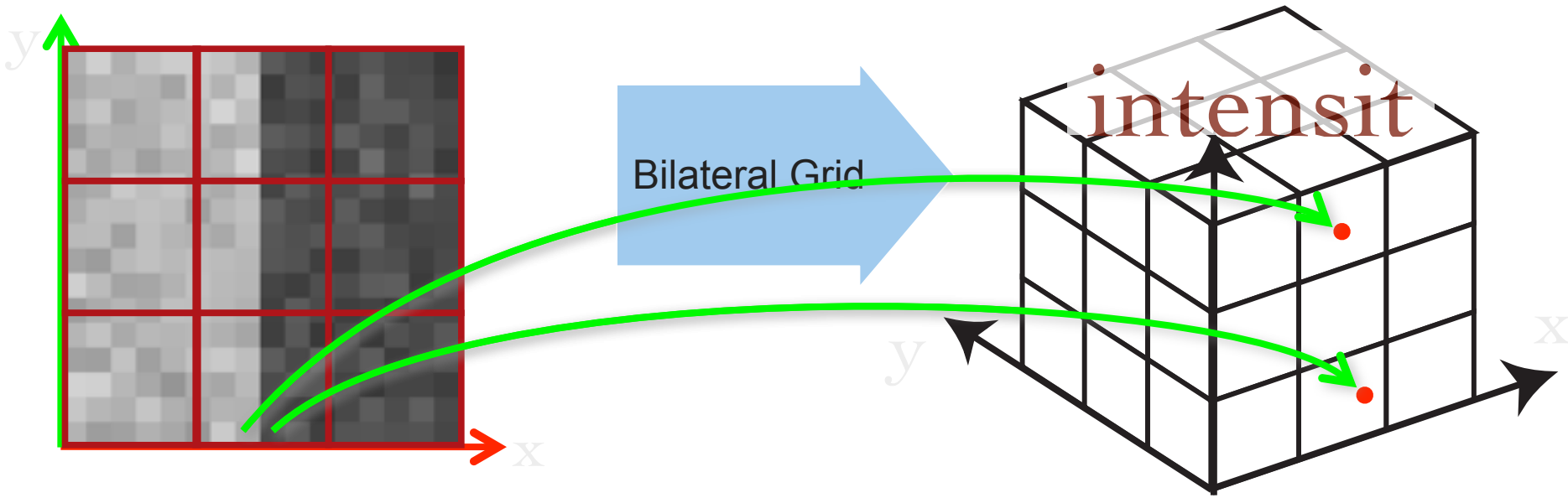
- **3D blur combines space and intensity terms**



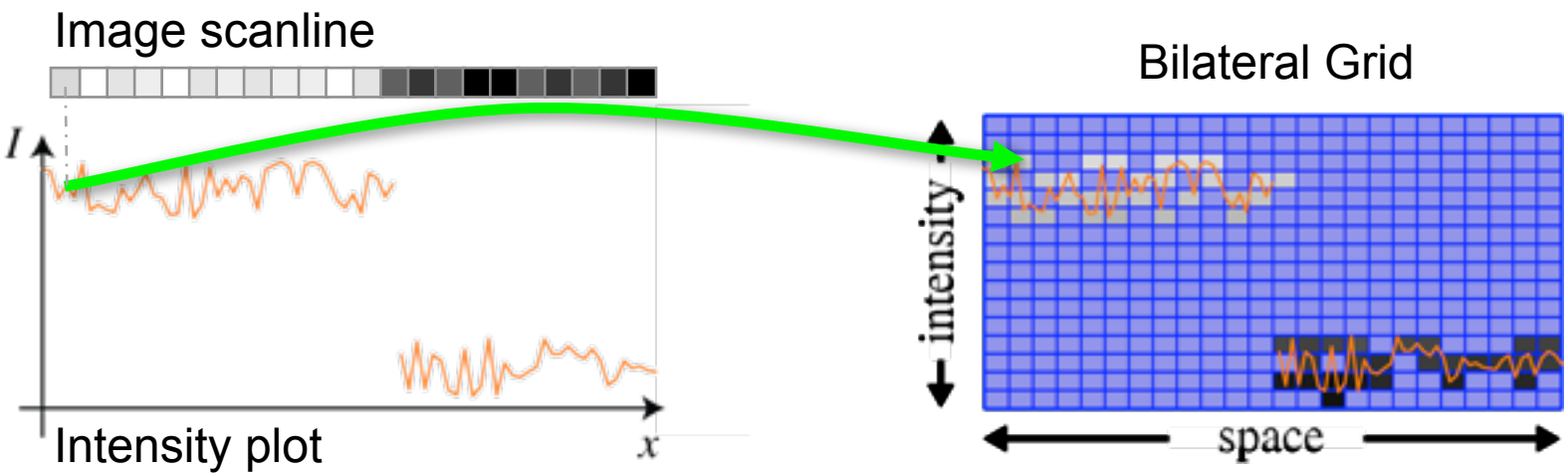
Questions?

Overview

- **Convert image to bilateral grid**
  - (x, y) pixel goes to x, y, I(x,y)
- **Blur the grid**
  - 3D Gaussian combines 2D x,y term f and I term g
- **Convert back to 2D image space**

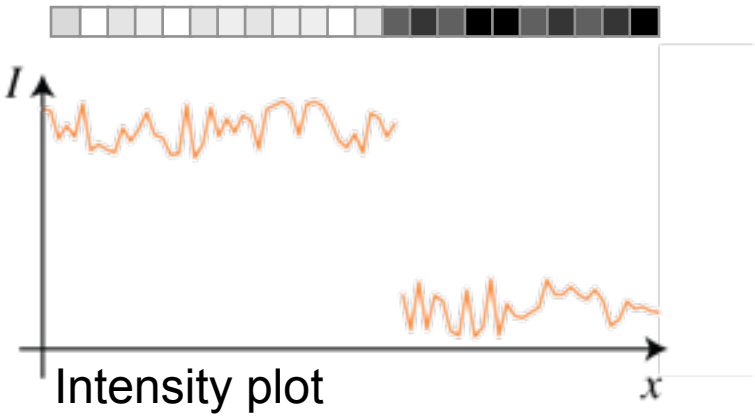


Bilateral Filter on the Bilateral Grid

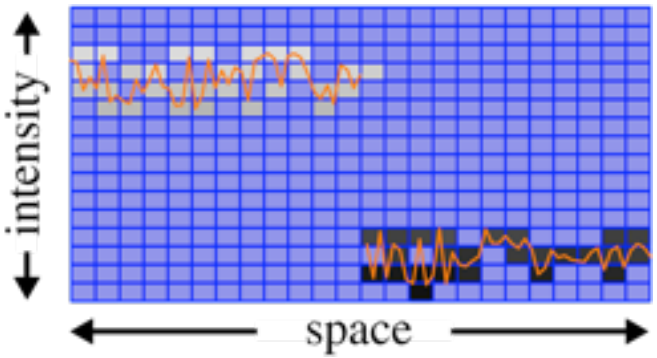


Bilateral Filter on the Bilateral Grid

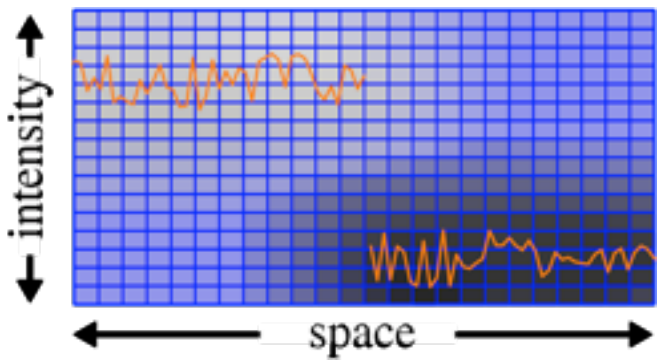
Image scanline



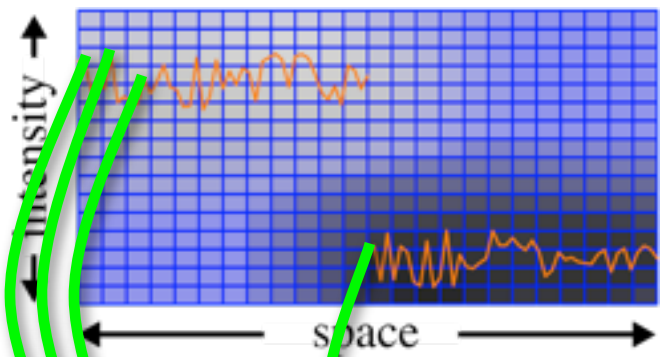
Bilateral Grid



Blurred bilateral grid (3D blur)



Blurred bilateral grid

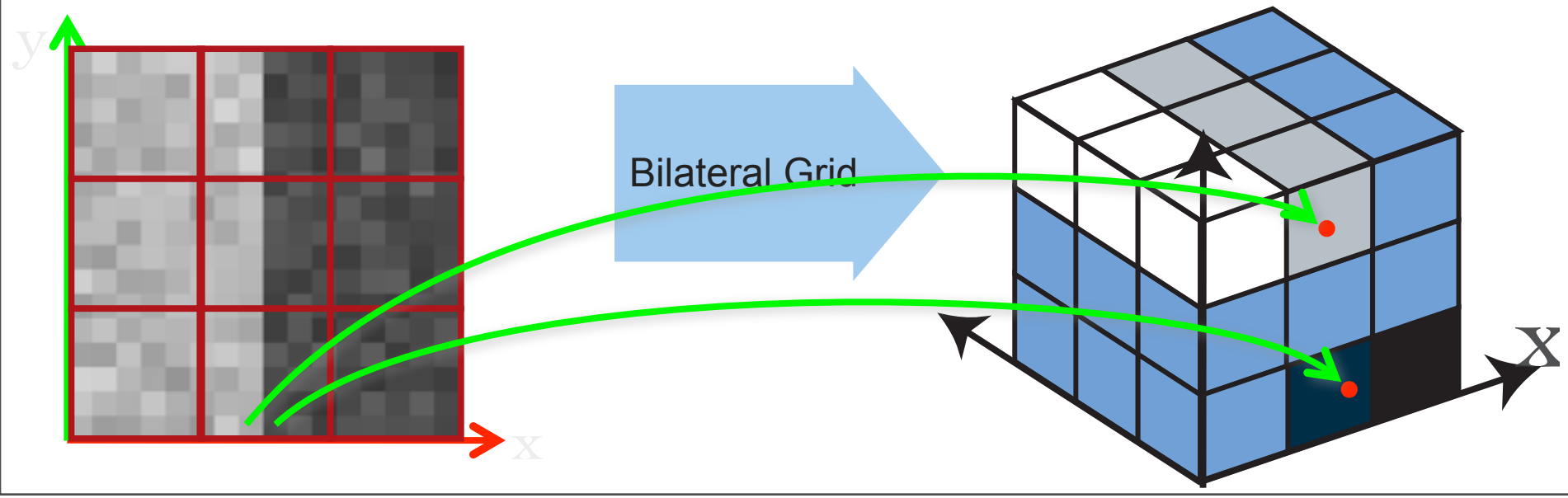


Query grid with input image

Filtered scanline

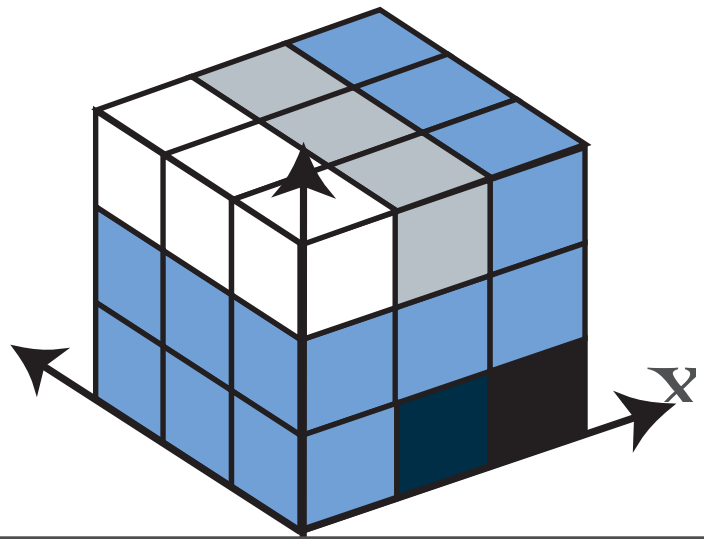
Grid creation

- **Convert image to bilateral grid**
  - (x, y) pixel goes to x, y, I(x,y)
- **Note that not all grid cells receive the same # of pixels**
  - empty cells shown in blue here
  - store a weight to keep track of #pixel
  - will give us normalization factor k in bilateral filter



Bilateral Grid data structure

- **3D array indexed by x, y, intensity**
- **Each cell stores**
  - a value (either RGB or just intensity)
  - a weight (keeps track of #pixels)
- **Resolution depends on application**
  - For bilateral filter, depends on σ_s and σ_r (σ should be ~ the width of a cell)



Implementation details

- **Probably a good idea to have helper functions to index grid directly from image x, y and I**
  - i.e. do the downsampling with appropriate scale factors (here σ_s and σ_r)

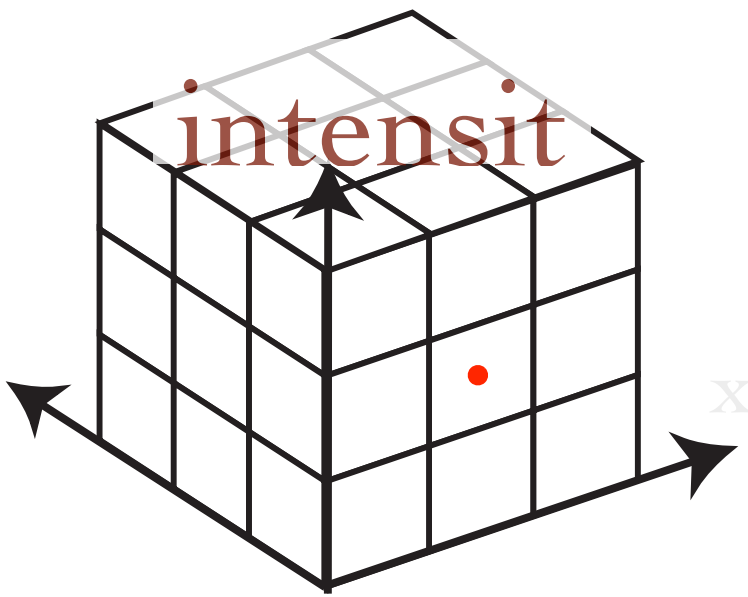
$$x, y, I \rightarrow \left[\frac{x}{\sigma_s}\right], \left[\frac{y}{\sigma_s}\right], \left[\frac{I}{\sigma_r}\right]$$

where [] denotes integer truncation

Blurring the grid

- **Same as in 2D**
- **Each cell replaced by Gaussian-weighted average of neighbors**
  - if v is the value in grid, the blurred output b is:

$$b(x,y,i) = \sum_{x',y',i'} G_{\sigma_f}(x-x')G_{\sigma_f}(y-y')G_{\sigma_g}(i-i')v(x,y,i)$$

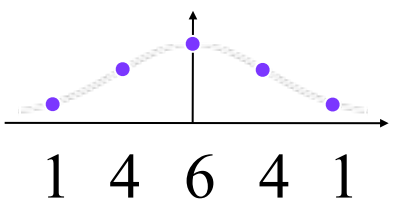


Even smarter: separable

- **Blur one axis at a time**
- **works because our blurring kernel is separable (defined as product along axes)**
- **e.g. blur along x axis:**

$$b(x, y, i) = \sum_{x'} G_{\sigma_s}(x - x')v(x, y, i)$$

- **If we have chosen sf = 1 cell width, then the Gaussian is simply [1 4 6 4 1]/16**



- JUSTIFY SEPARABLE

Blurring

- **Blur BOTH the values and the weights**
- **Recall original bilateral filter formulas**

$$J(x) = \frac{1}{k(x)} \sum_{\xi} f(x,\xi) \quad g(I(\xi) - I(x)) \quad I(\xi)$$

$$\mathrm{k(x)} = \sum_{\xi} f(x,\xi) \quad g(I(\xi) - I(x))$$

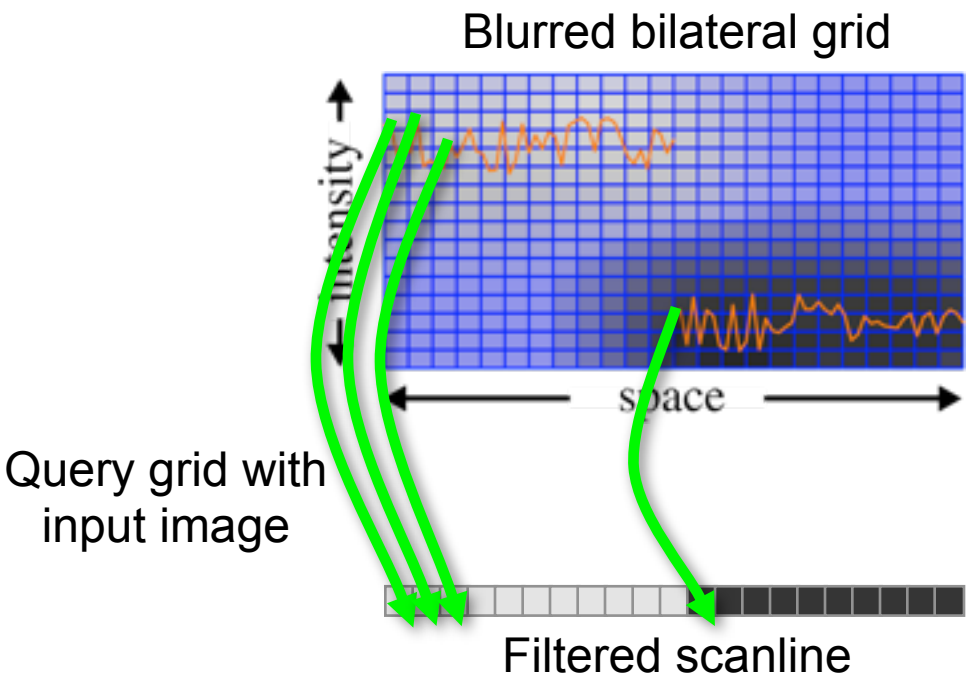
[Tomasi and Manduchi 1998]

- **k(x)=** $\sum_{\xi} f(x,\xi) \quad g(I(\xi) - I(x))$

$$J(x) = \frac{1}{k(x)} \sum_{\xi} f(x,\xi) \quad g(I(\xi) - I(x)) \quad I(\xi)$$

Slicing: critical step

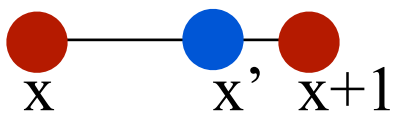
- **Read the grid at locations specified by input image**
  - Output at pixel x, y, is read from grid cell x, y, I(x, y)
- **Trilinear reconstruction**
  - because the grid is downsampled



Linear reconstruction

- **Say we only have values v at integer x**
- **We want to reconstruct at real-valued x'**
- **Linear reconstruction:**

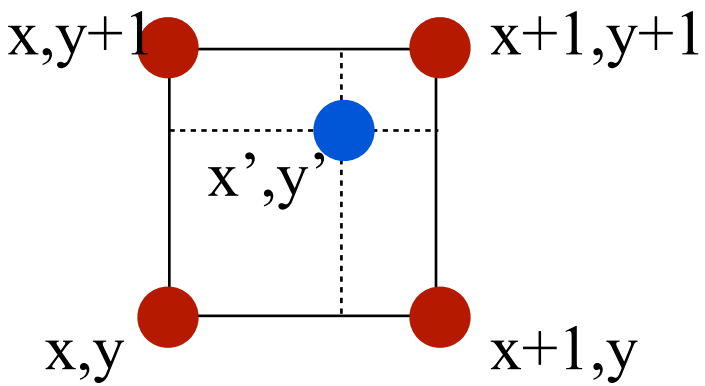
$$(1-x'+x)v(x, y)+(x'-x)v(x+1, y)]$$



BiLinear reconstruction

- **Say we only have values v at integer x & y**
- **We want to reconstruct at real-valued x', y'**
- **Bilinear reconstruction:**

$$(1-y'+y)[(1-x'+x)v(x, y)+(x'-x)v(x+1, y)] +$$
$$(y'-y)[(1-x'+x)v(x, y+1)+(x'-x)v(x+1, y+1)]$$



Bilinear: order does not matter

- **Linear along x followed by linear along y**

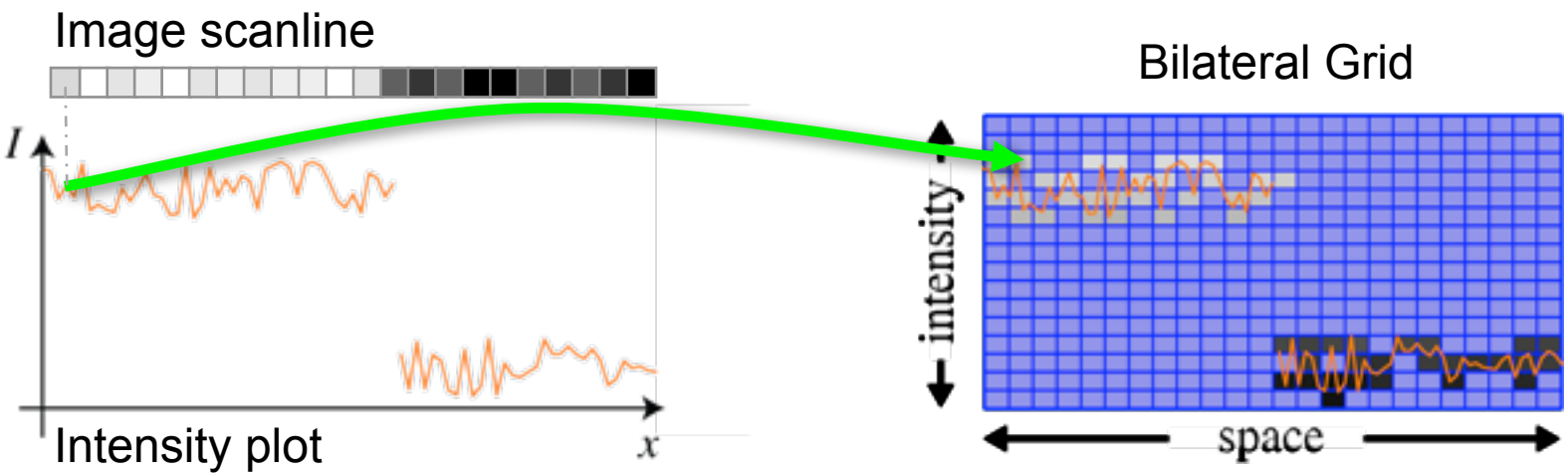
$$(1-y'+y)[(1-x'+x)v(x, y)+(x'-x)v(x+1, y)] +$$
$$(y'-y)[(1-x'+x)v(x, y+1)+(x'-x)v(x+1, y+1)]$$

- **Linear along y followed by linear along x**

$$(1-x'+x)[(1-y'+y)v(x, y)+(y'-y)v(x, y+1)] +$$
$$(x'-x)[(1-y'+y)v(x+1, y)+(y'-y)v(x+1, y+1)]$$

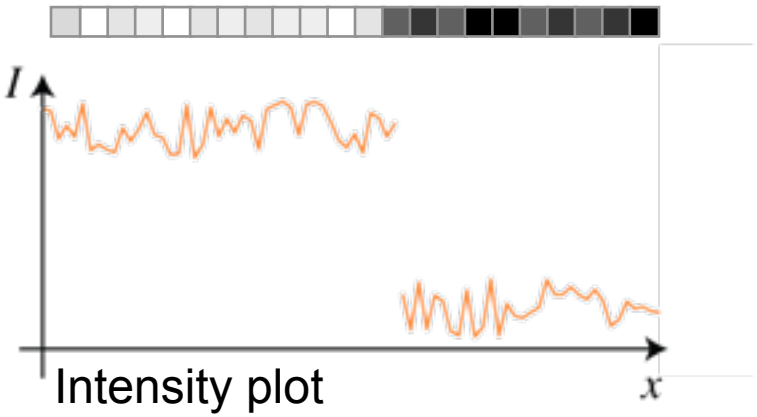
- **Reduces to the same terms**

Bilateral Filter on the Bilateral Grid

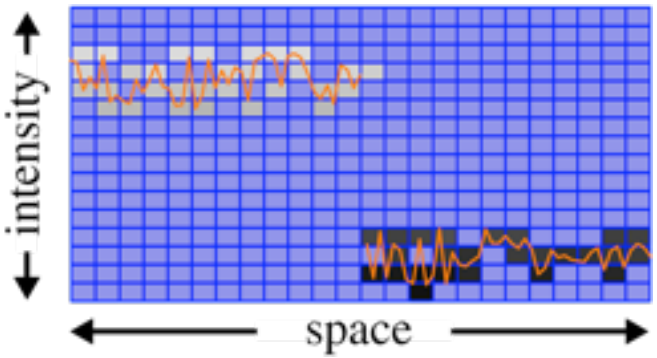


Bilateral Filter on the Bilateral Grid

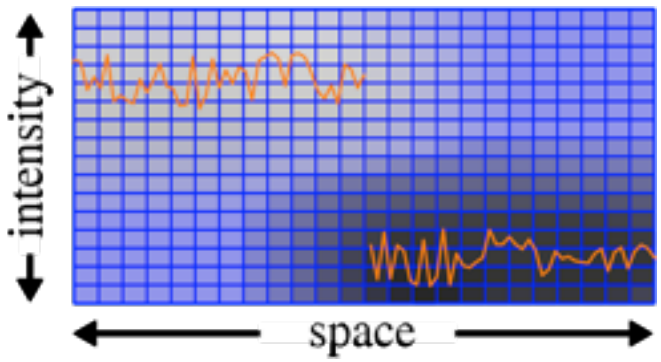
Image scanline



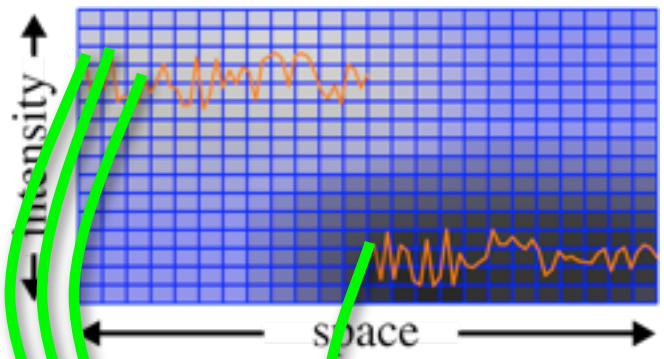
Bilateral Grid



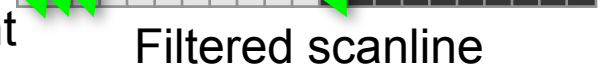
Blurred bilateral grid: both values & weights



Blurred bilateral grid



Query grid with
input image: output =
blurred value/blurred weight



Pseudo code

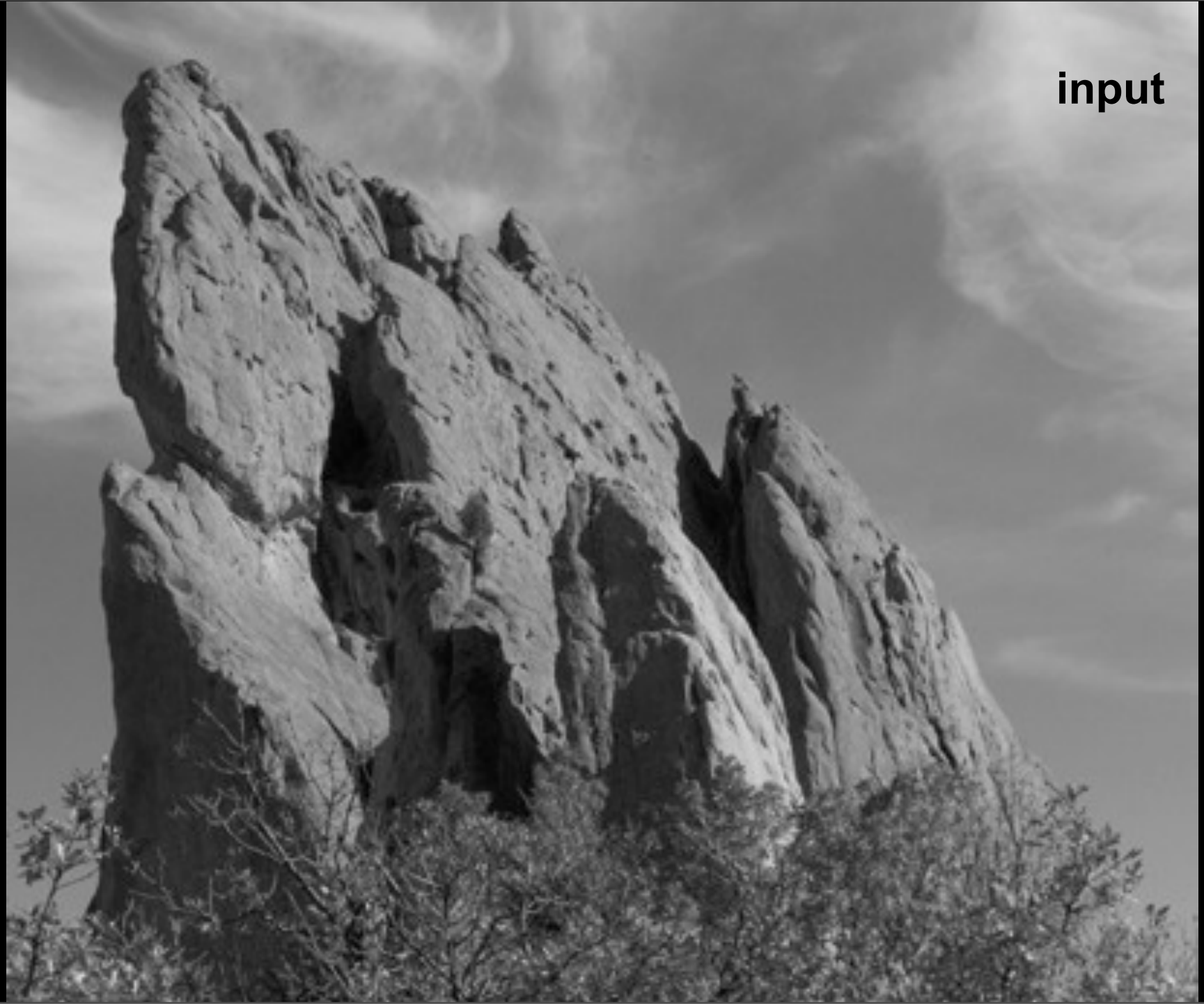
```
For each pixel x, y
   add I(x,y) to grid cell x/σ, y/σ, I(x,y)/σ
   add 1 to weight of grid cell x σ, y σ, I(x,y) σ
Blur values & weights along X axis
Blur values & weights along Y axis
Blur values & weights along I axis
For each pixel x, y
   output = value(x/σ, y/σ, I(x,y)/σ)
            / weight(x/σ,y/σ,I(x,y)/σ)
```

Grid creation

Grid Blur

Slicing

Questions?


input


brute-force
implementation


bilateral grid
visually similar

Performance

Image size: 2 MPixels

- Brute force: 10 minutes
- CPU Bilateral grid: **1 second**
- GPU bilateral grid
  - 2004 card (NV40): **28 ms** (36 Hz)
  - 2006 card (G80): **9 ms** (111 Hz)

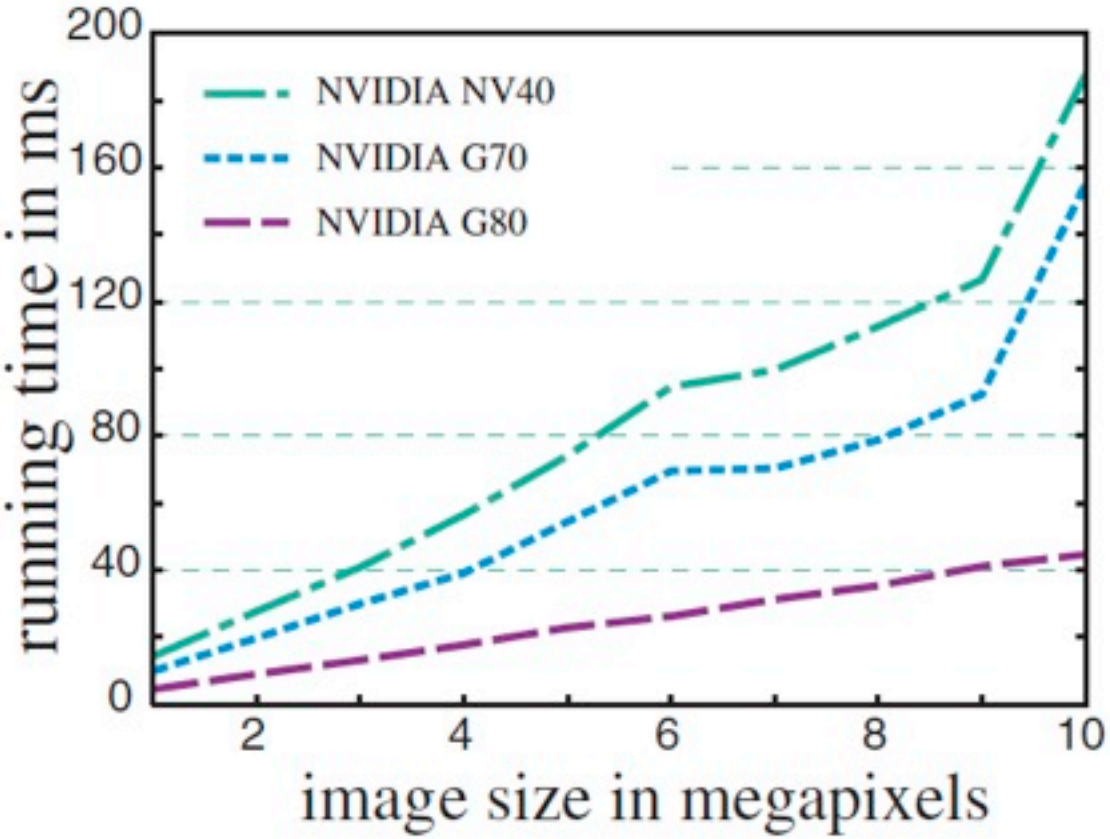


Figure 4: *Bilateral filter running times as a function of the image size (using $\sigma_s = 16$ and $\sigma_r = 0.1$). The memory requirements increase linearly from 625 kB at 1 megapixel to 6.25 MB at 10 megapixels.*

Discussion

number of pixels

number of 3D cells

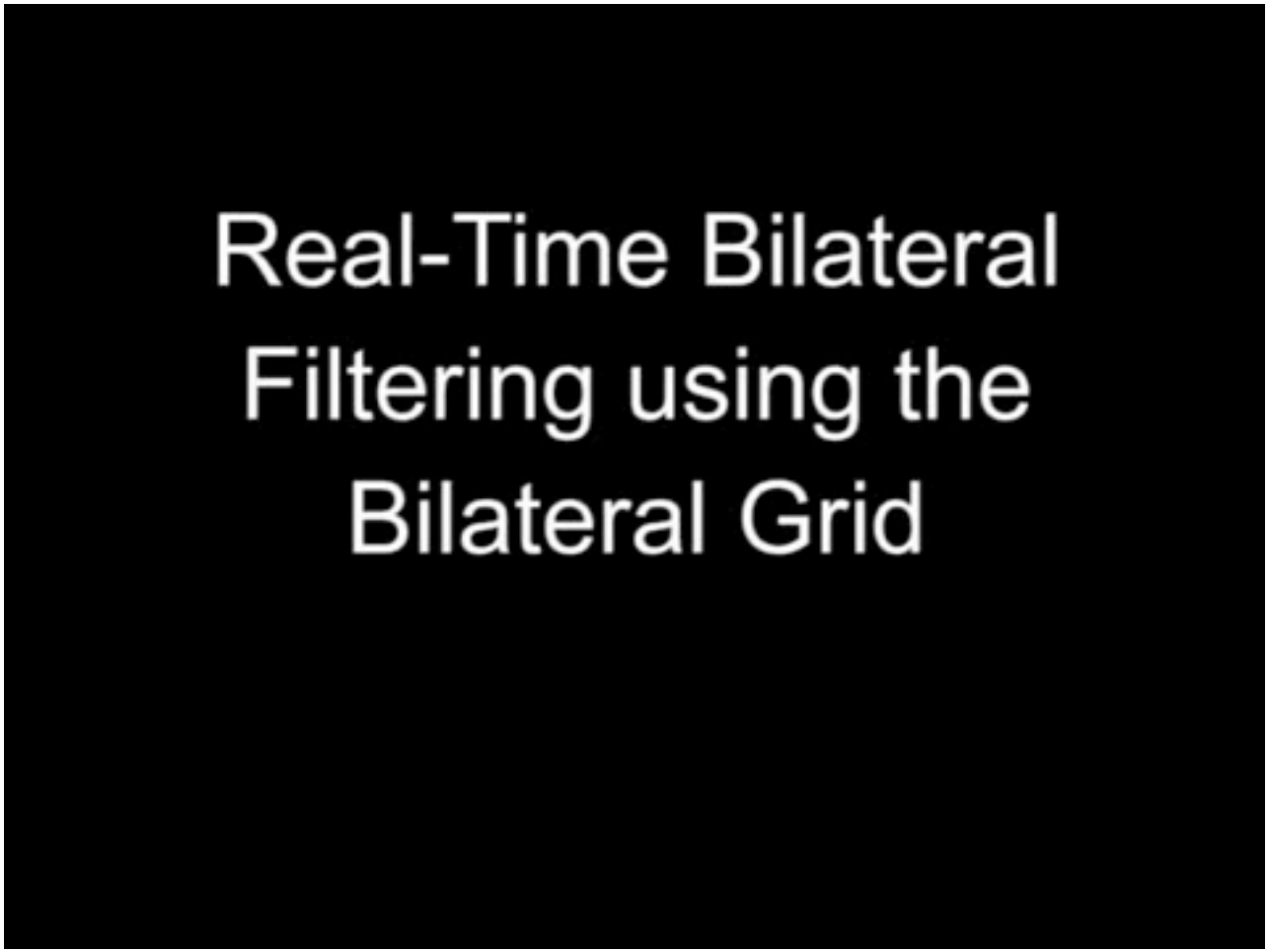
- Complexity: $O\left(|S| + \frac{|S|}{\sigma_s^2}\frac{|R|}{\sigma_r}\right)$

$|R|$: number of gray levels

- Fast for medium and large kernels
  - Can be ported on Graphics Processing Units (graphics cards) [Chen 07]: always very fast
- Can be extended to color images but slower
- Visually similar to brute-force computation

Surprising behavior

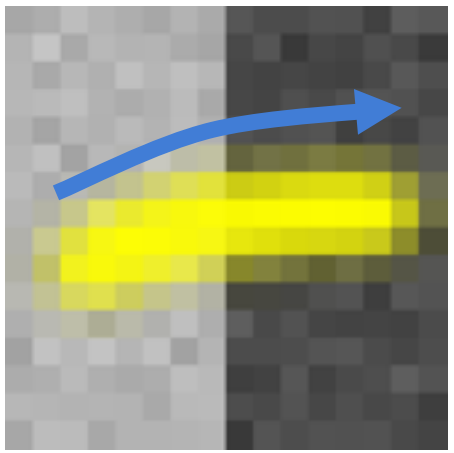
- Faster when spatial footprint gets bigger
  - Because we can downsample more aggressively


Real-Time Bilateral
Filtering using the
Bilateral Grid

More bilateral grid operations

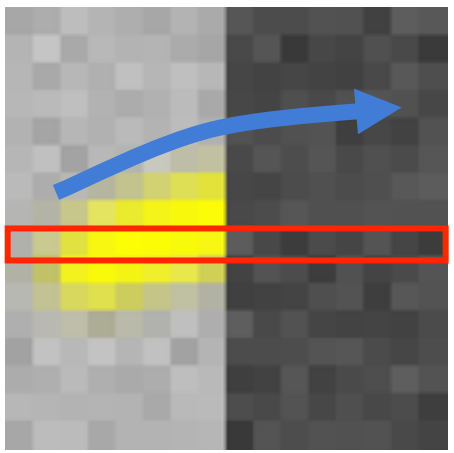
Edge-aware brush

- Classical paint brush
  - Ignores edges



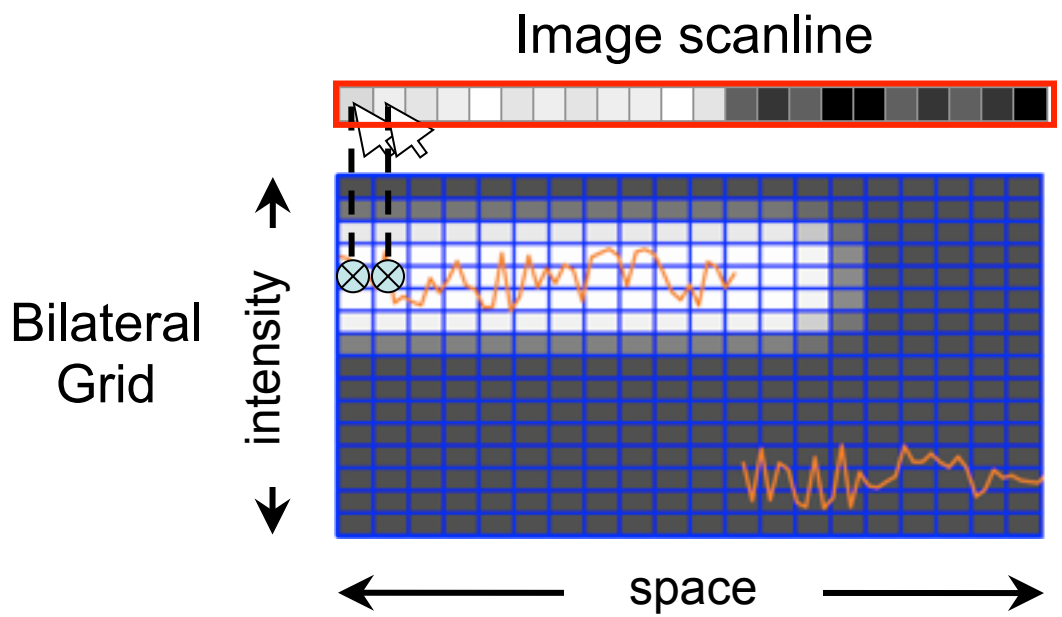
Stroke with classical brush

- Our edge-aware brush
  - Respects edges



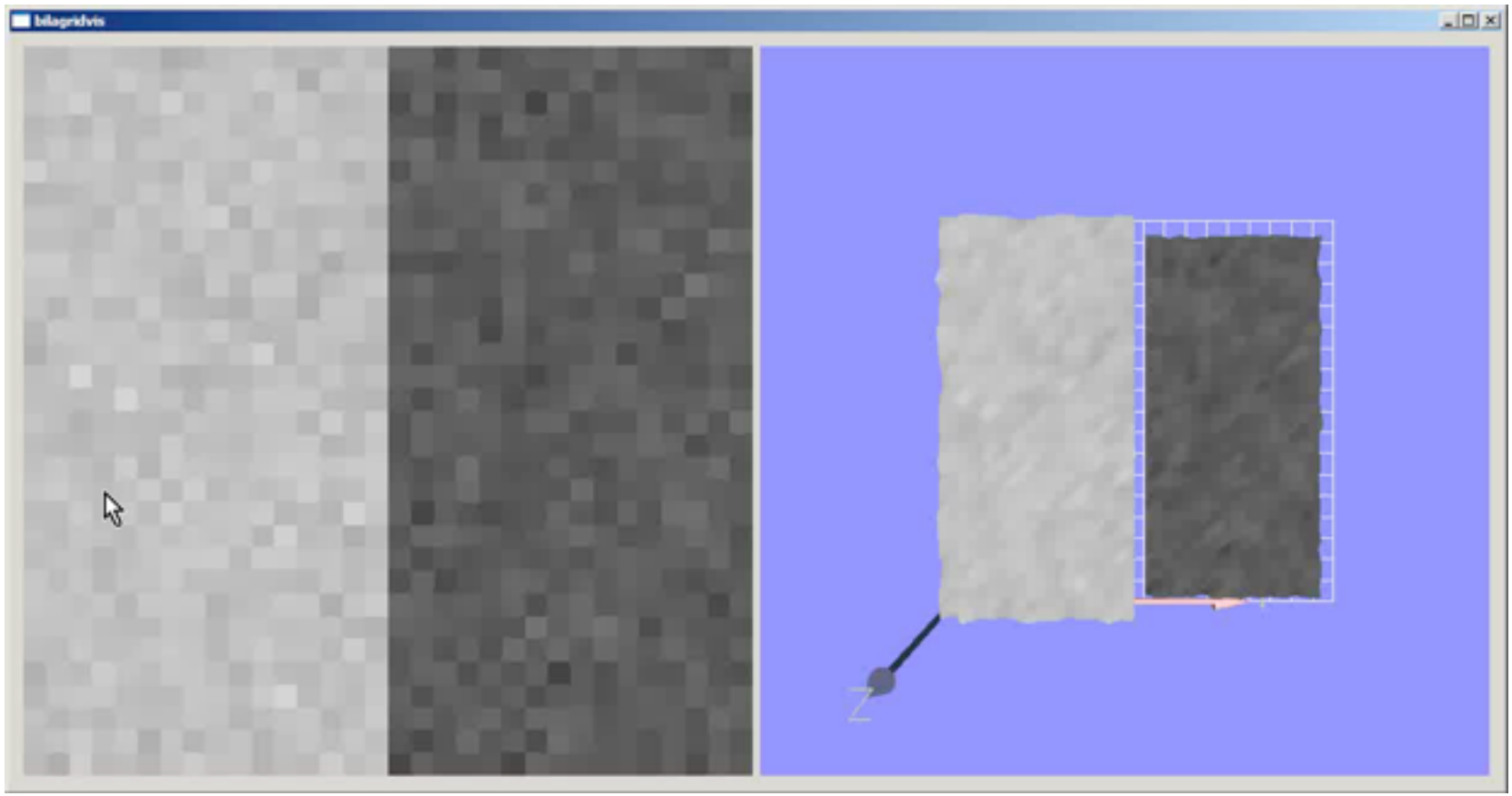
Stroke with bilateral brush

Bilateral Grid Painting



Bilateral Grid Painting

- When mouse is held down, paint only at intensity level of initial mouse click

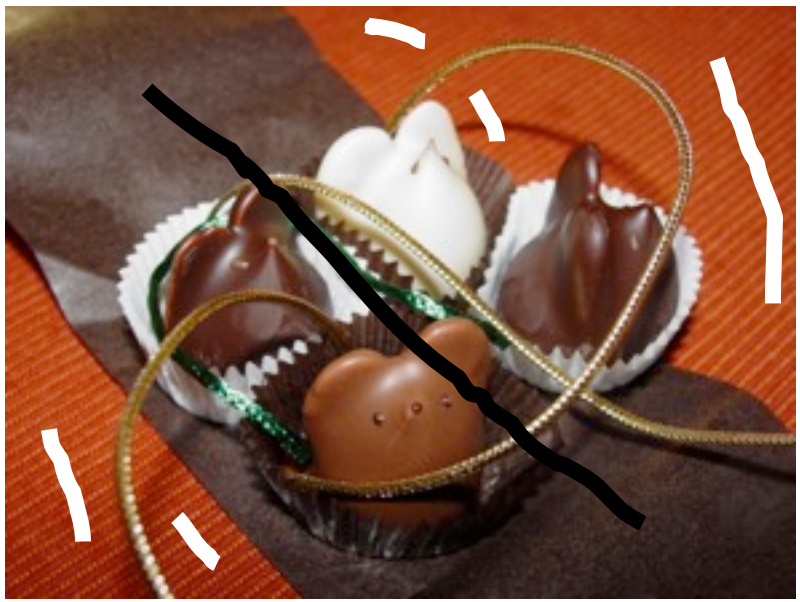


Input image

Bilateral Grid

Scribble-based Selection

- User scribbles to specify selection [Lischinski 06]
- Piecewise-smooth interpolation to get full selection
  - Respects intensity discontinuities

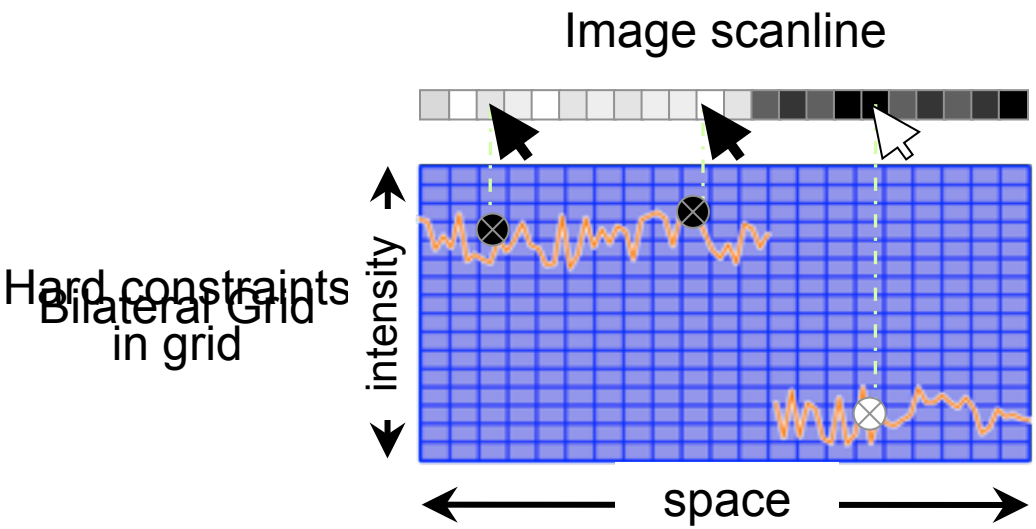


Input image

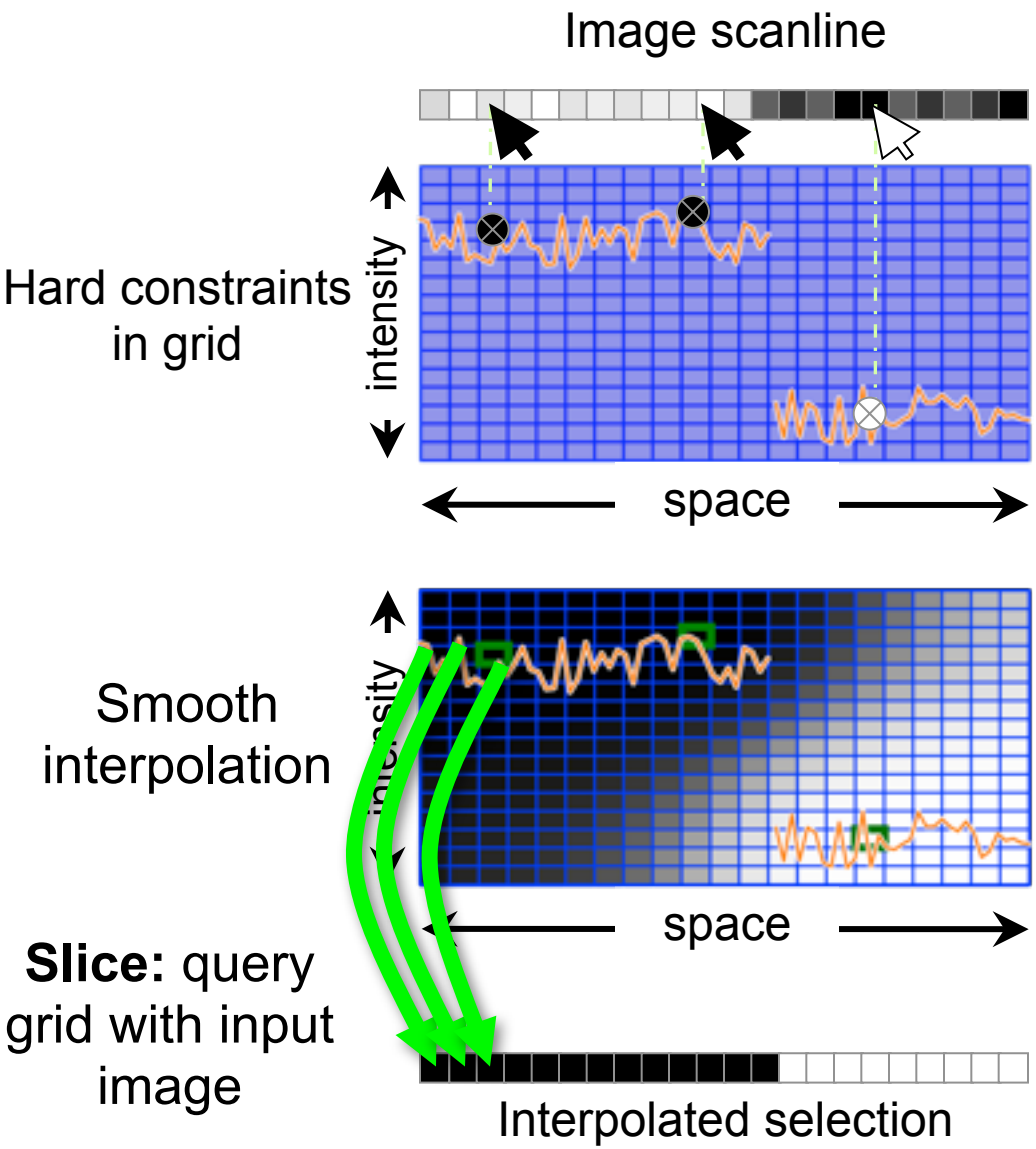
Input with scribbles

Our interpolated selection

Scribble-based Selection



Scribble-based Selection

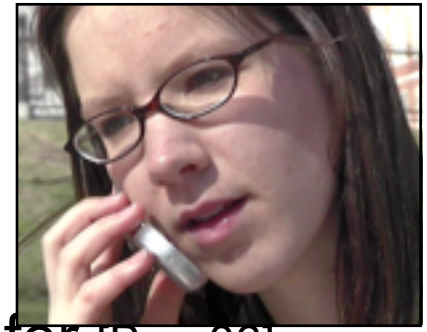


More about the use of scribbles & optimization next week

Many Operations and Applications

- Local histogram equalization
- Interactive tone mapping

- Video abstraction
  [Winnemoller 06, DeCarlo 02]



- Photographic style transfer [Bae 06]

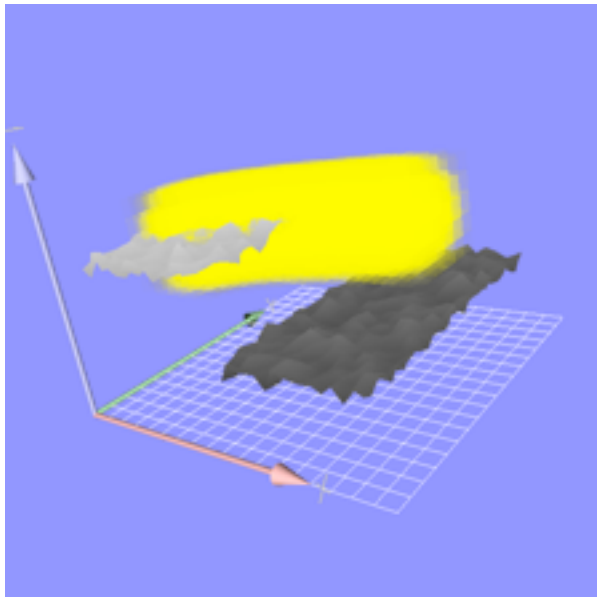
Questions?

Discussion

- Respects luminance edges
- Color bilateral grid would be 5D
  - high memory cost
  - Might not fit on current graphics hardware
  - Luminance edges are often sufficient

Bilateral brush crosses thin lines

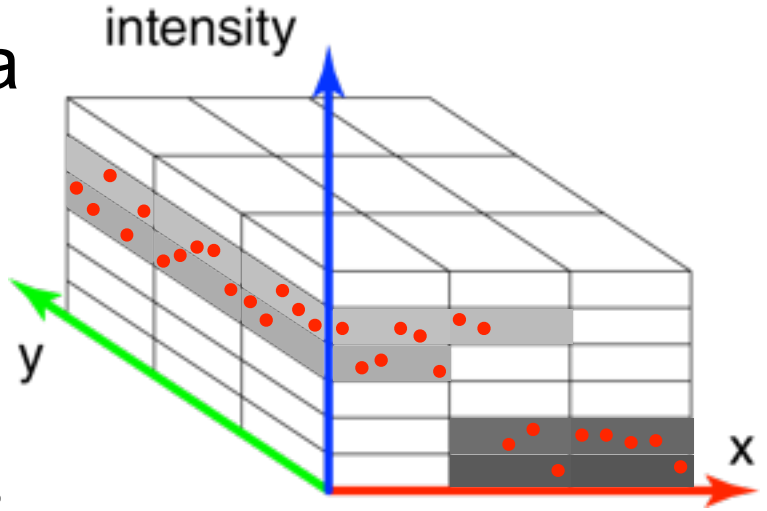
- Grid resolution depends on the operator
  - E.g., for edge-aware brush:
    space sampling rate ~ brush radius
    intensity sampling rate ~ edge-awareness



Edge-aware brush

Summary: the Bilateral Grid

- 3D representation for 2D data
- Intelligent downsampling
- Many edge-aware operations
  - Painting, scribble interpolation, bilateral filter, local histogram equalization
- Real-time for HD video



Refs

- A Gentle Introduction to Bilateral Filtering and its Applications
  - http://people.csail.mit.edu/sparis/bf_course/
- A Fast Approximation of the Bilateral Filter using a Signal Processing Approach. Sylvain Paris and Frédo Durand. International Journal of Computer Vision (IJCV'09)
  - http://people.csail.mit.edu/sparis/publi/2009/ijcv/Paris_09_Fast_Approximation.pdf
- Real-time Edge-Aware Image Processing with the Bilateral Grid. Jiawen Chen, Sylvain Paris, Frédo Durand. In ACM Transactions on Graphics (Proceedings of the ACM SIGGRAPH 2007 conference)
  - http://groups.csail.mit.edu/graphics/bilagrid/
- Fast Bilateral Filtering for the Display of High-Dynamic-Range Images. Frédo Durand and Julie Dorsey. SIGGRAPH 2002
  - http://people.csail.mit.edu/fredo/PUBLI/Siggraph2002/

Questions ?

Alternative

- Merge exposures & tone map in one single step
- Burt

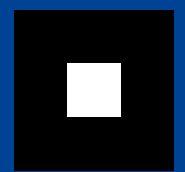
EXTRA MATERIAL

- Another acceleration technique

Box Kernel [Weiss 06]

- Bilateral filter with a square box window [Yarovlasky 85]

$$Y[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} B_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) I_{\mathbf{q}}$$



box window

restrict the sum

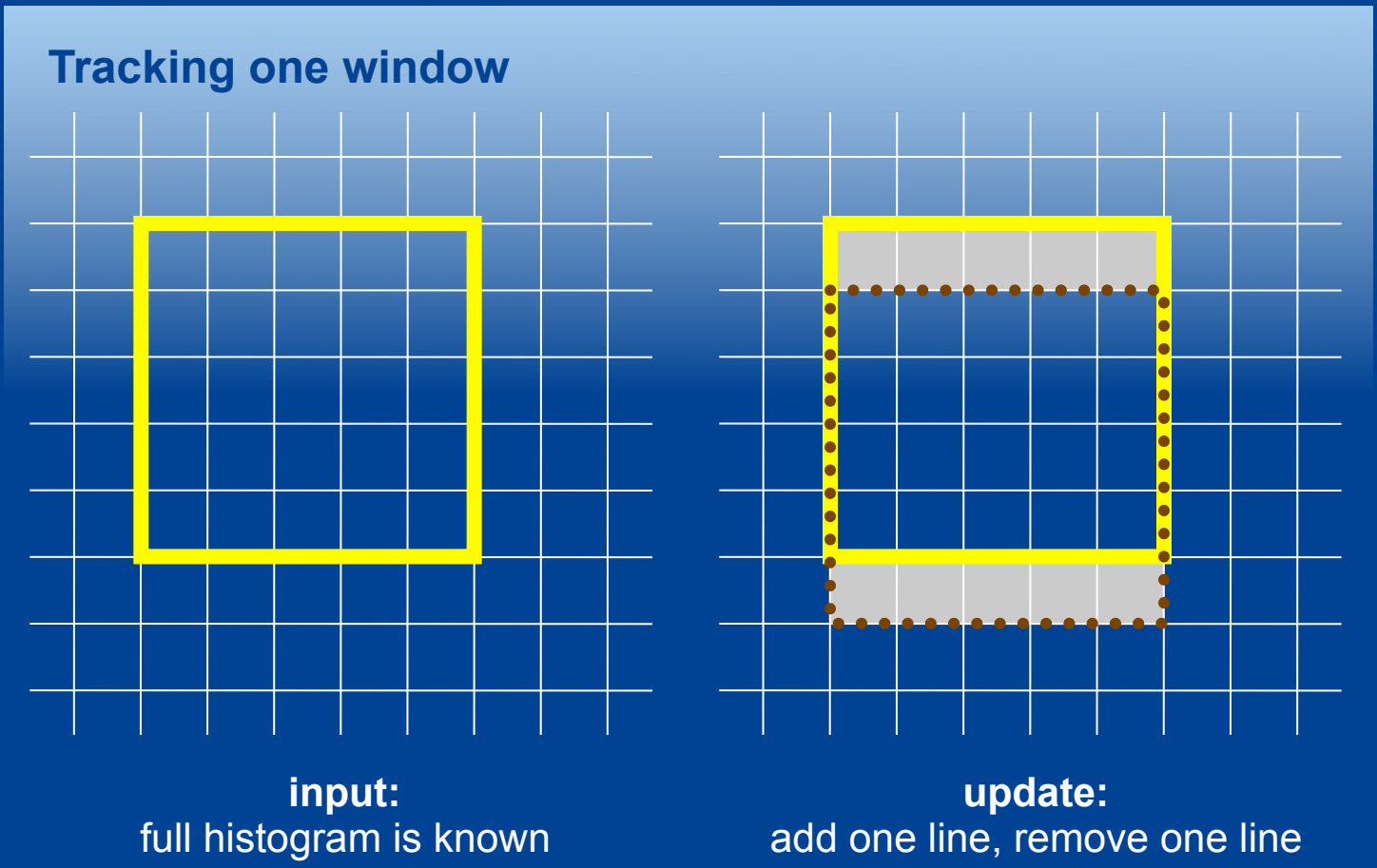
$$Y[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in B_{\sigma_s}} G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|) I_{\mathbf{q}}$$

independent of position $\mathbf{q}$

- The bilateral filter can be computed only from the list of pixels in a square neighborhood.

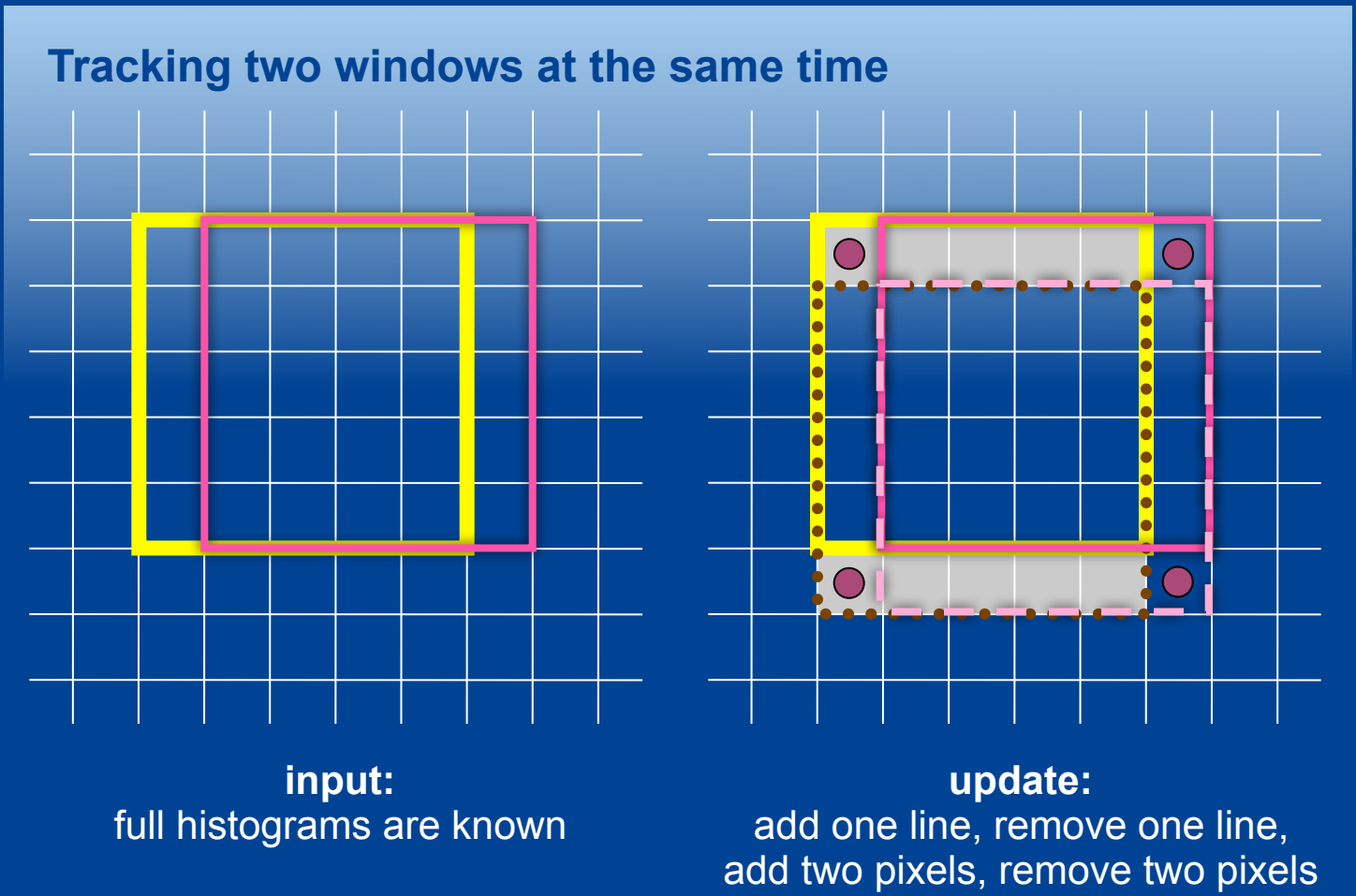
Box Kernel [Weiss 06]

- Idea: fast histograms of square windows



Box Kernel [Weiss 06]

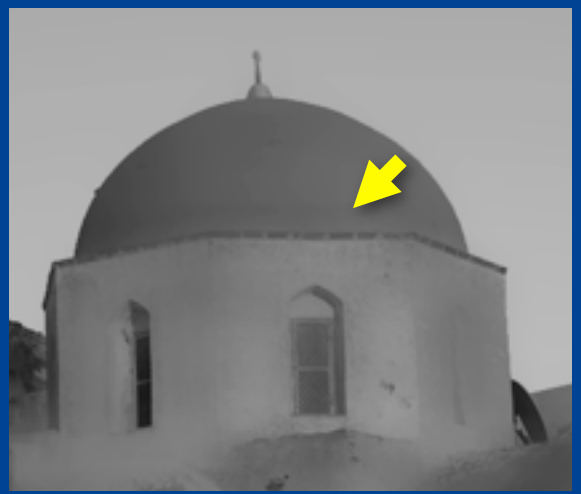
- Idea: fast histograms of square windows



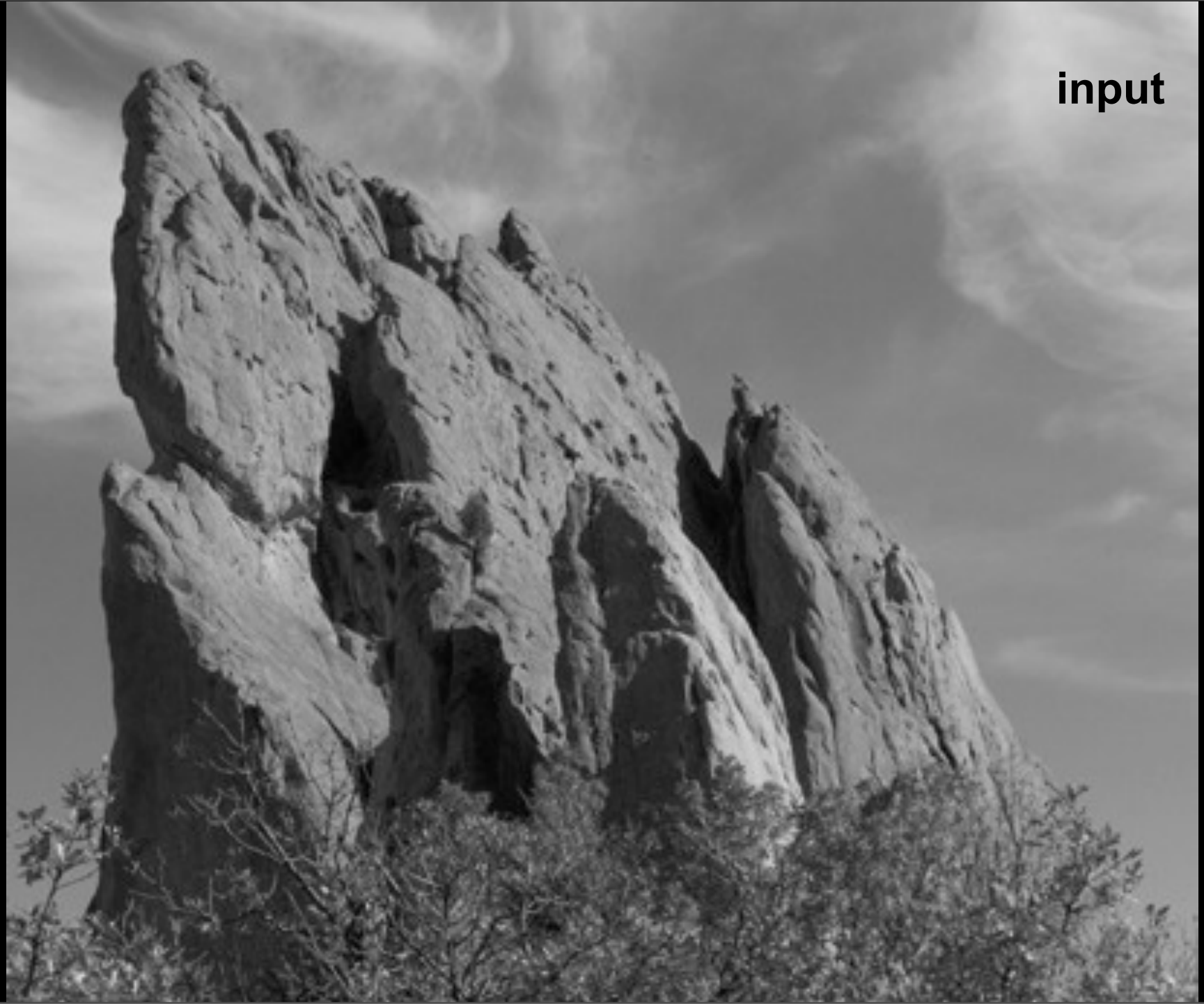
Discussion

- Complexity: $\ddot{I}$ $(|S| \times \log \sigma_s)$
  - always fast
- Only single-channel images
- Exploit vector instructions of CPU
- Visually satisfying results (no artifacts)
  - 3 passes to remove artifacts due to box windows (Mach bands)

1 iteration



3 iterations


input


brute-force
implementation


box kernel
visually different,
yet no artifacts