A Frequency Analysis of Light Transport

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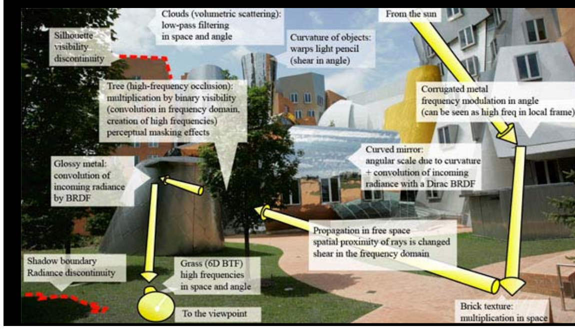
Artis Gravir/Imag-Inria & MIT CSAIL



Our research

- 3D rendering
  - Light transport
  - Material appearance
  - Real time rendering, hardware
- Computational Photography & Video
  - Image enhancement, dynamic range, relighting
  - Data-rich imaging
  - Image decomposition and manipulation
- In all cases, it's all about complicated signals

Understanding, manipulating and computing signals

- Discontinuities
  - Where things change
- Gradient
  - Useful for interpolation, criterion
- ***Frequency content (today's talk)***
  - Useful for sampling
  - Useful for inverse problems
  - Sometimes useful as basis function
- Statistics

And all these capture perceptual properties

Bilateral filter

Signal decomposition that characterizes multiscale content and preserves discontinuities

- Tone mapping
- Flash no flash
- Meshes

Visibility

- Singularity approach (discontinuities)
- Fake shadow blurriness (signal characteristics are right, not values)

A Frequency Analysis of Light Transport

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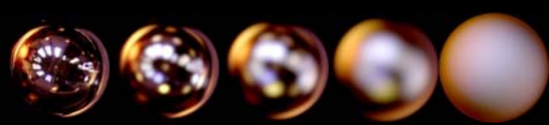
Illumination effects

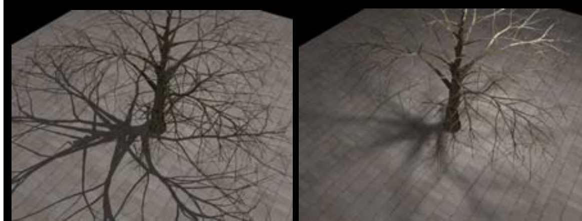
- Blurry reflections:



From [Ramamoorthi and Hanrahan 2001]

Illumination effects

- Shadow boundaries:



Point light source — Area light source

Illumination effects

- Indirect lighting is usually blurry:

Direct lighting only — Indirect lighting only

Frequency aspects of light transport

- Blurriness = frequency content
  - Sharp variations: high frequency
  - Smooth variations: low frequency
- All effects are expressed as frequency content:
  - Diffuse shading: low frequency
  - Shadows: introduce high frequencies
  - Indirect lighting: tends to be low frequency

Frequency content matters in graphics

- Sampling, antialiasing
  - Texture prefiltering
  - Light field sampling
- Fourier-like basis
  - Precomputed Radiance Transfer
  - Wavelet radiosity, SH
- Low-frequency assumption
  - Irradiance caching

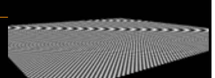
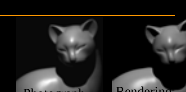


Image M. Zwicker

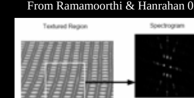
Image H. Wann Jensen

Frequency content matters in vision

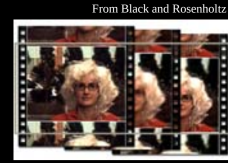
- Inverse lighting
  - And illumination invariants
- Shape from texture
- Shape from (de)focus
  - See our Defocus Matting [McGuire et al. Siggraph 2005]



From Ramamoorthi & Hanrahan 01



From Black and Rosenholtz 97



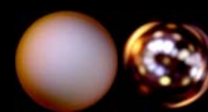
Problem statement

- How does light interaction in a scene explain the frequency content?
- Theoretical framework:
  - Understand the frequency spectrum of the radiance function
  - From the equations of light transport

Unified framework:

- Spatial frequency (e.g. shadows, textures)

- Angular frequency (e.g. blurry highlight)



Disclaimer: not Fourier optics

- We do **not** consider wave optics, interference, diffraction
- Only geometrical optics

From [Hecht]

Overview

- Previous work
- Our approach:
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- Case studies
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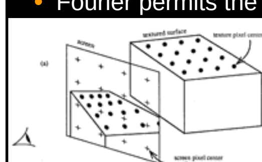
Previous work

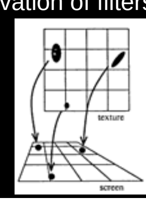
- Vast body of literature:
  - Light field sampling
  - Perceptually-based rendering
  - Wavelets for Computer Graphics
  - Irradiance caching
  - Photon mapping
  - ...
- We focus on frequency analysis in graphics & vision:
  - Texture antialiasing
  - Light field sampling
  - Reflection as a convolution

Texture pre-filtering [Heckbert 89]

- Input signal: texture map
- Perspective: transforms signal
- Image: resampling
- Fourier permits the derivation of filters



From [Heckbert 1989]



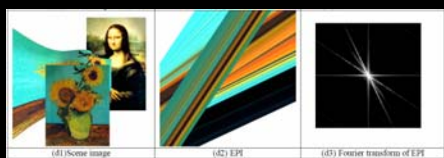
minification

magnification

Light field sampling

[Chai et al. 00, Isaksen et al. 00, Stewart et al. 03]

- Light field spectrum as a function of object distance
- No BRDF, occlusion ignored



From [Chai et al. 2000]

Signal processing for reflection

[Ramamoorthi & Hanrahan 01, Basri & Jacobs 03]

- Reflection on a curved surface is a convolution
- Direction only

From [Ramamoorthi and Hanrahan 2001]

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Our approach

- Light sources are input signal
- Interactions are filters/transforms
  - Transport
  - Visibility
  - BRDF
  - Etc.

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Light source signal

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Light source signal
↓
Transport

Our approach

- Light sources are input signal
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  - Etc.

Light source signal → Transport → Signal 1

Our approach

- Light sources are input signal
- Interactions are filters/transforms
  - Transport
  - Visibility
  - BRDF
  - Etc.

Light source signal → Transport → Occlusion → Signal 2

Our approach

- Light sources are input signal
- Interactions are filters/transforms
  - Transport
  - Visibility
  - BRDF
  - Etc.

Light source signal → Transport → Occlusion → Transport → Signal 3

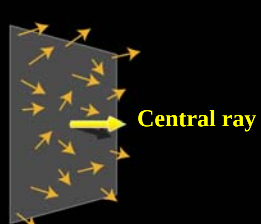
Our approach

- Light sources are input signal
- Interactions are filters/transforms
  - Transport
  - Visibility
  - BRDF
  - Etc.

Light source signal → Transport → Occlusion → Transport → BRDF → Signal 4

Local light field

- 4D light field, around a *central ray*



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- We study its spectrum during transport

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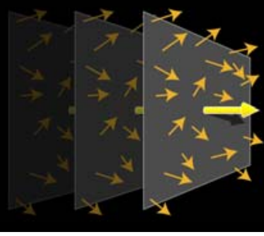

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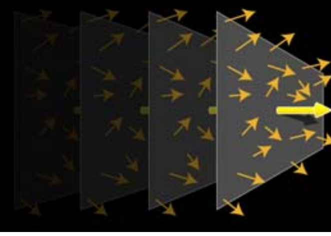
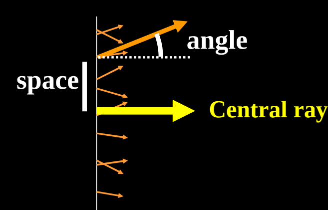


Local light field

- We give explanations in 2D
- Local light field is therefore 2D
- See paper for extension to 3D

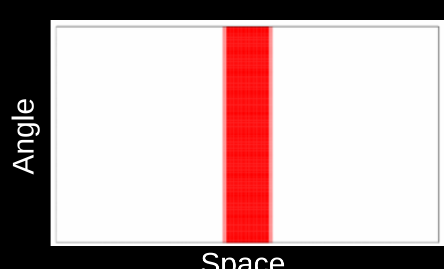
Local light field parameterization

- Space and angle



Local light field representation

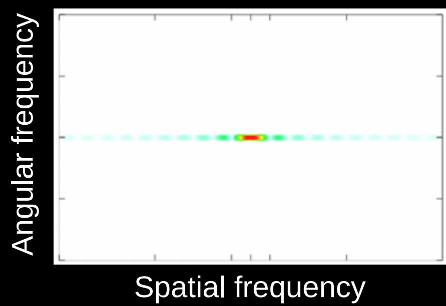
- Density plot:



Local light field Fourier spectrum

- We are interested in the Fourier spectrum of the local light field
- Also represented as a density plot

Local light field
Fourier spectrum



Fourier analysis 101

- Spectrum corresponds to blurriness:
  - Sharpest feature has size ~ $1/F_{max}$
- Convolution theorem:
  - Multiplication of functions: spectrum is convolved
  - Convolution of functions: spectrum is multiplied
- Classical spectra:
  - Box ↔ sinc
  - Dirac ↔ constant

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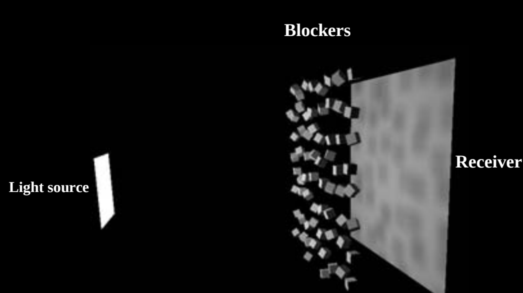
Example scene



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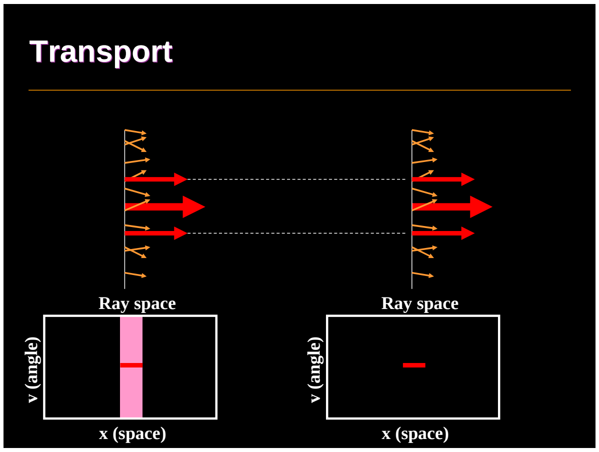
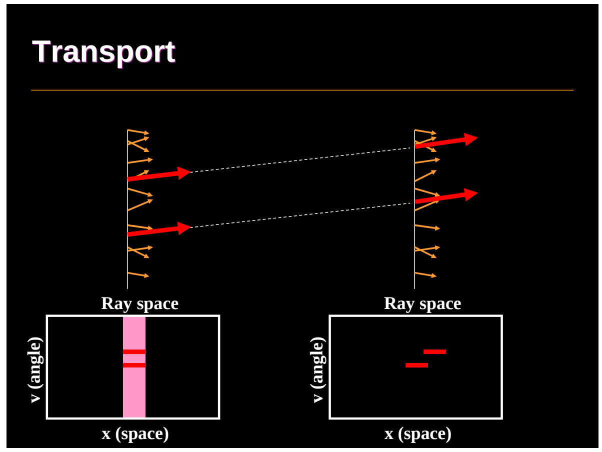

Transport
Ray space
Ray space
v (angle)
x (space)
v (angle)
x (space)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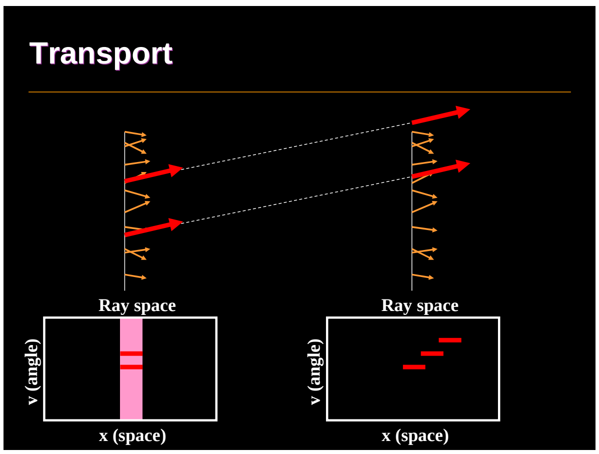
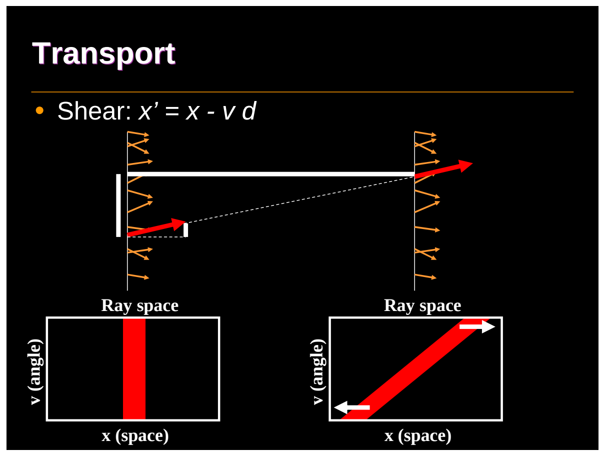
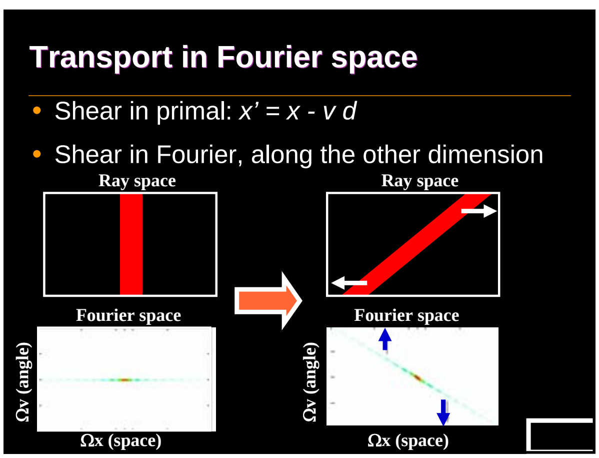
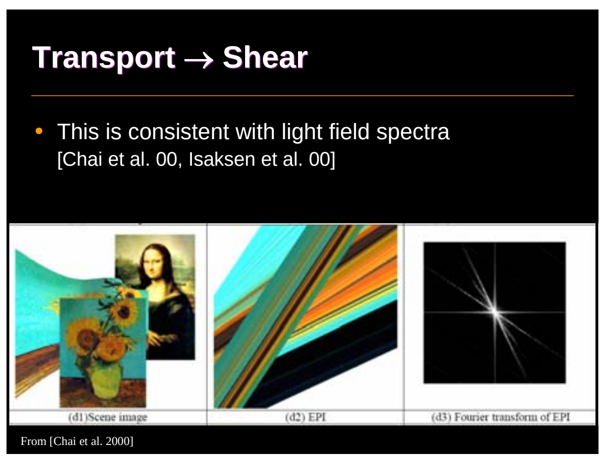

Transport
Ray space
Ray space
v (angle)
x (space)
v (angle)
x (space)


Transport
Shear: x' = x - v d
Ray space
Ray space
v (angle)
x (space)
v (angle)
x (space)


Transport in Fourier space
Shear in primal: x' = x - v d
Shear in Fourier, along the other dimension
Ray space
Ray space
Fourier space
Fourier space
Ωv (angle)
Ωx (space)
Ωv (angle)
Ωx (space)


Transport → Shear
This is consistent with light field spectra
[Chai et al. 00, Isaksen et al. 00]
(d1) Scene image
(d2) EPI
(d3) Fourier transform of EPI
From [Chai et al. 2000]

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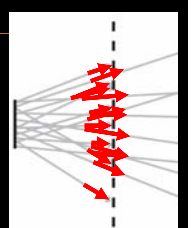
Occlusion: multiplication

- Occlusion is a multiplication in ray space
  - Convolution in Fourier space
- Creates new spatial frequency content
  - Related to the spectrum of the blockers

Occlusion

- Consider planar occluder
- Multiplication by binary function
  - Mostly in space



Occlusion

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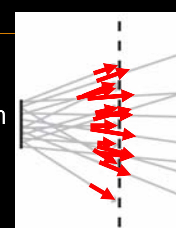
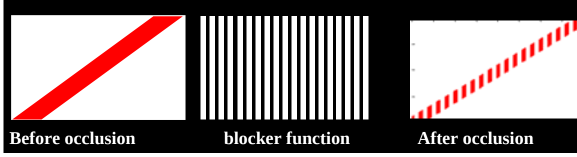
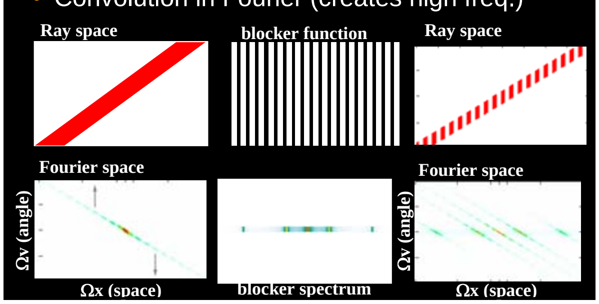




Occlusion in Fourier

- Multiplication in primal
- Convolution in Fourier (creates high freq.)



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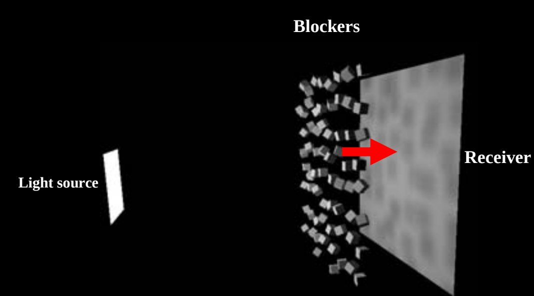
Another propagation



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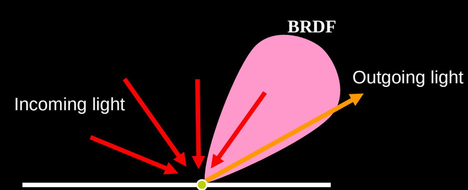
BRDF integration

- Outgoing light:
  - Integration of incoming light times BRDF



BRDF integration

- Ray-space: ***convolution***
  - Outgoing light: convolution of incoming light and BRDF
  - For rotationally-invariant BRDFs

- Fourier domain: ***multiplication***
  - Outgoing spectrum: multiplication of incoming spectrum and BRDF spectrum

BRDF in Fourier: multiplication

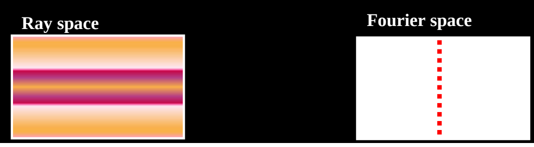
- BRDF is bandwidth-limiting in angle

Example: diffuse BRDF

- Convolve by constant:
  - multiply by horizontal window
  - Only spatial frequencies remain

Relation to previous work

- [Ramamoorthi & Hanrahan 01, Basri & Jacob 03]
- They consider a spatially-constant illumination
- But essentially, nothing much changed



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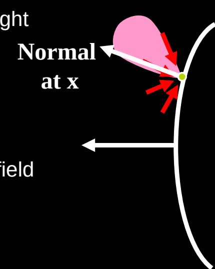
Curvature

- Ray reaches curved surface:
  - Transform it into planar surface
  - “Unroll” curved surface
- Equivalent to changing angular content:
  - Linear effect on angular dimension
  - No effect on spatial dimension
- Shear in the angular dimension

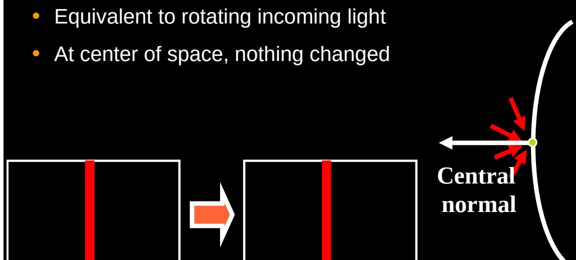
Curvature

- For each point x, the normal has a different angle
- Equivalent to rotating incoming light
- We will reparameterize the light field



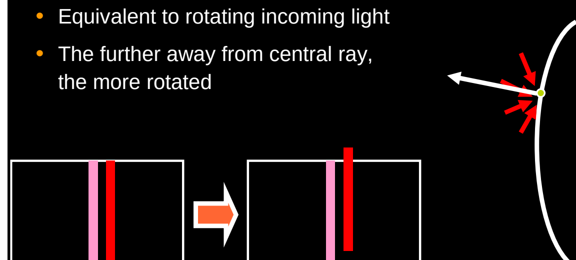
Curvature

- For each point x, the normal has a different angle
- Equivalent to rotating incoming light
- At center of space, nothing changed



Curvature

- For each point x, the normal has a different angle
- Equivalent to rotating incoming light
- The further away from central ray, the more rotated



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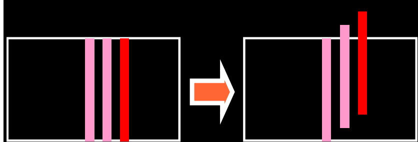
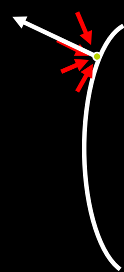
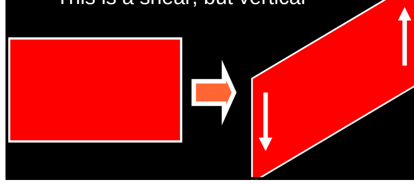
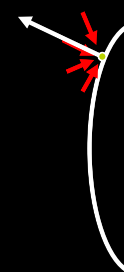



Curvature

- For each point x, the normal has a different angle
- Equivalent to rotating incoming light
- The further away from central ray, the more rotated
- This is a shear, but vertical

Main transforms: summary

	Radiance/Fourier	Effect
Transport	Shear	
Occlusion	Multiplication/Convolution	Adds spatial frequencies
BRDF	Convolution/Multiplication	Removes angular frequencies
Curvature	Shear	

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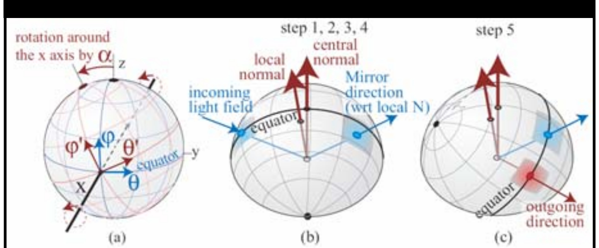
Extension to 3D

- It works. See paper:



Even more effects in paper

- Various technical details
  - Cosine/Fresnel term:
  - Central incidence angle:
- Texture mapping (multiplication/convolution)
- Separable BRDF
- Spatially varying BRDF (semi-convolution)
- …and full extension to 3D

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  - Solar oven
  - Adaptive shading sampling
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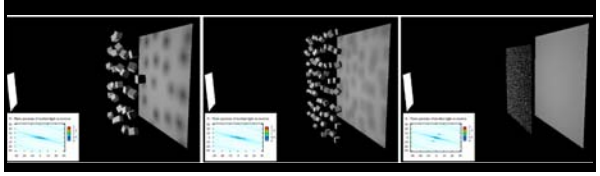
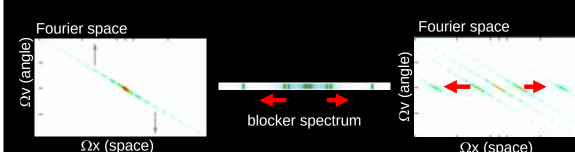
Diffuse soft shadows

- Decreasing blockers size:
  - First high-frequencies increase
  - Then only low frequency
  - Non-monotonic behavior



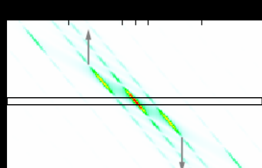
Diffuse soft shadows (2)

- Occlusion : convolution in Fourier
  - creates high frequencies
  - Blockers scaled down → spectrum scaled up



Diffuse soft shadows (3)

- Transport after occlusion:
  - Spatial frequencies moved to angular dimension
- Diffuse reflector:
  - Angular frequencies are cancelled



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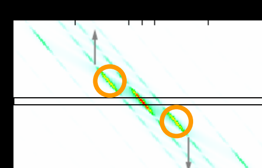
- Diffuse reflector:
  - Angular frequencies are cancelled

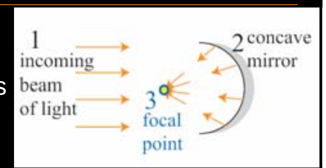


Overview

- Previous work
- Our approach:
  - Local light field
  - Transformations on local light field
- Case studies:
  - Diffuse soft shadows
  - Solar oven
  - Adaptive shading sampling
- Conclusions and future directions

Solar oven

- Curved surface
- In: parallel light rays
- Out: focal point



Solar oven

- Curved surface
- In: parallel light rays
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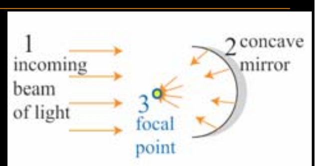

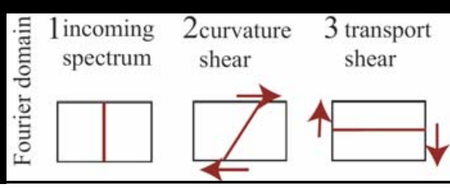

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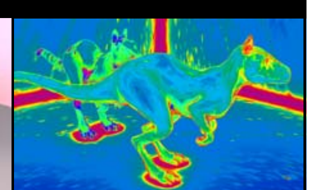
Adaptive shading sampling

- Monte-Carlo ray tracing
- Blurry regions need fewer shading samples

Adaptive shading sampling

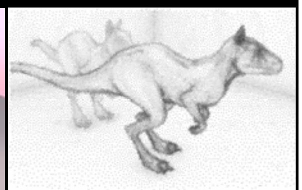
- Per-pixel prediction of max. frequency (bandwidth)
  - Based on curvature, BRDF, distance to occluder, etc.
  - No spectrum computed, just estimate max frequency



Per-pixel bandwidth criterion

Adaptive shading sampling

- Per-pixel prediction of max. frequency (bandwidth)
  - Based on curvature, BRDF, distance to occluder, etc.
  - No spectrum computed, just estimate max frequency



Shading samples

Uniform sampling

Adaptive sampling

Overview

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Conclusions

- Unified framework:
  - For frequency analysis of radiance
  - In both space and angle
  - Simple mathematical operators
  - Extends previous analyses
- Explains interesting lighting effects:
  - Soft shadows, caustics
- Proof-of-concept:
  - Adaptive sampling

Future work

- More experimental validation on synthetic scenes
- Extend the theory:
  - Bump mapping, microfacet BRDFs, sub-surface scattering…
  - Participating media
  - Wave optics
- Applications to rendering:
  - Photon mapping, spatial sampling for PRT
  - Revisit traditional techniques
- Vision and shape from shading
- 3D displays aliasing
- Optics and computational photography

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  - INRIA Équipe associée
  - Realreflect EU IST project
  - MIT-France

Why *Local* Light Field?

- Linearization:
  - $\theta \approx \tan \theta$
  - Curvature
  - Extension to 3D
- *Local* information is what we need:
  - Local frequency content, for local sampling
  - Based on local properties of the scene (occluders)

Other bases?

- We're not using Fourier as a function basis
  - Don't recommend it, actually
  - Just used for analysis, understanding, predictions
- Results are useable with any other base:
  - Wavelets, Spherical Harmonics, point sampling, etc
  - Max. frequency translates in sampling rate
- Analysis relies on Fourier properties:
  - Especially the convolution theorem

Reflection on a surface: Full summary

- Angle of incidence
- Curvature
- Cosine/Fresnel term
- Mirror re-parameterization
- BRDF
- Curvature

Reflection on a surface: Full summary

- Angle of incidence: scaling
- Curvature: shear in angle
- Cosine/Fresnel term: multiplication/convolution
- Mirror re-parameterization
- BRDF: convolution/multiplication
- Curvature: shear in angle