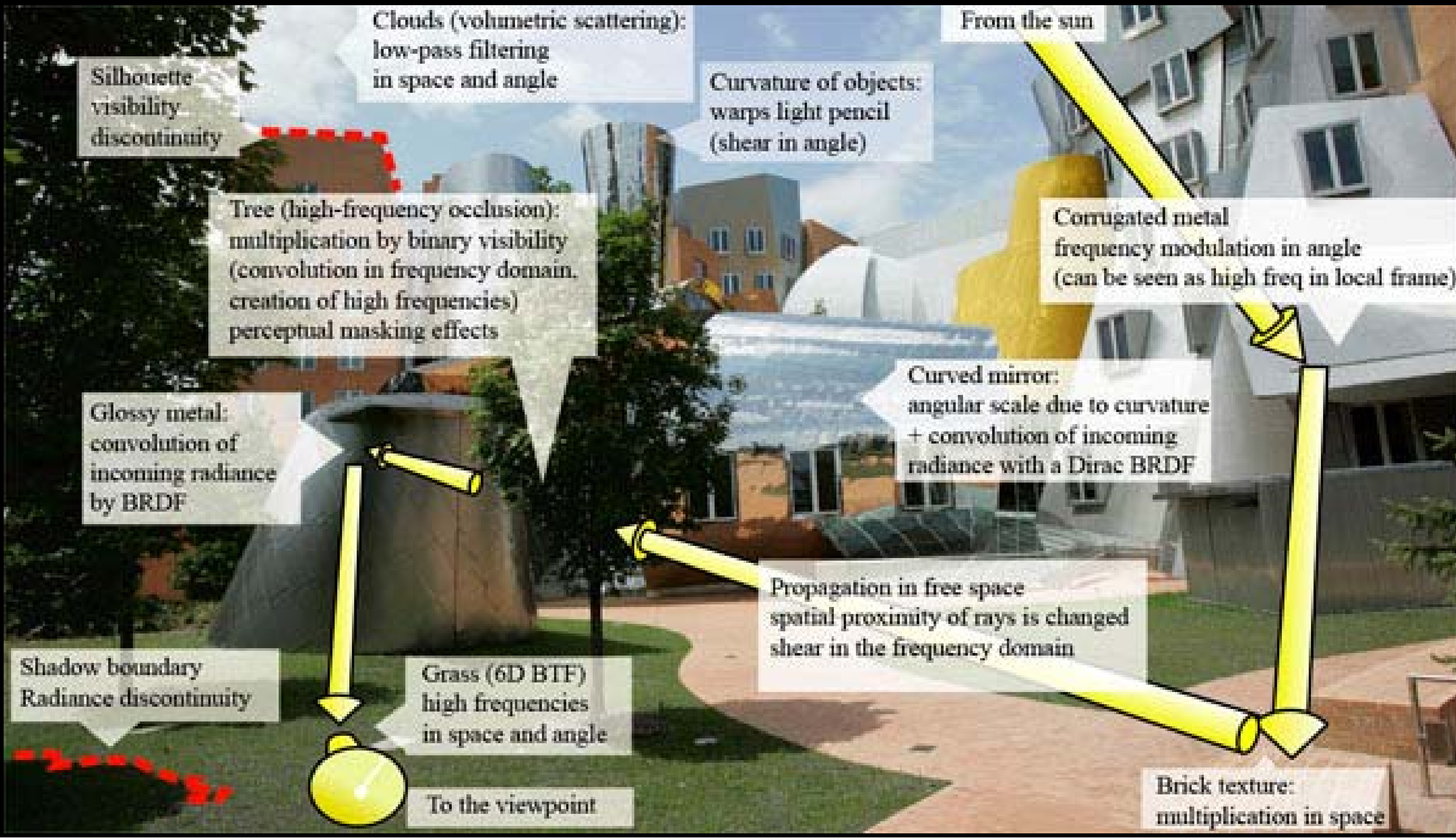
# A Frequency Analysis of Light Transport

Frédo Durand – MIT CSAIL

With Nicolas Holzschuch, Cyril Soler, Eric Chan & Francois Sillion

Artis Gravir/Imag-Inria & MIT CSAIL



# Our research

- 3D rendering
  - Light transport
  - Material appearance
  - Real time rendering, hardware
- Computational Photography & Video
  - Image enhancement, dynamic range, relighting
  - Data-rich imaging
  - Image decomposition and manipulation
- In all cases, it's all about complicated signals

# Understanding, manipulating and computing signals

- Discontinuities
  - Where things change
- Gradient
  - Useful for interpolation, criterion
- ***Frequency content (today's talk)***
  - Useful for sampling
  - Useful for inverse problems
  - Sometimes useful as basis function
- Statistics

And all these capture perceptual properties

# Bilateral filter

Signal decomposition that characterizes multiscale content and preserves discontinuities

- Tone mapping

- Flash no flash

- Meshes

# Visibility

- Singularity approach (discontinuities)

- Fake shadow blurriness (signal characteristics are right, not values)

# A Frequency Analysis of Light Transport

F. Durand, MIT CSAIL

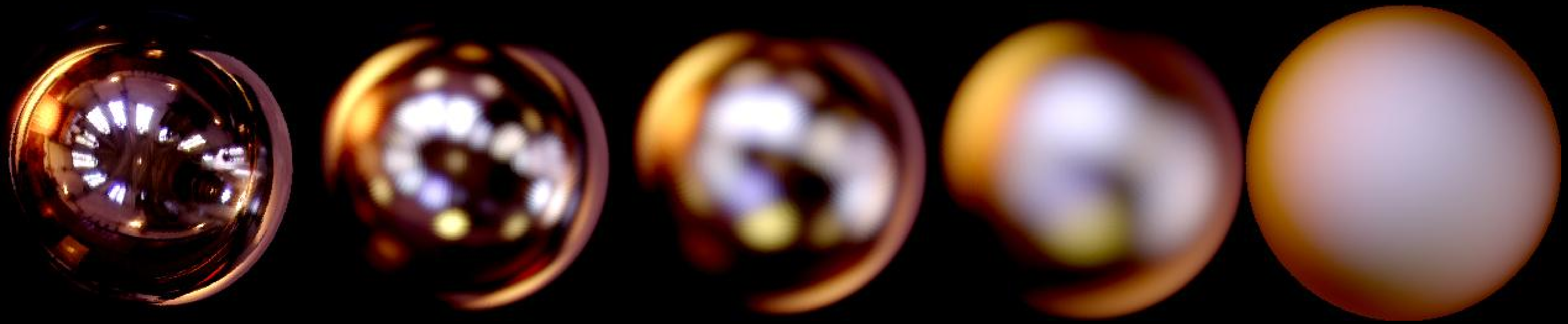
N. Holzschuch, C. Soler, ARTIS/GRAVIR-IMAG INRIA

E. Chan, MIT CSAIL

F. Sillion, ARTIS/GRAVIR-IMAG INRIA

# Illumination effects

- Blurry reflections:



From [Ramamoorthi and Hanrahan 2001]

# Illumination effects

- Shadow boundaries:

Point light source

Area light source

# Illumination effects

- Indirect lighting is usually blurry:

Direct lighting only

Indirect lighting only

# Frequency aspects of light transport

- Blurriness = frequency content
  - Sharp variations: high frequency
  - Smooth variations: low frequency
- All effects are expressed as frequency content:
  - Diffuse shading: low frequency
  - Shadows: introduce high frequencies
  - Indirect lighting: tends to be low frequency

# Frequency content matters in graphics

- Sampling, antialiasing
  - Texture prefiltering
  - Light field sampling
- Fourier-like basis
  - Precomputed Radiance Transfer
  - Wavelet radiosity, SH
- Low-frequency assumption
  - Irradiance caching

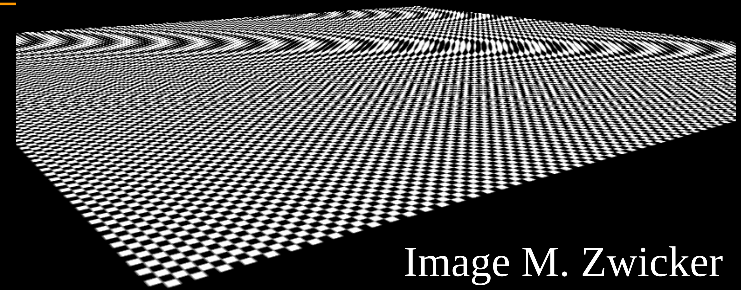
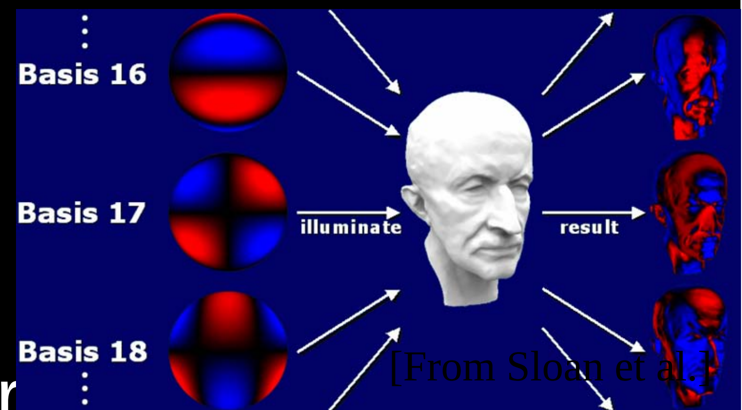


Image M. Zwicker



[From Sloan et al.]

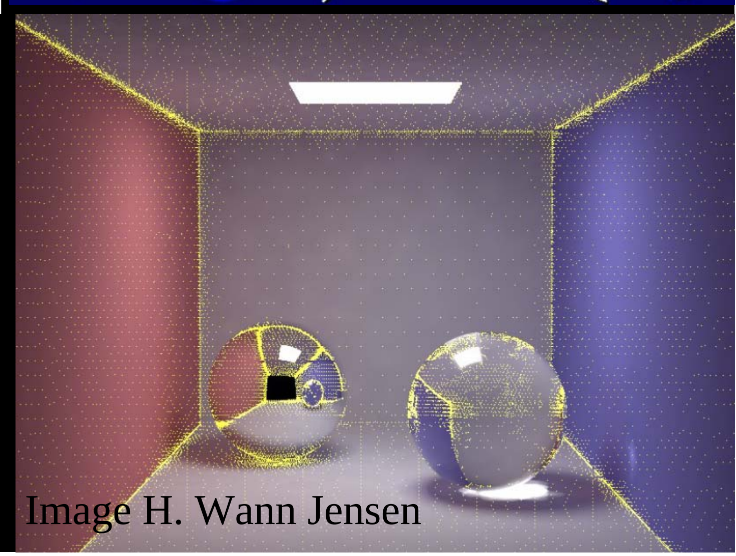
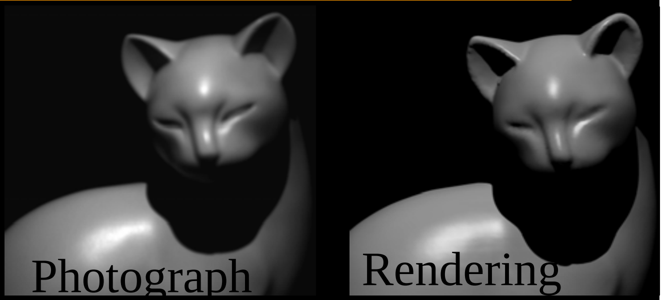


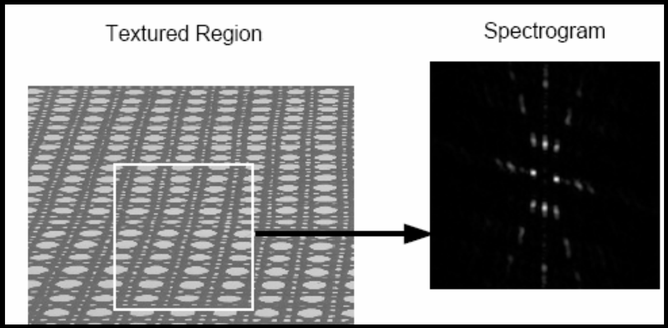
Image H. Wann Jensen

# Frequency content matters in vision

- Inverse lighting
  - And illumination invariants
- Shape from texture
- Shape from (de)focus
  - See our Defocus Matting [McGuire et al. Siggraph 2005]



From Ramamoorthi & Hanrahan 01



From Black and Rosenholtz 97

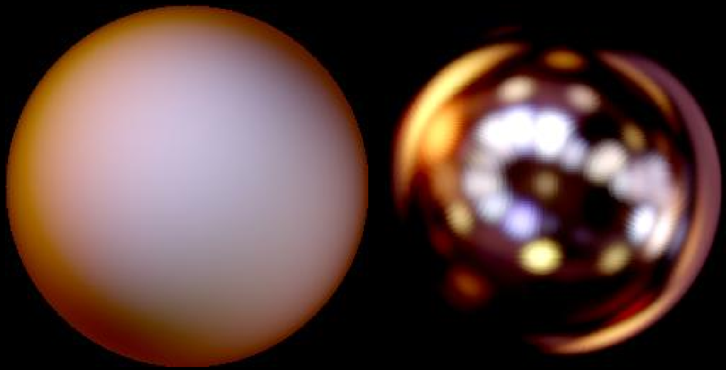
# Problem statement

- How does light interaction in a scene explain the frequency content?
- Theoretical framework:
  - Understand the frequency spectrum of the radiance function
  - From the equations of light transport

# Unified framework:

- Spatial frequency
  (e.g. shadows, textures)

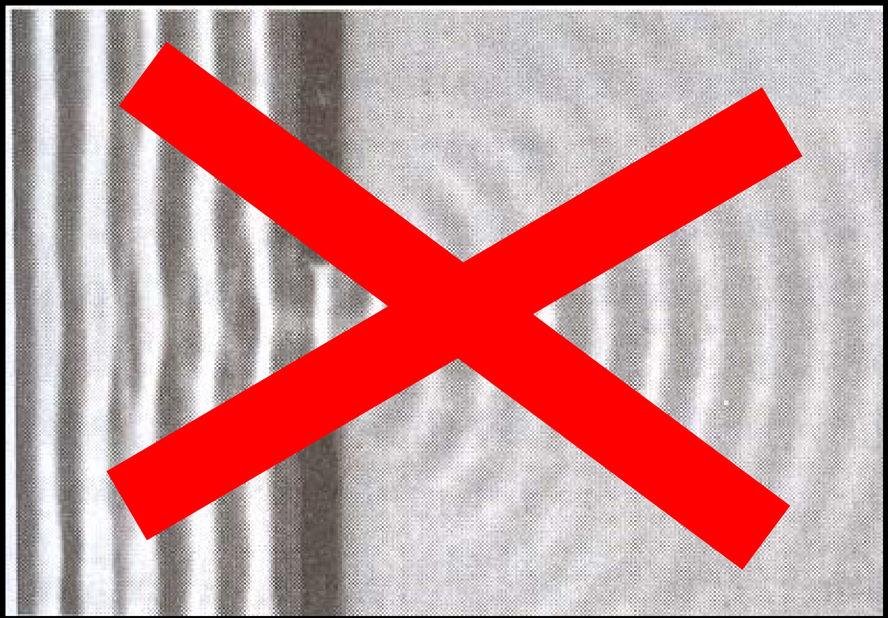
- Angular frequency
  (e.g. blurry highlight)

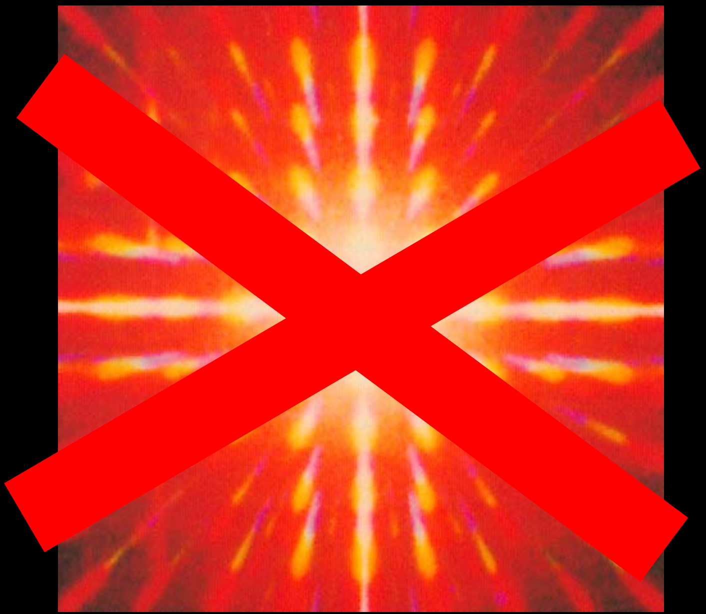


# Disclaimer: not Fourier optics

- We do **not** consider wave optics, interference, diffraction
- Only geometrical optics

From [Hecht]





# Overview

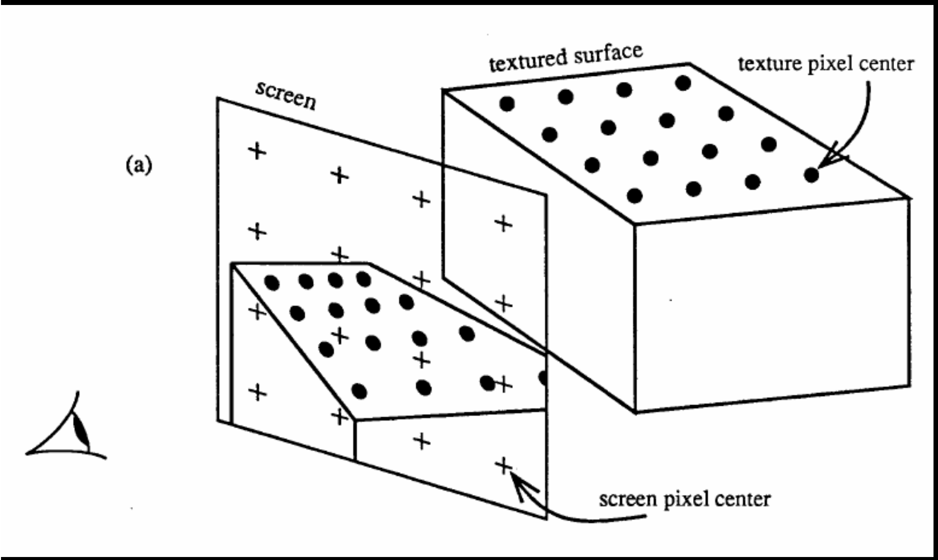
- Previous work
- Our approach:
  - Local light field
  - Transformations on local light field
- Case studies
- Conclusions and future directions

# Previous work

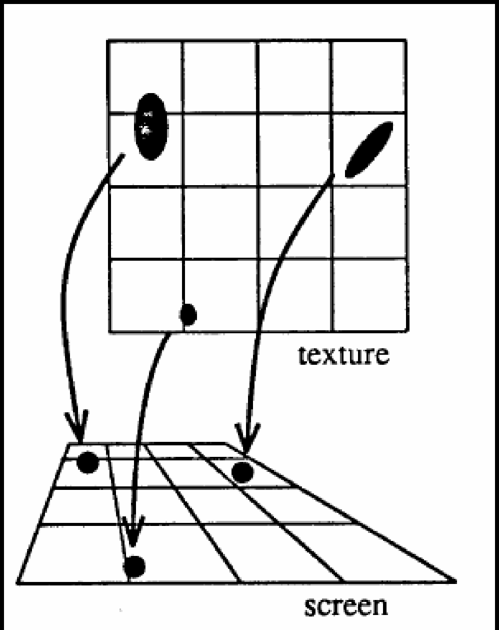
- Vast body of literature:
  - Light field sampling
  - Perceptually-based rendering
  - Wavelets for Computer Graphics
  - Irradiance caching
  - Photon mapping
  - …
- We focus on frequency analysis in graphics & vision:
  - Texture antialiasing
  - Light field sampling
  - Reflection as a convolution

# Texture pre-filtering [Heckbert 89]

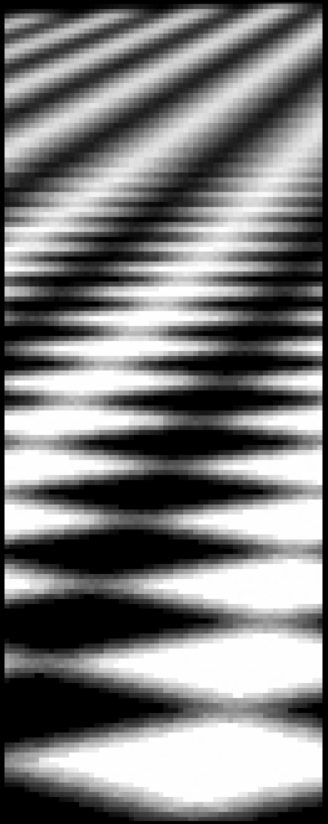
- Input signal: texture map
- Perspective: transforms signal
- Image: resampling
- Fourier permits the derivation of filters




From [Heckbert 1989]




minification

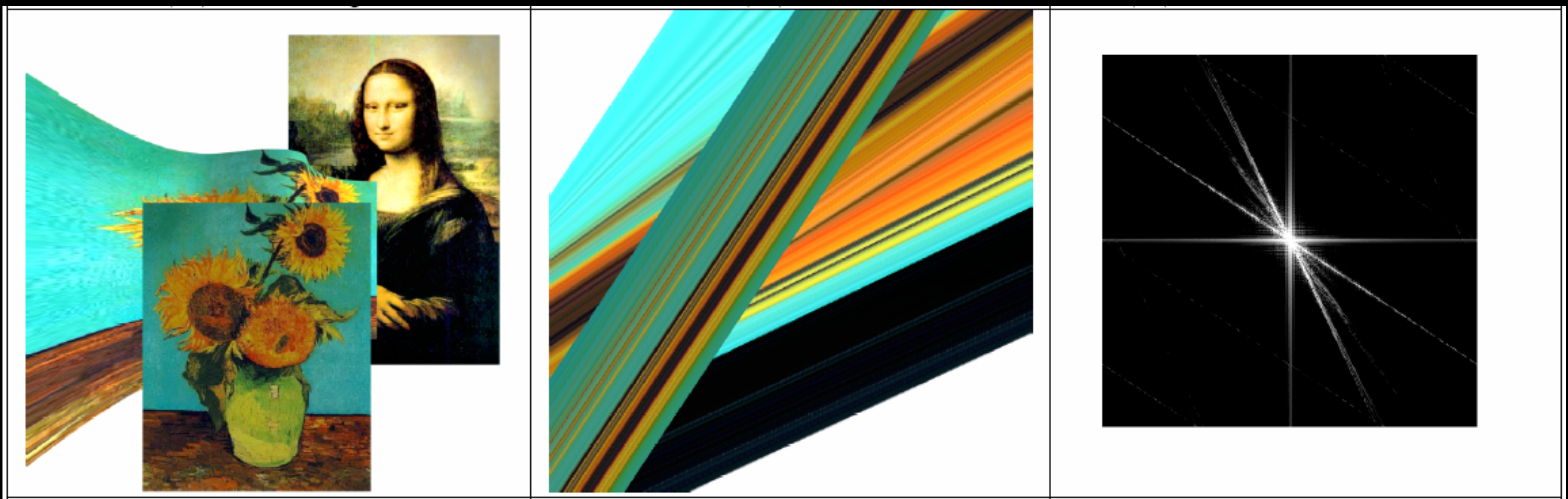


magnification

# Light field sampling

[Chai et al. 00, Isaksen et al. 00, Stewart et al. 03]

- Light field spectrum as a function of object distance
- No BRDF, occlusion ignored



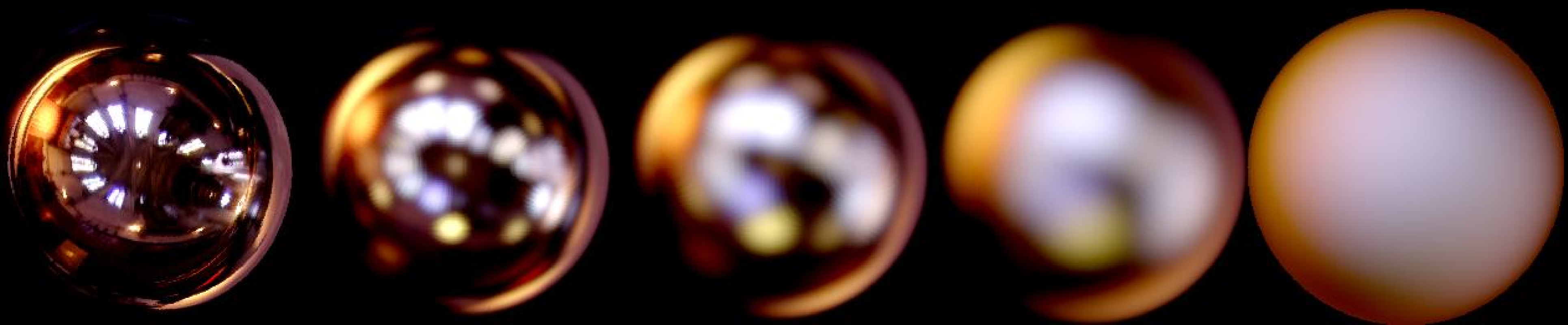
(d1)Scene image | (d2) EPI | (d3) Fourier transform of EPI

From [Chai et al. 2000]

# Signal processing for reflection

[Ramamoorthi & Hanrahan 01, Basri & Jacobs 03]

- Reflection on a curved surface is a convolution
- Direction only



From [Ramamoorthi and Hanrahan 2001]

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# Our approach

- Light sources are input signal
- Interactions are filters/transforms
  - Transport
  - Visibility
  - BRDF
  - Etc.

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**Light source signal**

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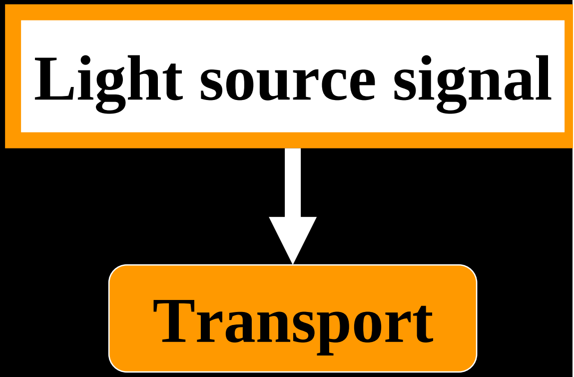

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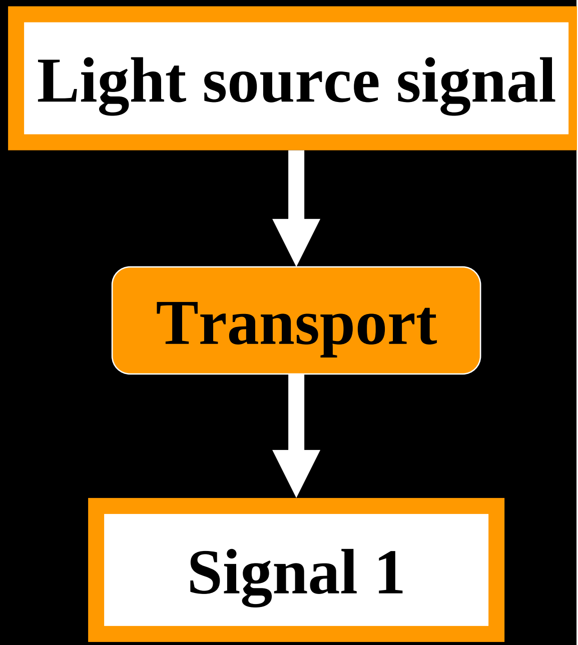

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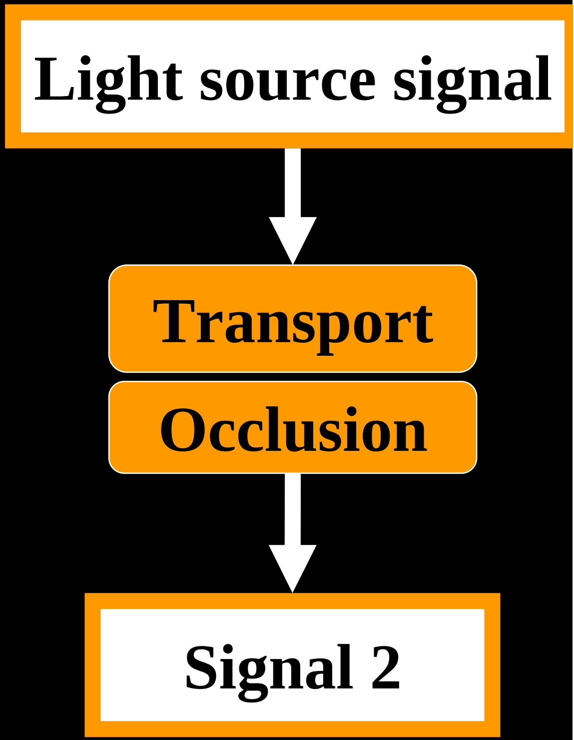

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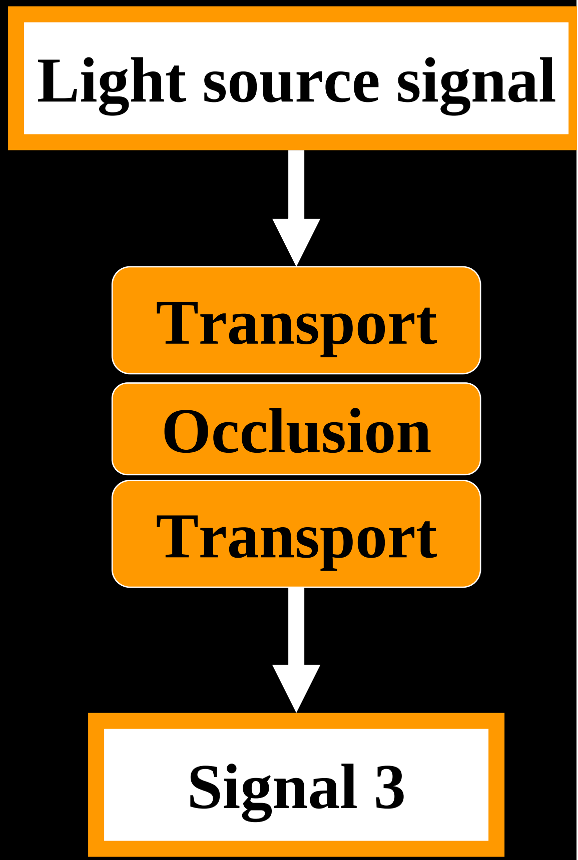

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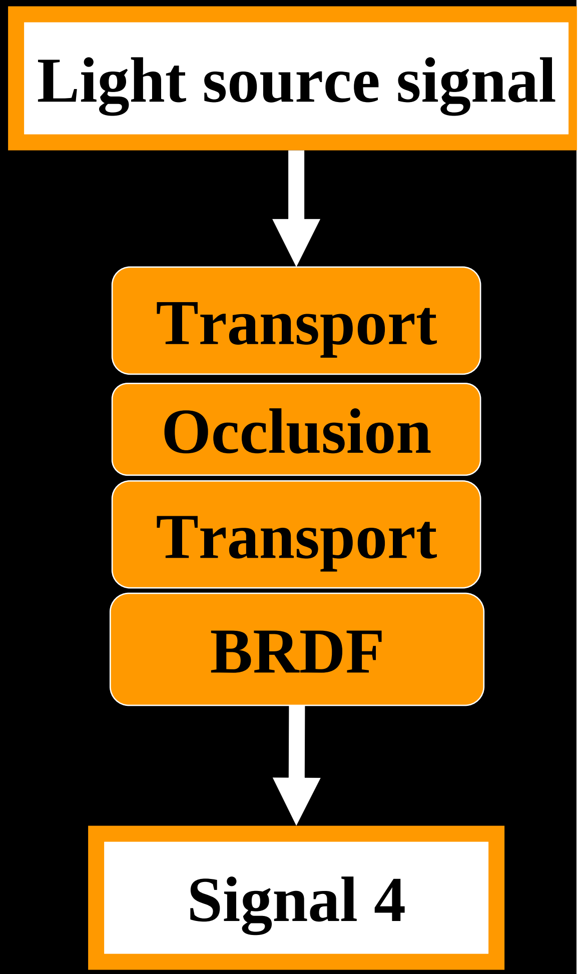
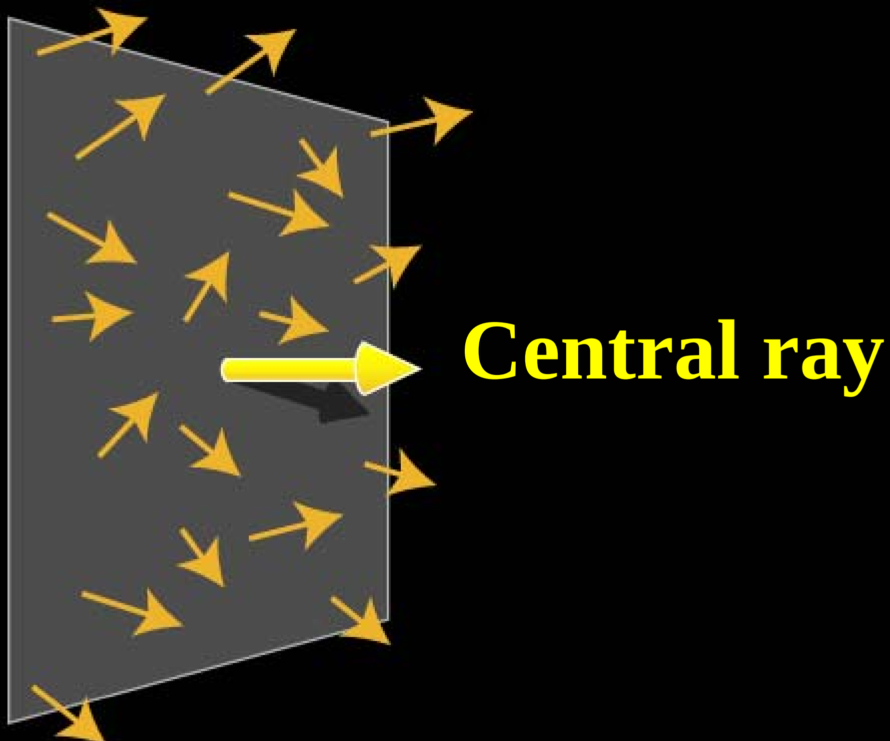


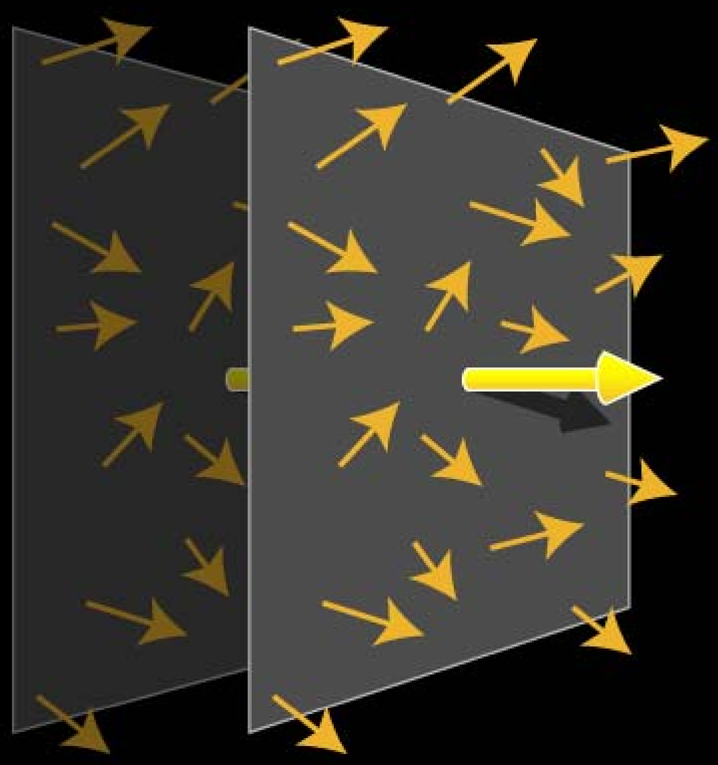
# Local light field

- 4D light field, around a *central ray*



# Local light field

- 4D light field, around a *central ray*
- We study its spectrum during transport



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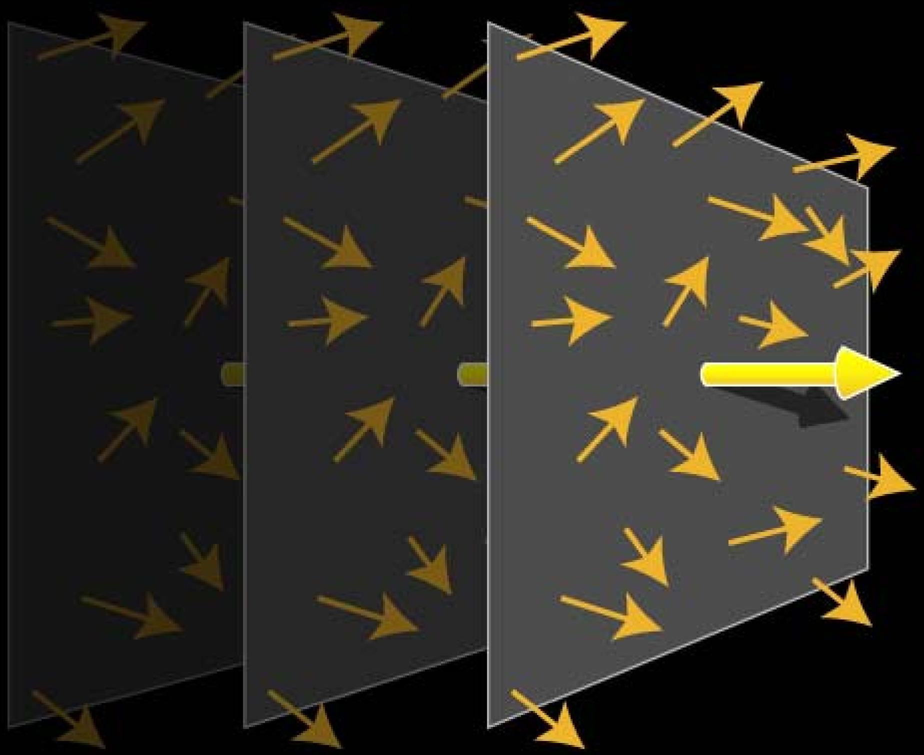

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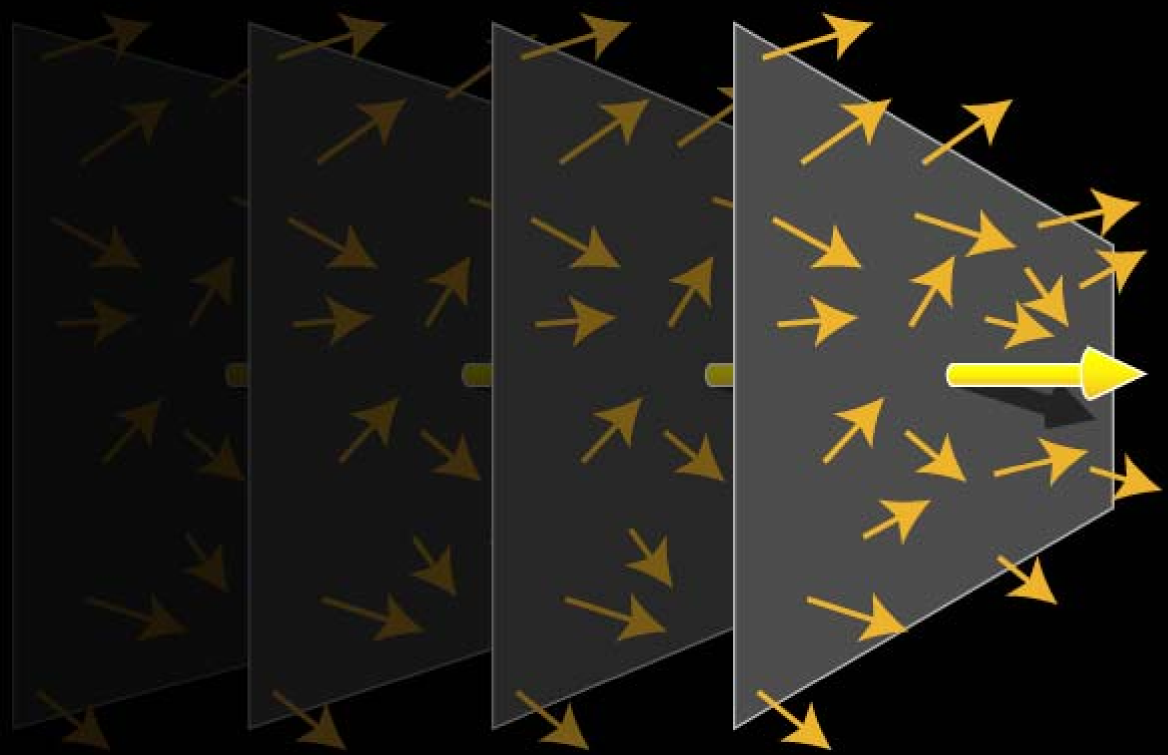
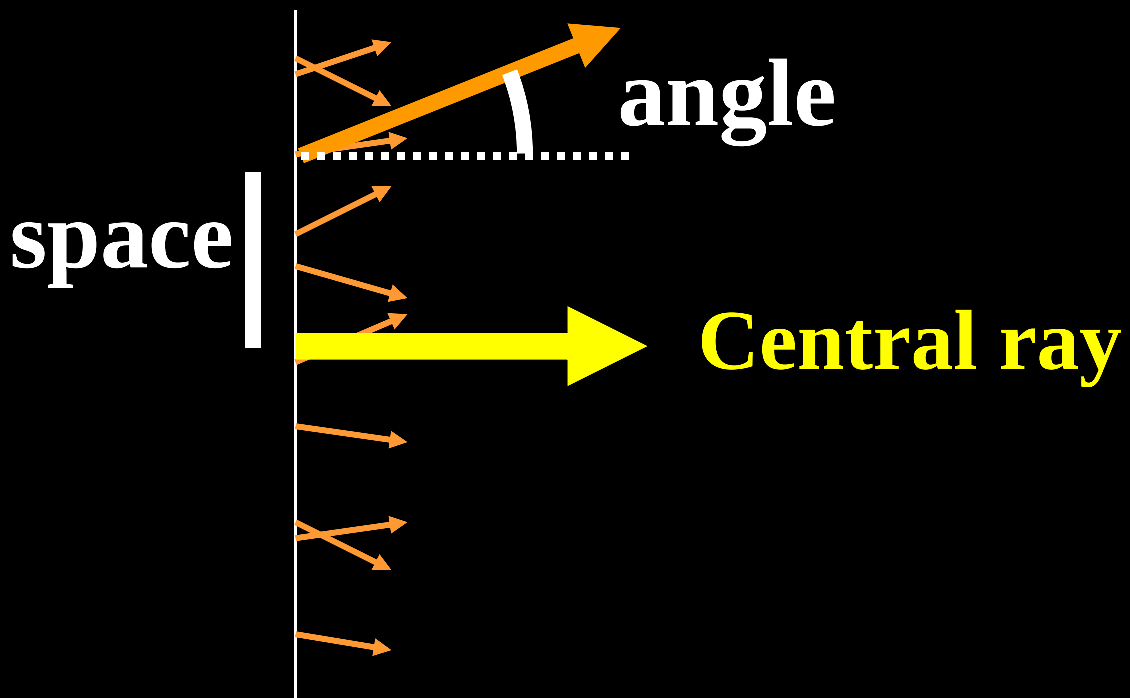


# Local light field

- We give explanations in 2D
- Local light field is therefore 2D
- See paper for extension to 3D

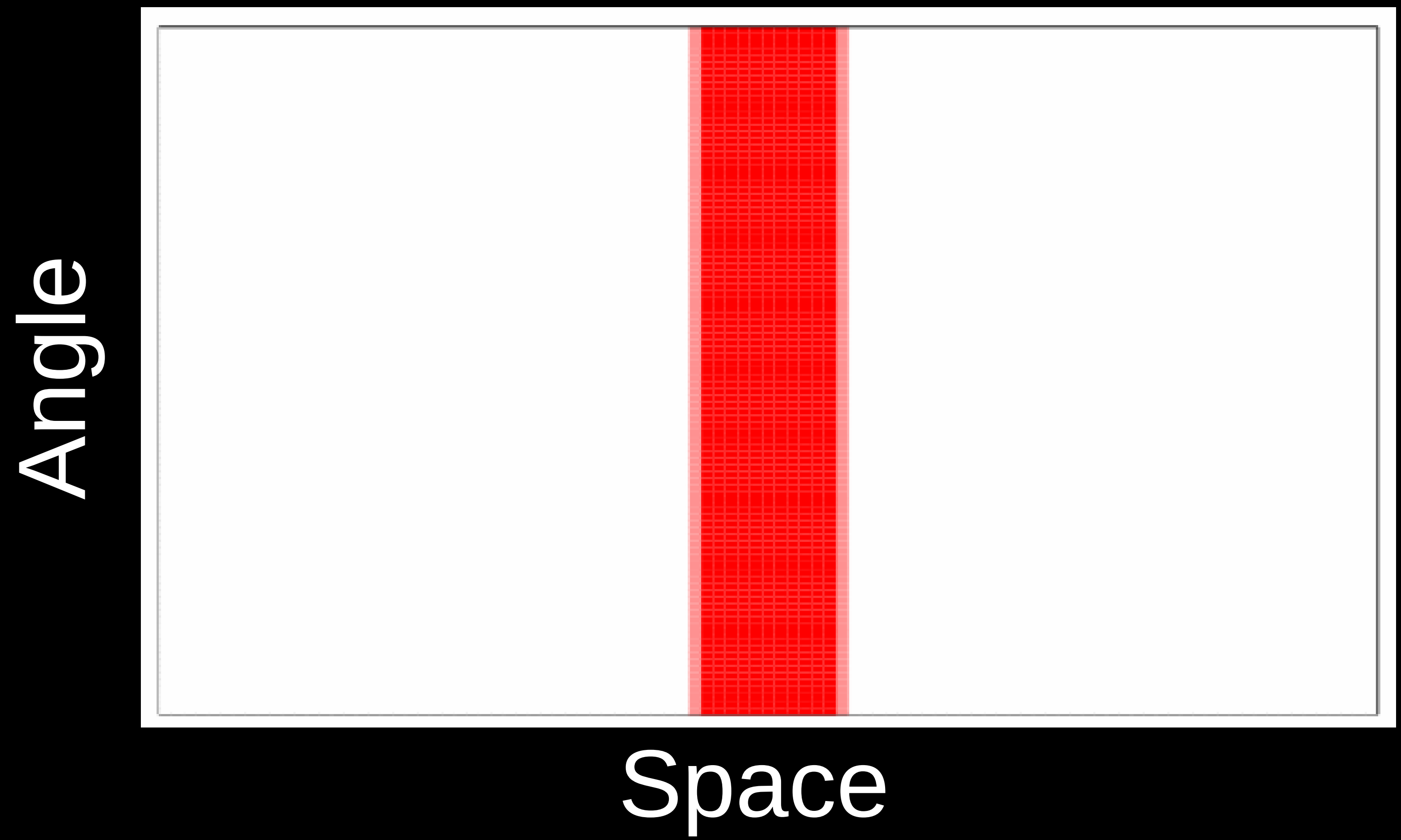
# Local light field parameterization

- Space and angle



# Local light field representation

- Density plot:

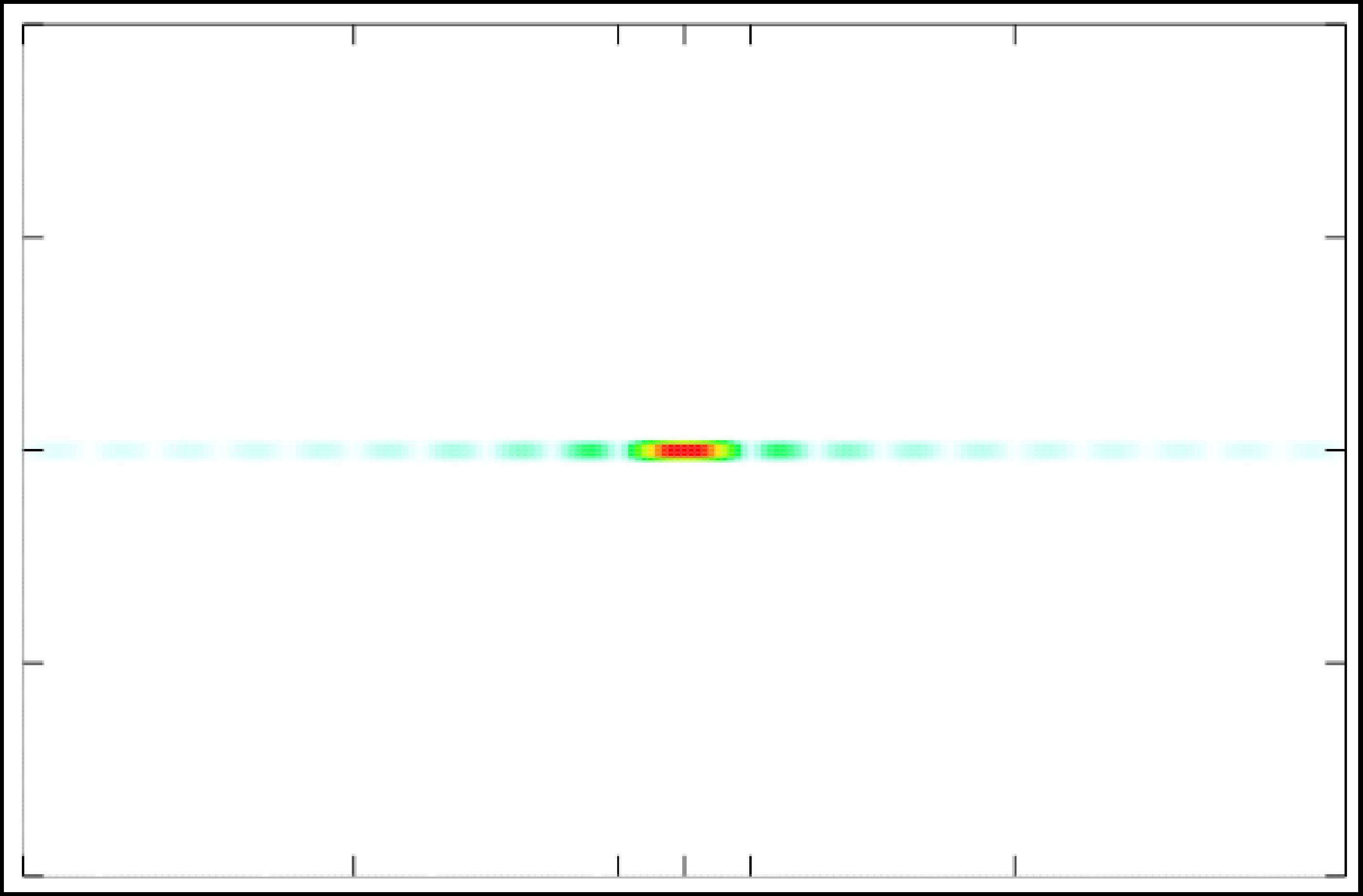


# Local light field
# Fourier spectrum

- We are interested in the Fourier spectrum of the local light field
- Also represented as a density plot

# Local light field Fourier spectrum

Angular frequency



Spatial frequency

# Fourier analysis 101

- Spectrum corresponds to blurriness:
  - Sharpest feature has size ~ $1/F_{max}$
- Convolution theorem:
  - Multiplication of functions: spectrum is convolved
  - Convolution of functions: spectrum is multiplied
- Classical spectra:
  - Box ↔ sinc
  - Dirac ↔ constant

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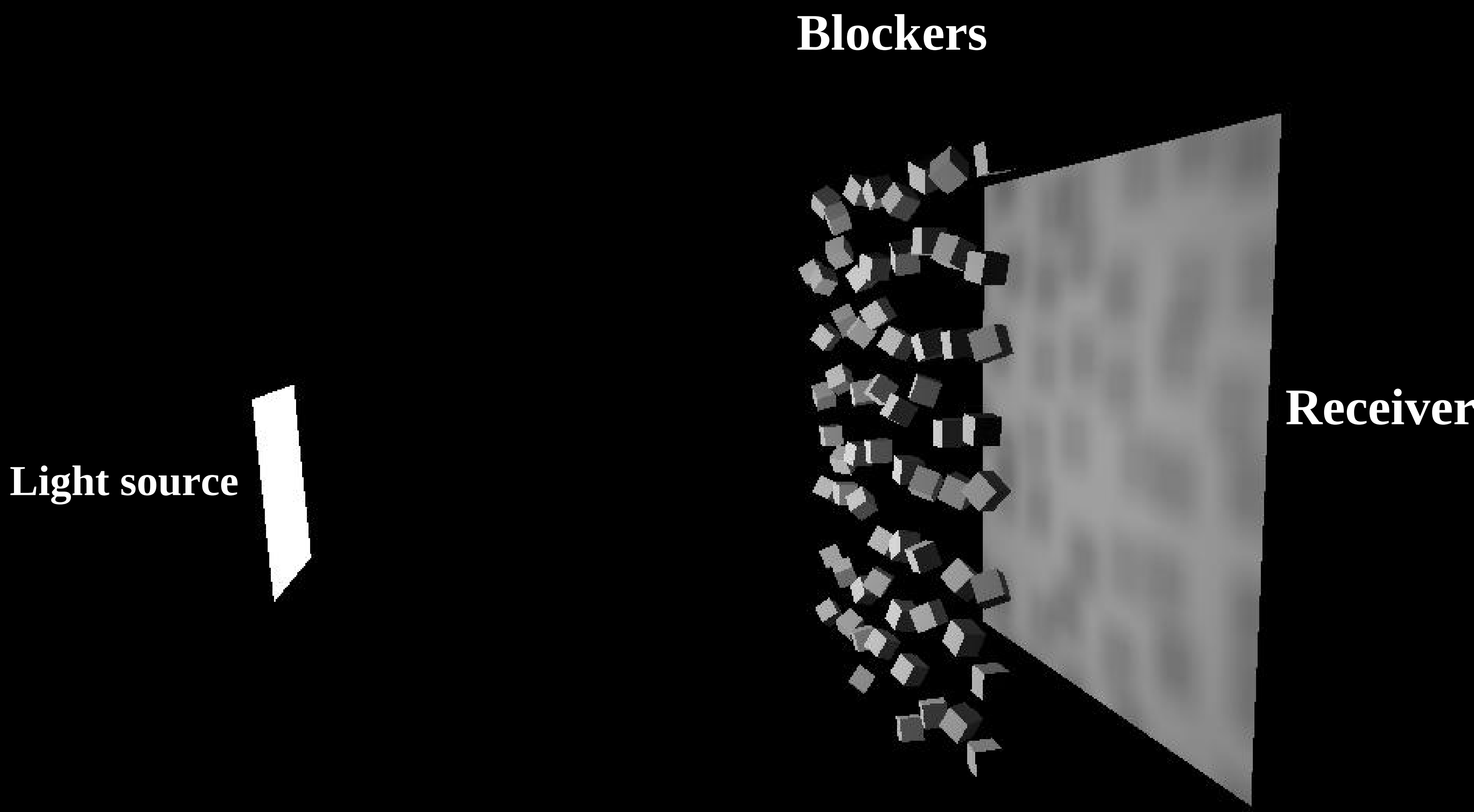
# Example scene



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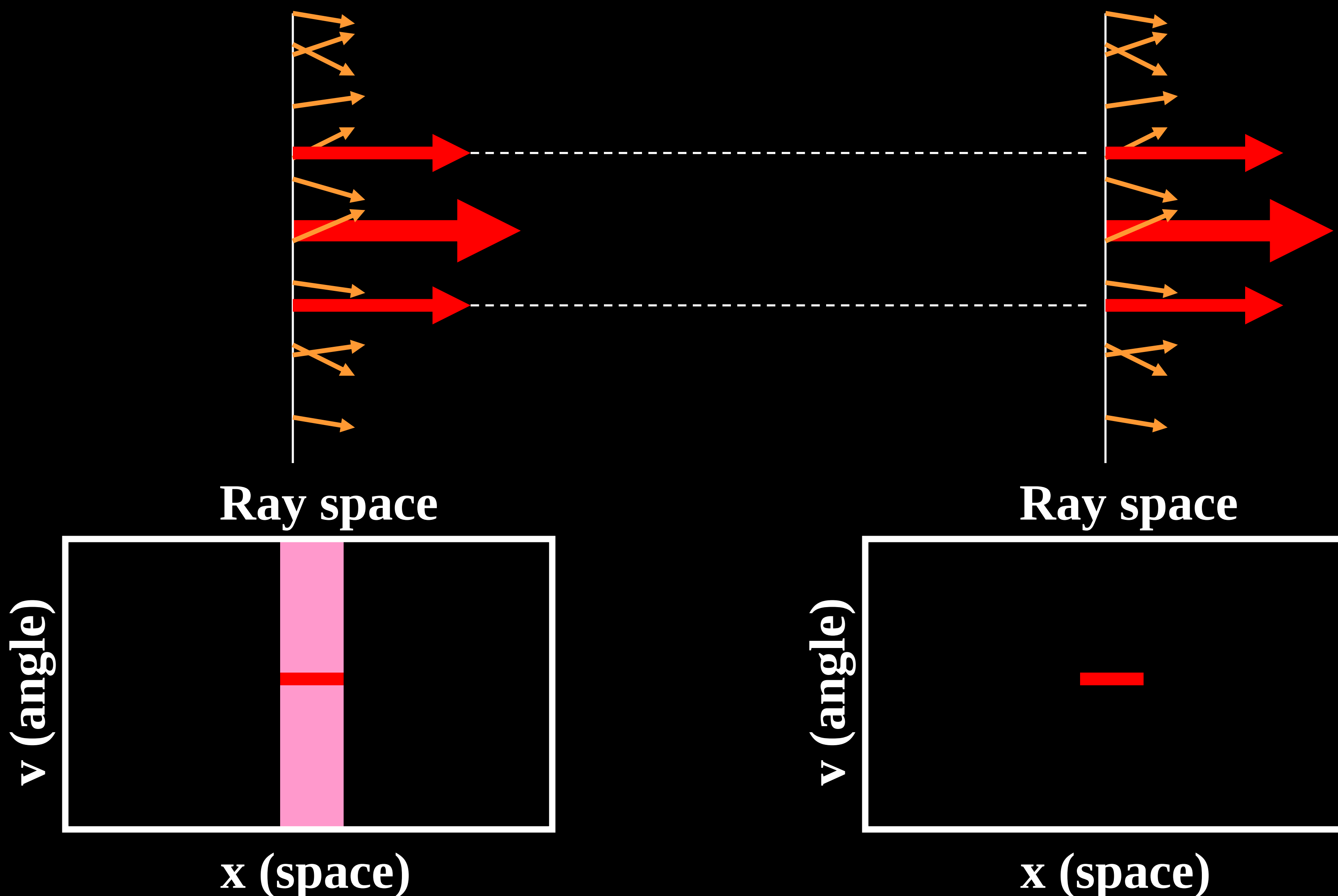
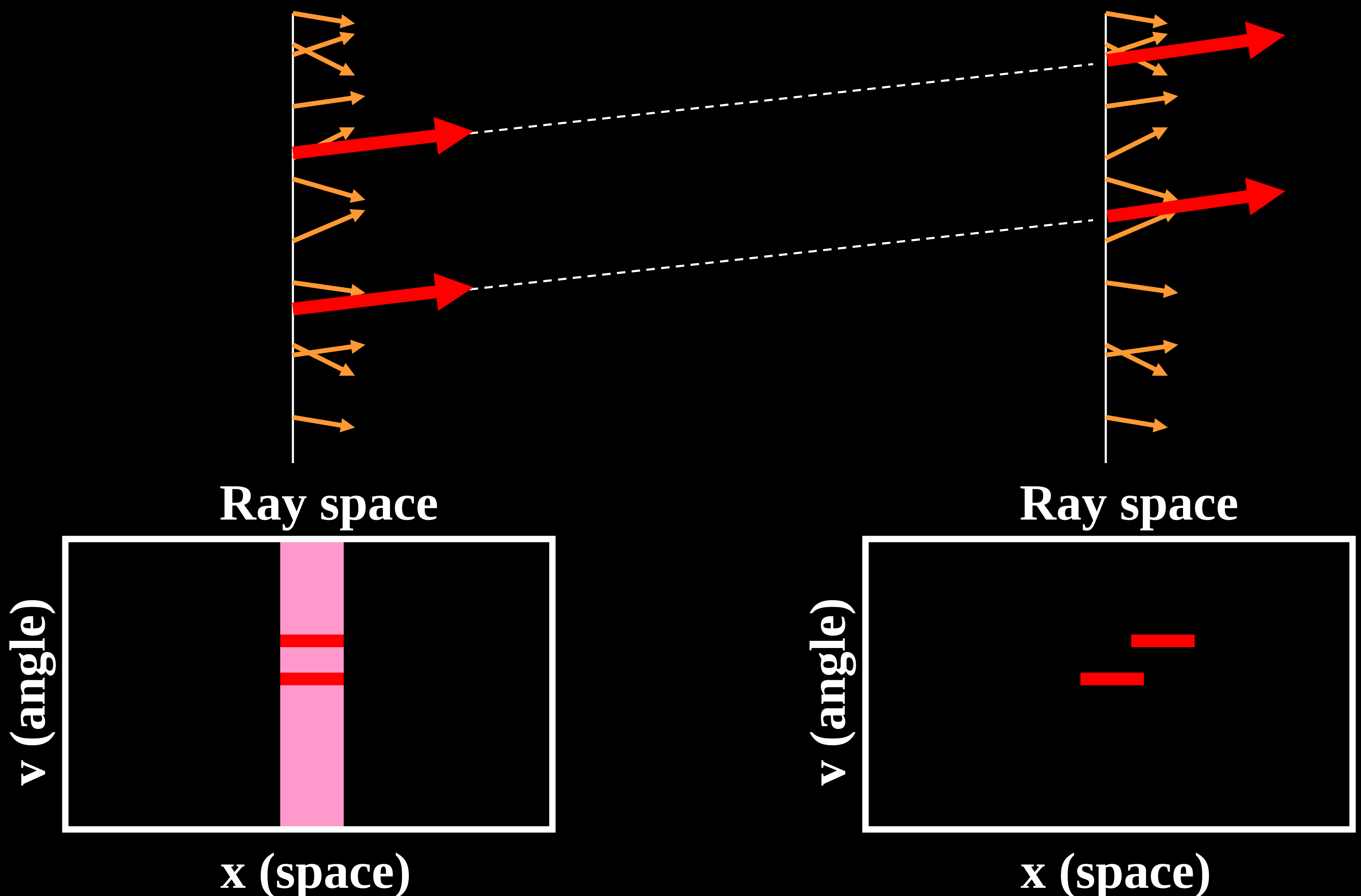
# Transport



# Transport



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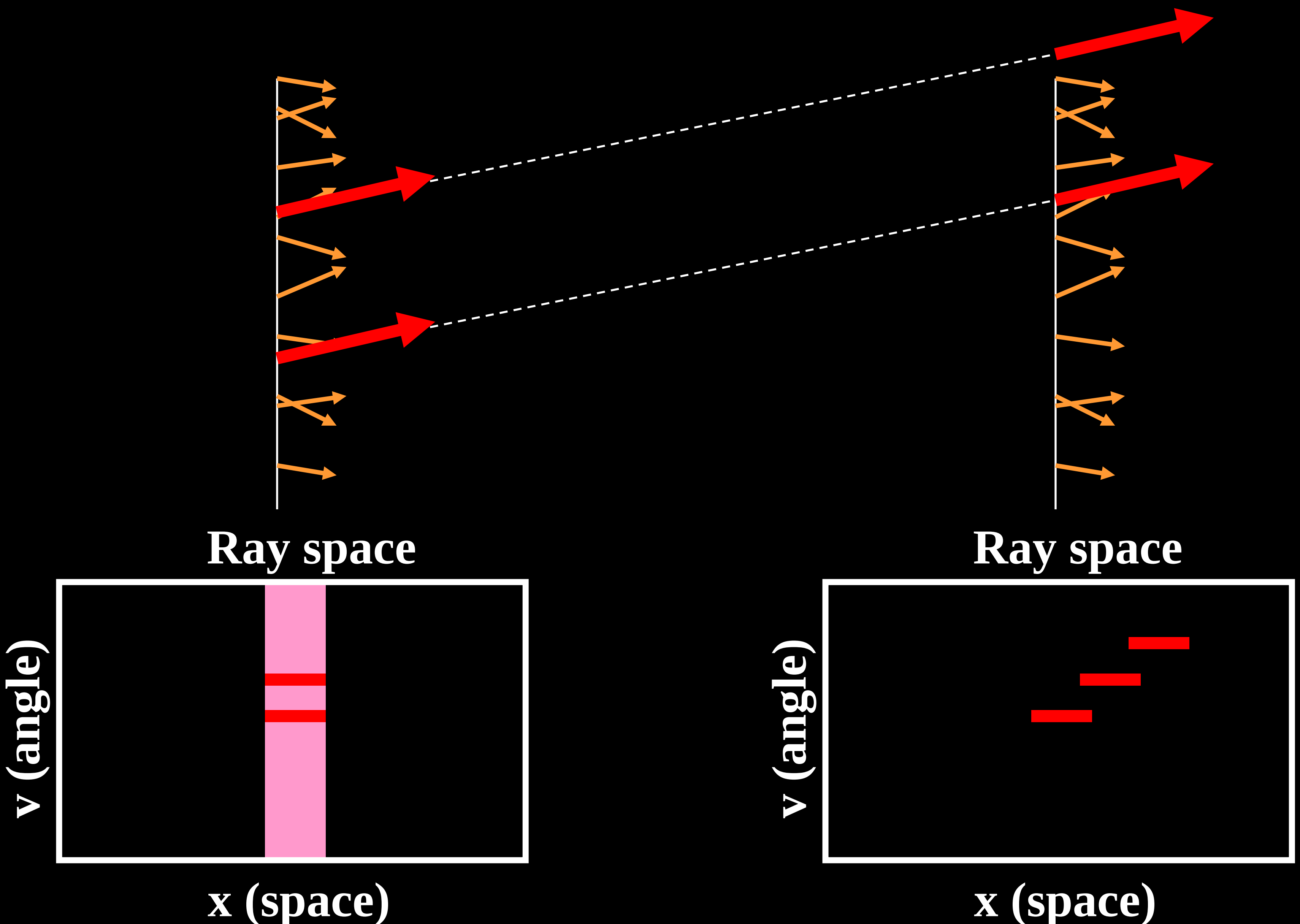
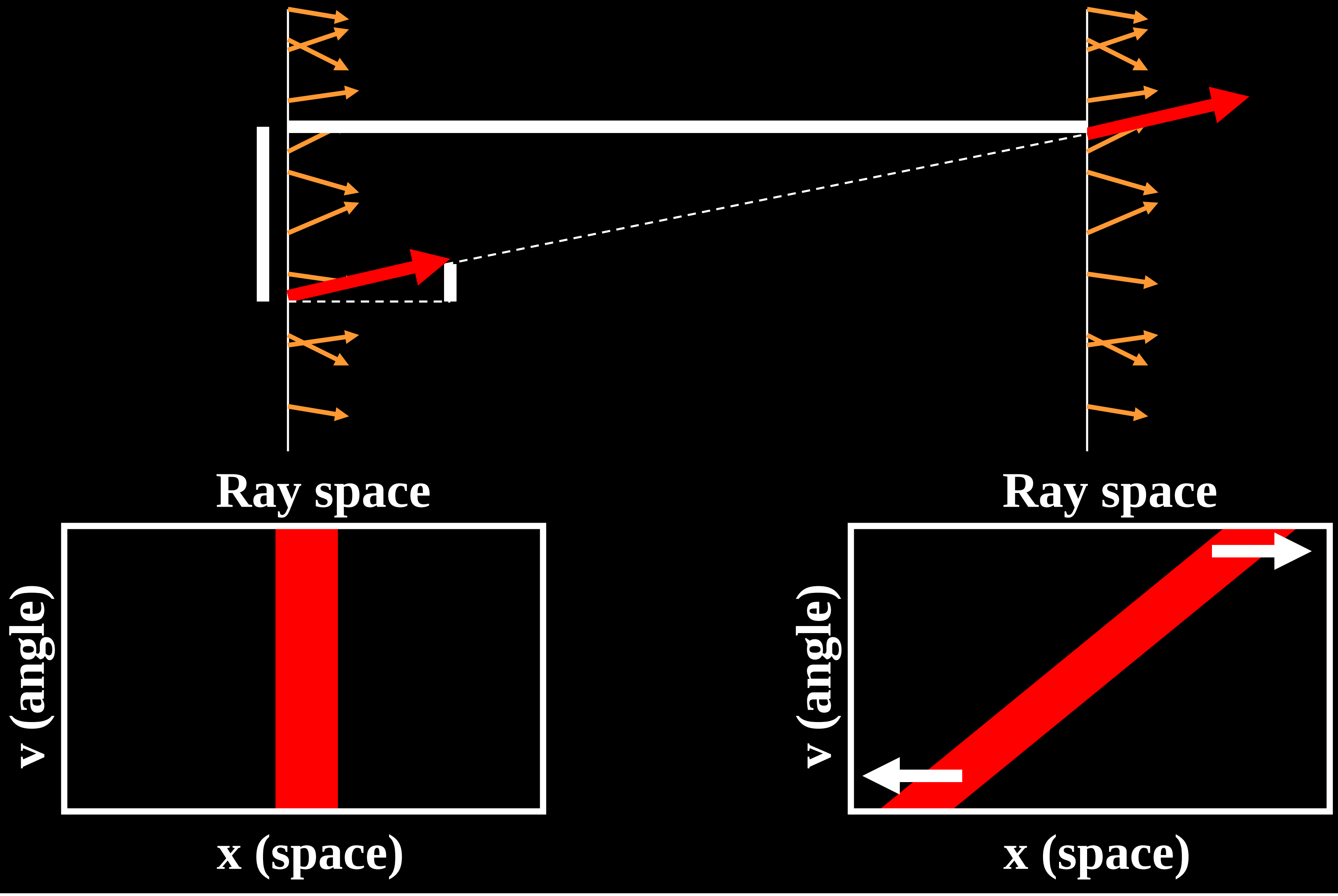


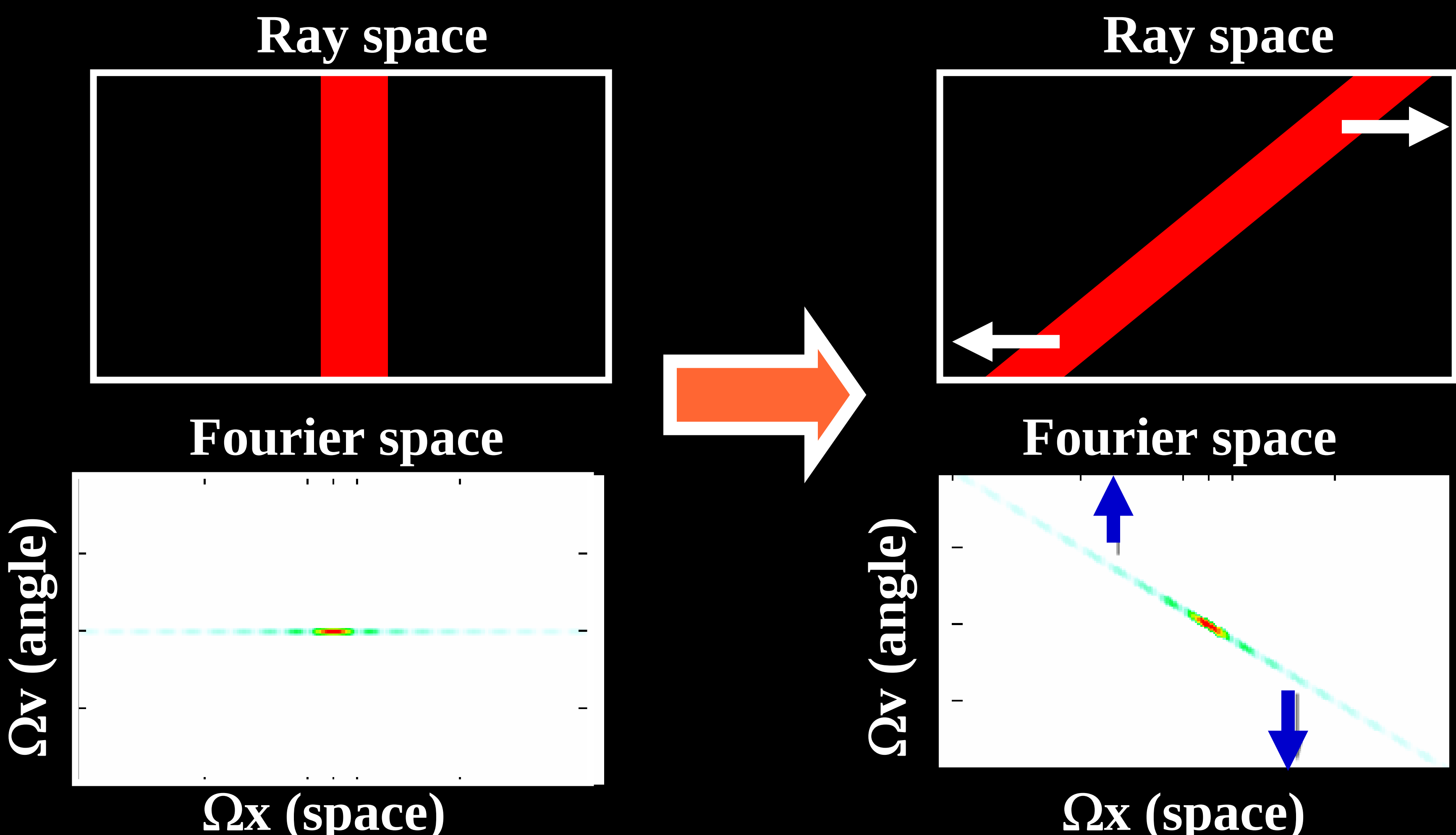
# Transport

- Shear: $x' = x - v\,d$



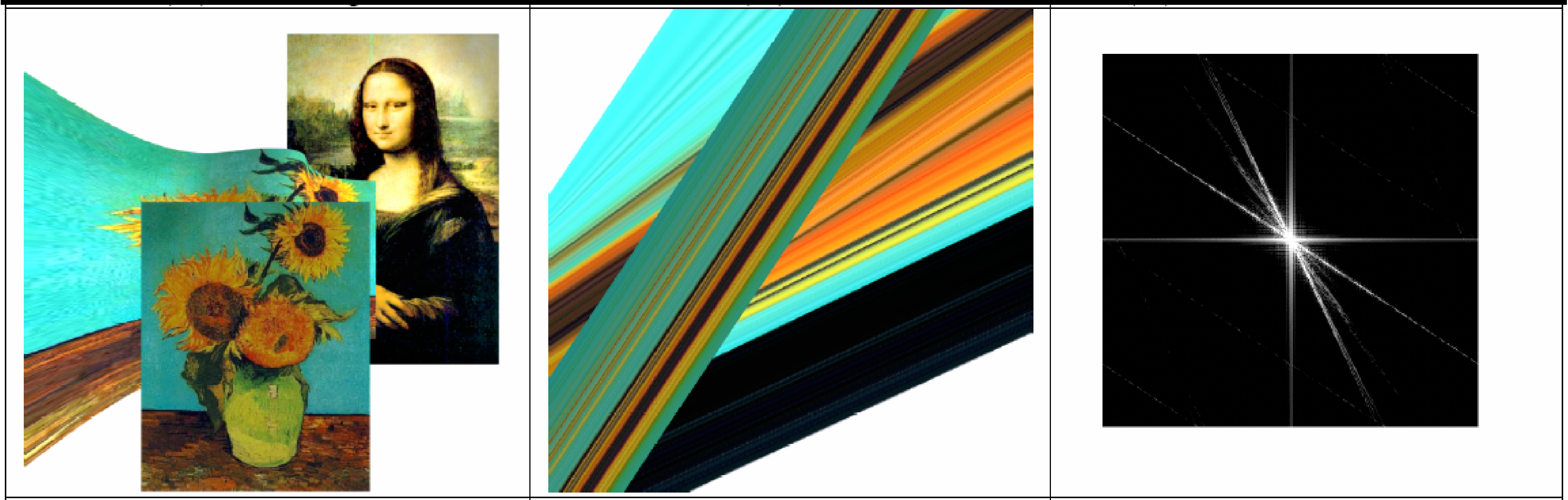
# Transport in Fourier space

- Shear in primal: $x' = x - v\,d$
- Shear in Fourier, along the other dimension



# Transport → Shear

- This is consistent with light field spectra
  [Chai et al. 00, Isaksen et al. 00]



| (d1)Scene image | (d2) EPI | (d3) Fourier transform of EPI |
|---|---|---|

From [Chai et al. 2000]

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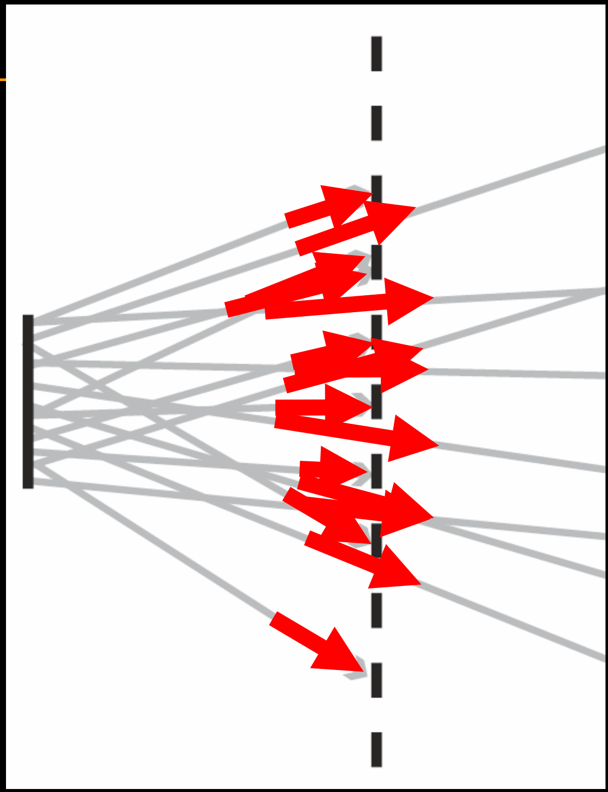
# Occlusion: multiplication

- Occlusion is a multiplication in ray space
  - Convolution in Fourier space
- Creates new spatial frequency content
  - Related to the spectrum of the blockers

# Occlusion

- Consider planar occluder
- Multiplication by binary function
  - Mostly in space



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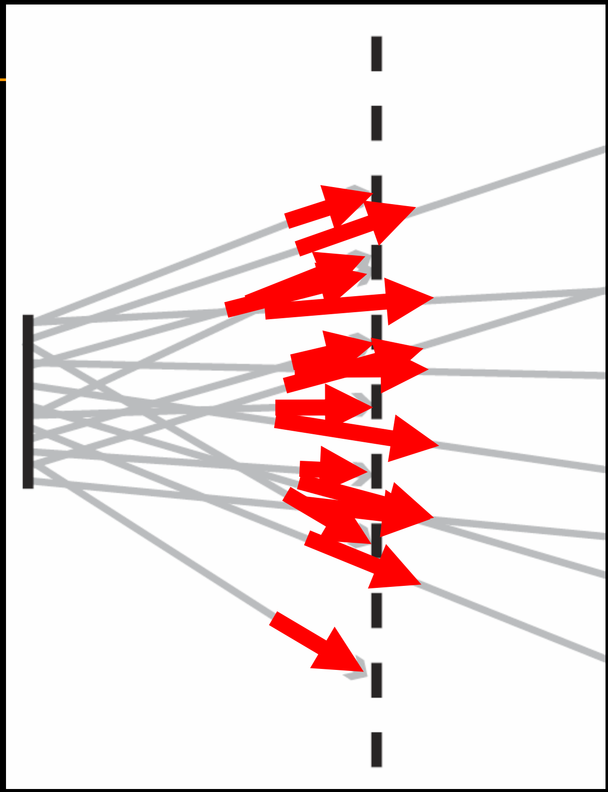
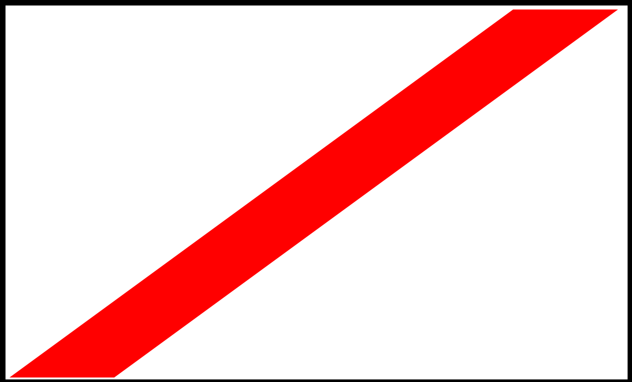
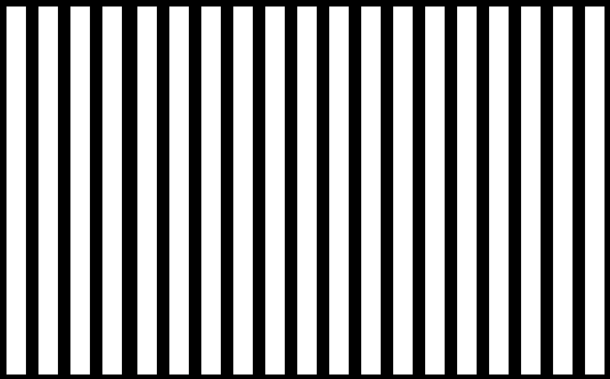
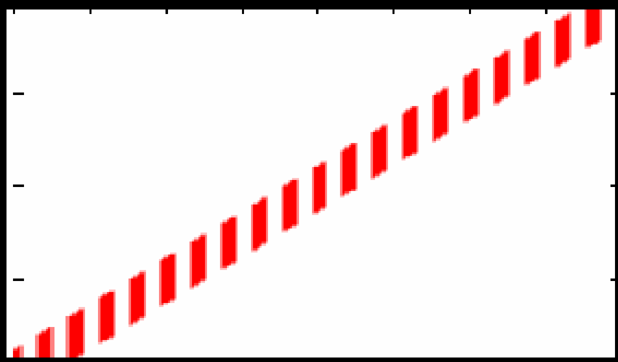




Before occlusion



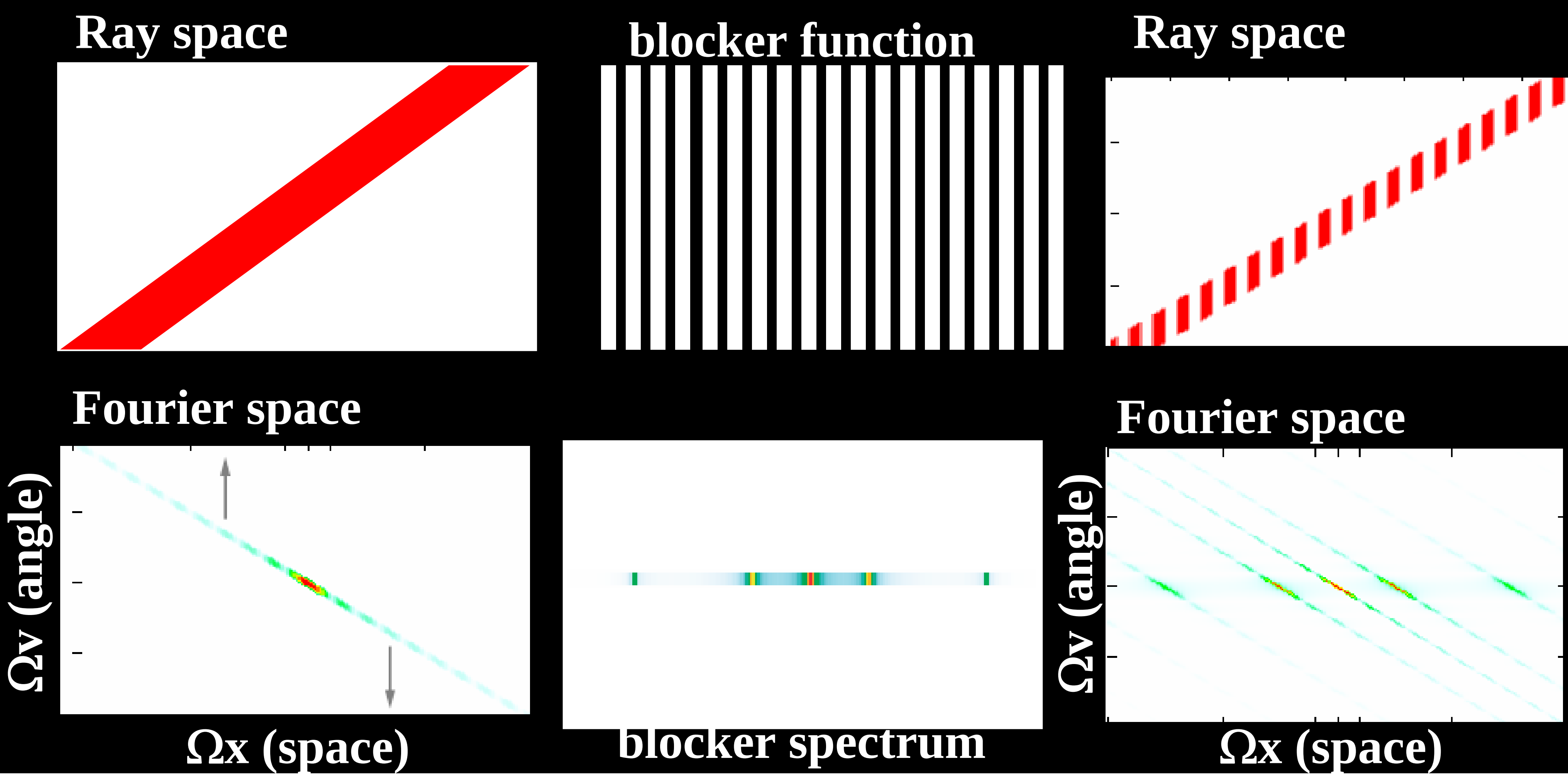
blocker function



After occlusion

# Occlusion in Fourier

- Multiplication in primal
- Convolution in Fourier (creates high freq.)



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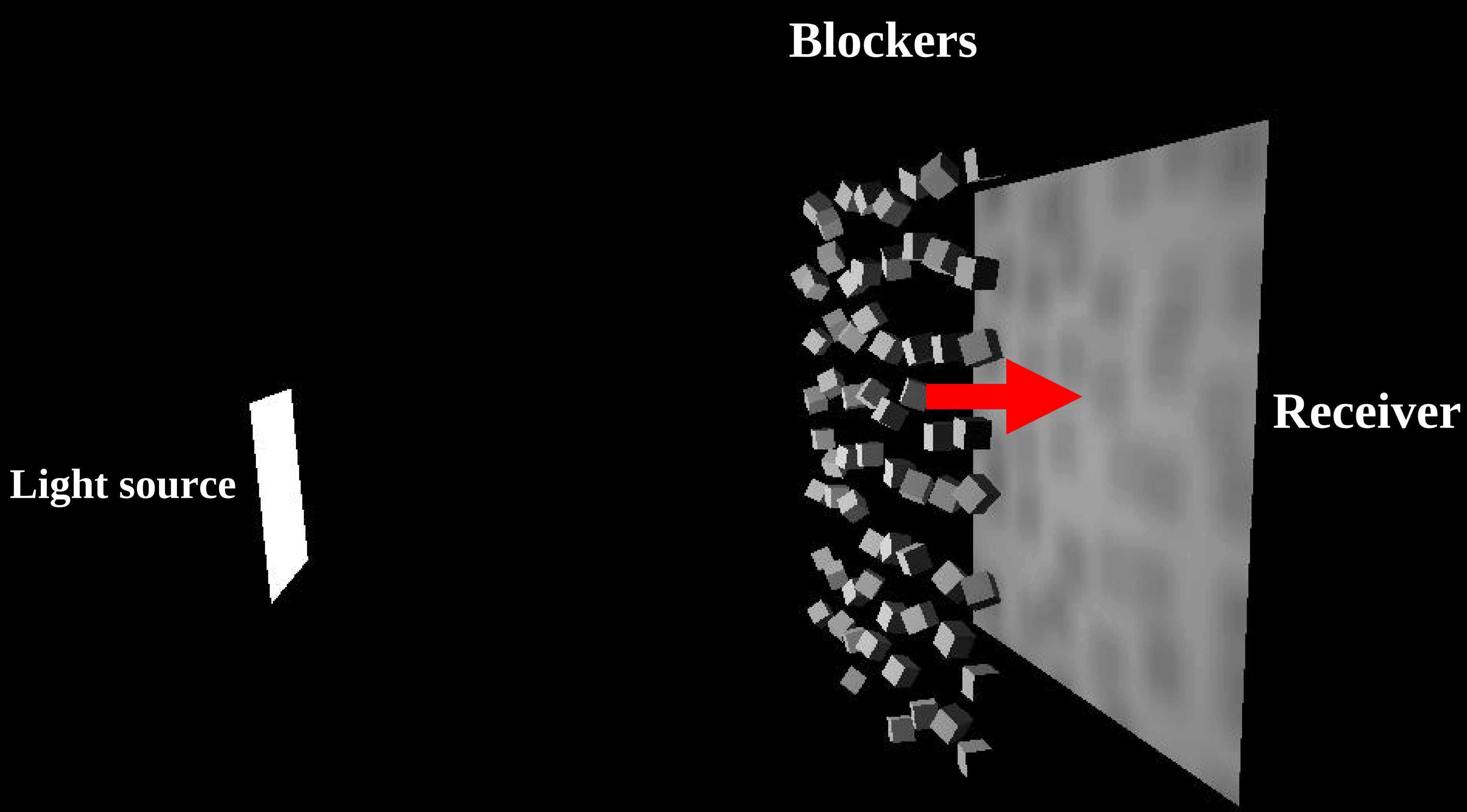
# Another propagation



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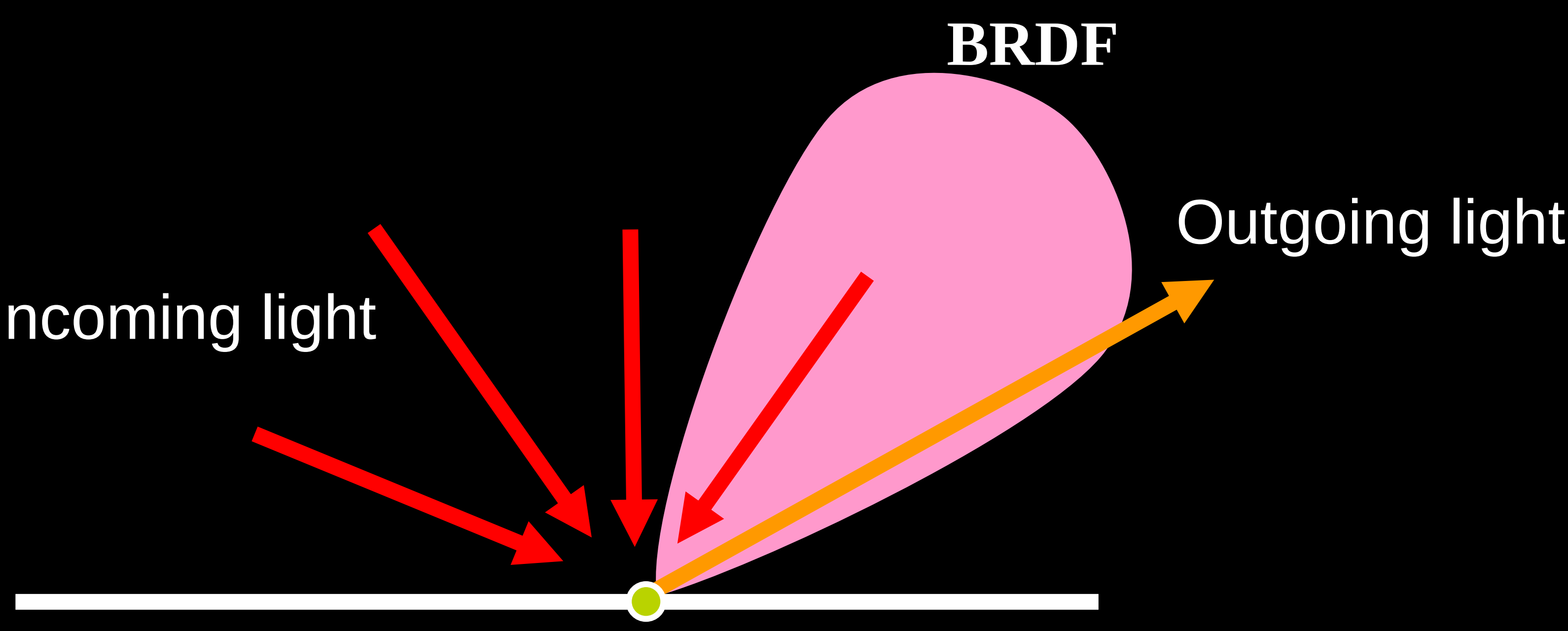
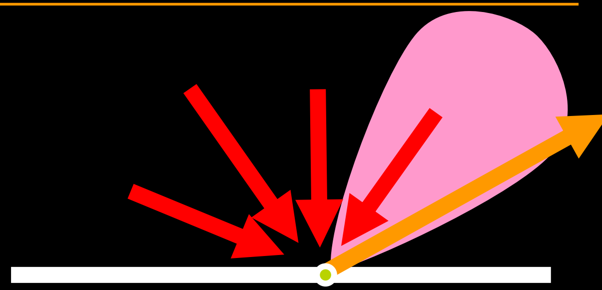
# BRDF integration

- Outgoing light:
  - Integration of incoming light times BRDF

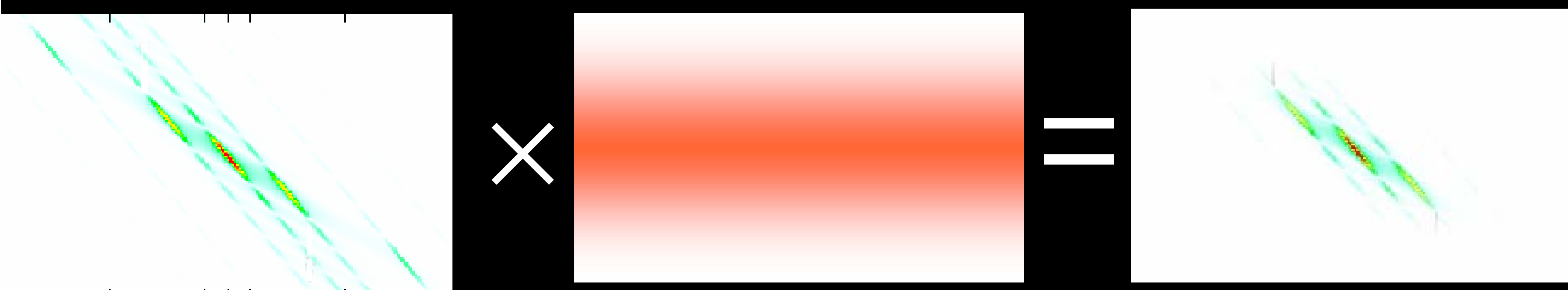


# BRDF integration



- Ray-space: ***convolution***
  - Outgoing light:
    convolution of incoming light and BRDF
  - For rotationally-invariant BRDFs
- Fourier domain: ***multiplication***
  - Outgoing spectrum: multiplication of incoming spectrum and BRDF spectrum

# BRDF in Fourier: multiplication



- BRDF is bandwidth-limiting in angle

# Example: diffuse BRDF

- Convolve by constant:
  - multiply by horizontal window
  - Only spatial frequencies remain

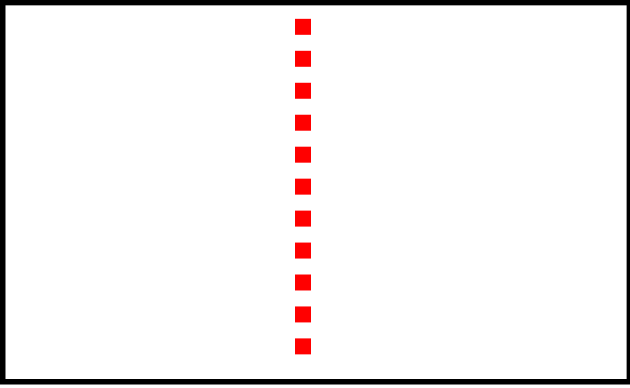
# Relation to previous work

- [Ramamoorthi & Hanrahan 01, Basri & Jacob 03]
- They consider a spatially-constant illumination

- But essentially, nothing much changed

Ray space

Fourier space



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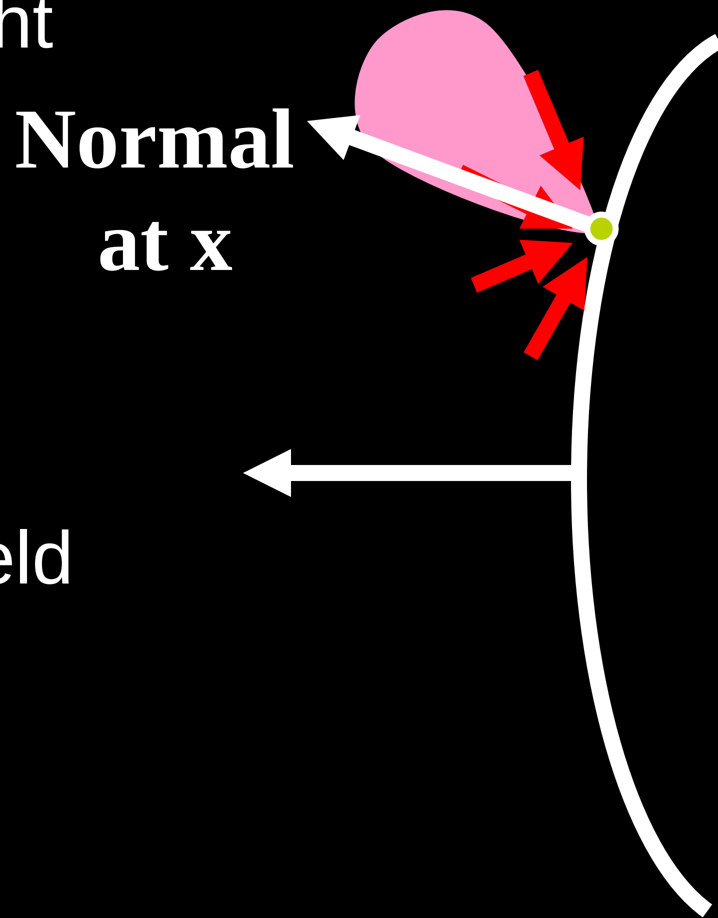
# Curvature

- Ray reaches curved surface:
  - Transform it into planar surface
  - “Unroll” curved surface
- Equivalent to changing angular content:
  - Linear effect on angular dimension
  - No effect on spatial dimension
- Shear in the angular dimension

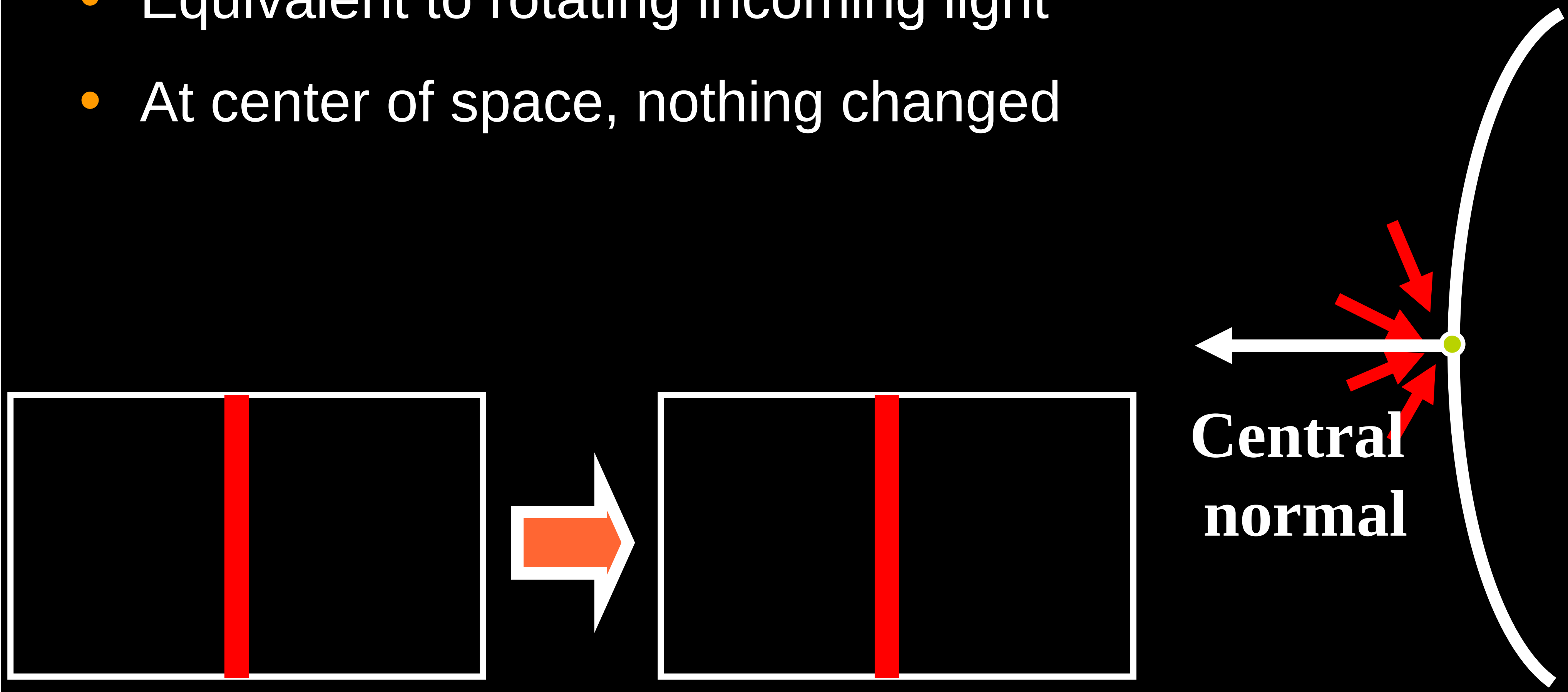
# Curvature

- For each point x, the normal has a different angle
- Equivalent to rotating incoming light
- We will reparameterize the light field



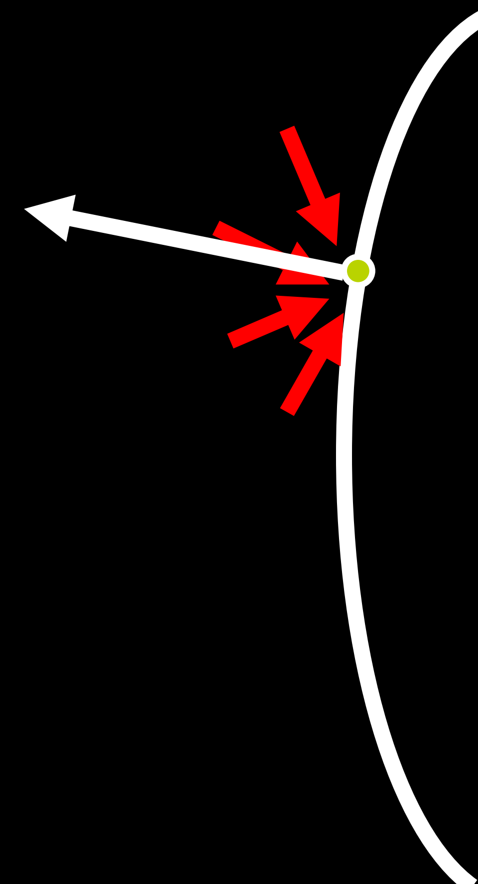
# Curvature

- For each point x, the normal has a different angle
- Equivalent to rotating incoming light
- At center of space, nothing changed



# Curvature

- For each point x, the normal has a different angle
- Equivalent to rotating incoming light
- The further away from central ray, the more rotated



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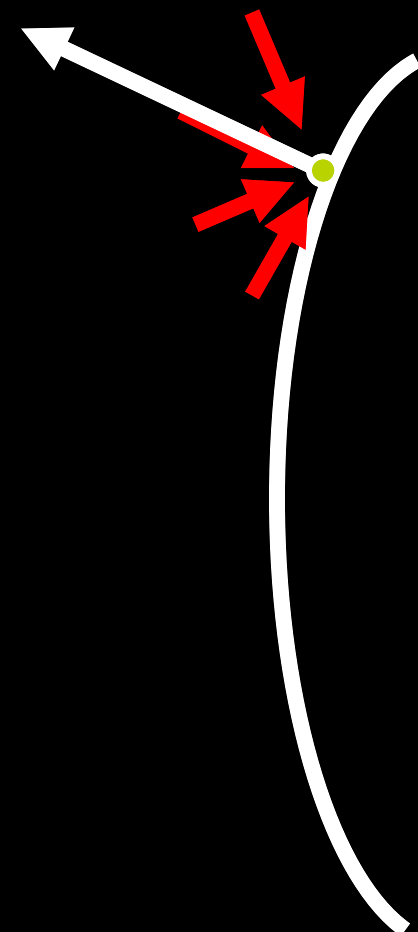
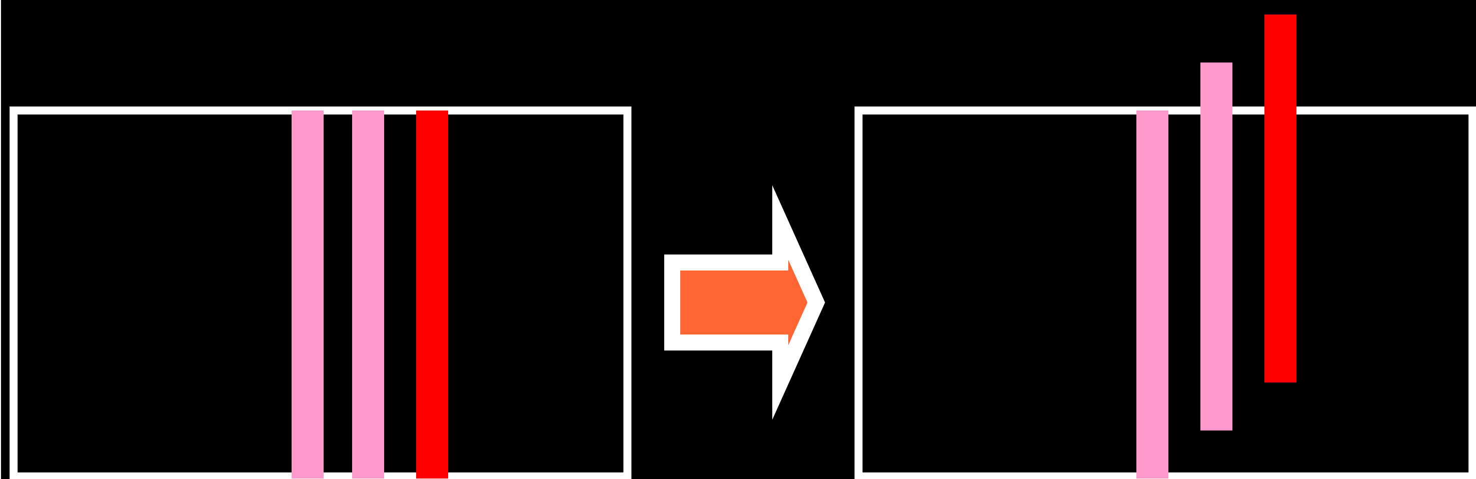
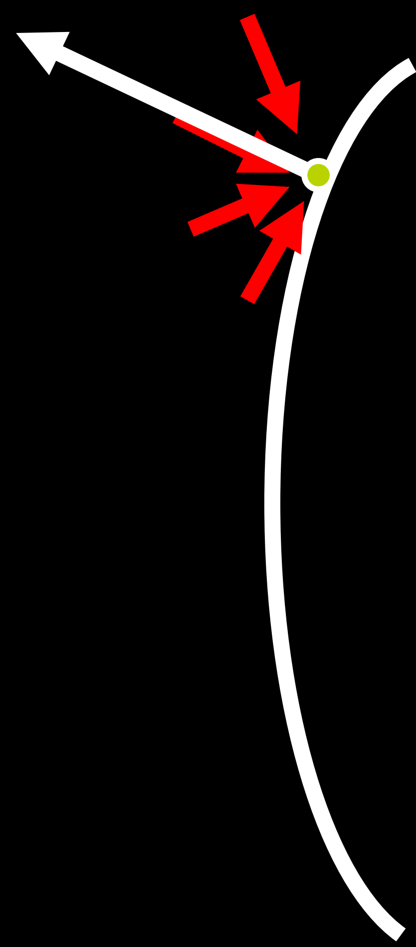
- Equivalent to rotating incoming light
- The further away from central ray, the more rotated

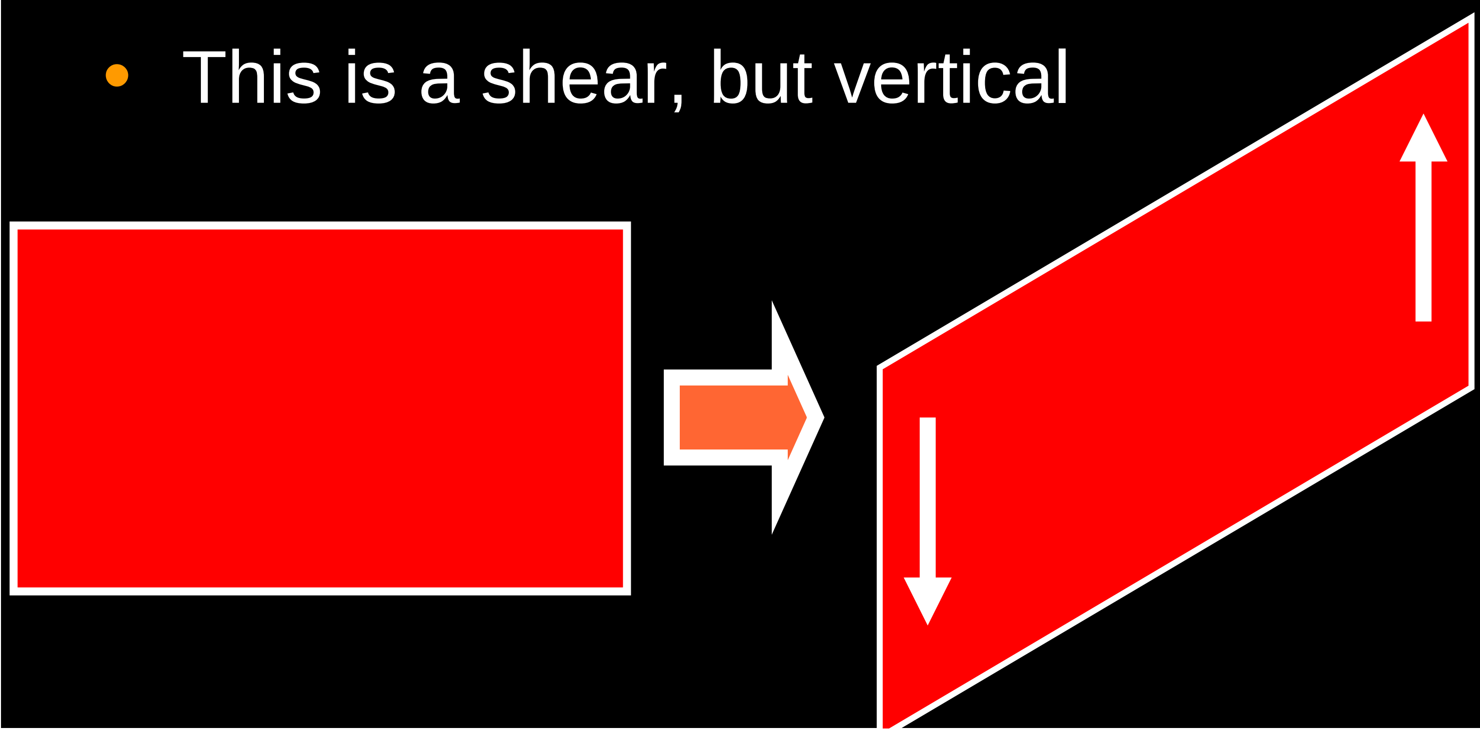




# Curvature

- For each point x, the normal has a different angle
- Equivalent to rotating incoming light
- The further away from central ray, the more rotated
- This is a shear, but vertical





# Main transforms: summary

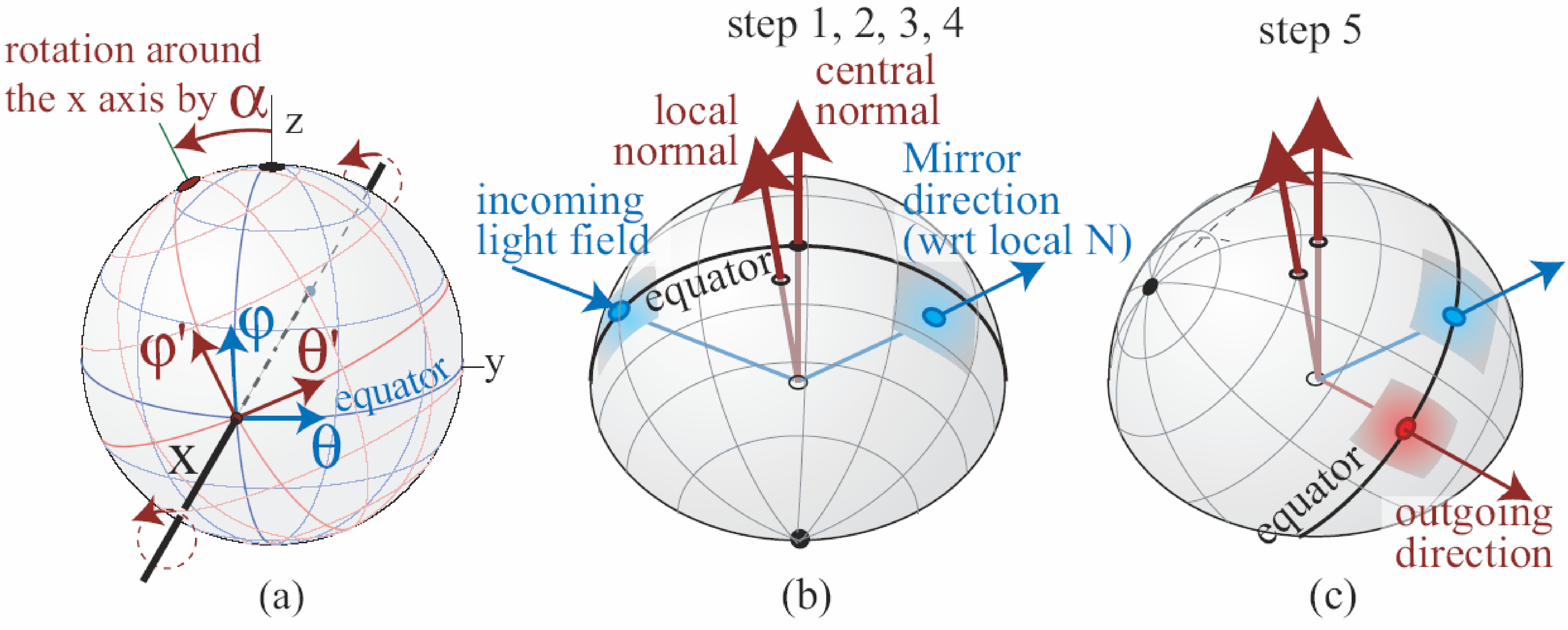
| | Radiance/Fourier | Effect |
|---|---|---|
| Transport | Shear | |
| Occlusion | Multiplication/Convolution | Adds spatial frequencies |
| BRDF | Convolution/Multiplication | Removes angular frequencies |
| Curvature | Shear | |

# Overview

- Previous work
- Our approach:
  - Local light field
  - Transformations on local light field
    - Transport
    - Occlusion
    - BRDF
    - Curvature
    - Other transforms
- Case studies
- Conclusions and future directions

# Extension to 3D

- It works. See paper:



# Even more effects in paper

- Various technical details
  - Cosine/Fresnel term:
  - Central incidence angle:
- Texture mapping (multiplication/convolution)
- Separable BRDF
- Spatially varying BRDF (semi-convolution)

- …and full extension to 3D

# Overview

- Previous work
- Our approach:
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- Case studies:
  - Diffuse soft shadows
  - Solar oven
  - Adaptive shading sampling
- Conclusions and future directions

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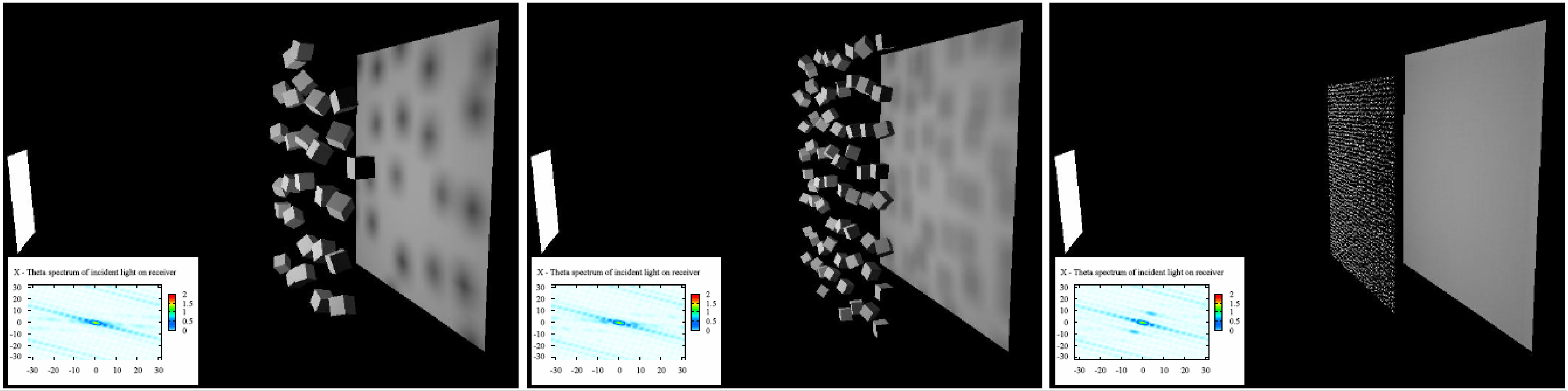
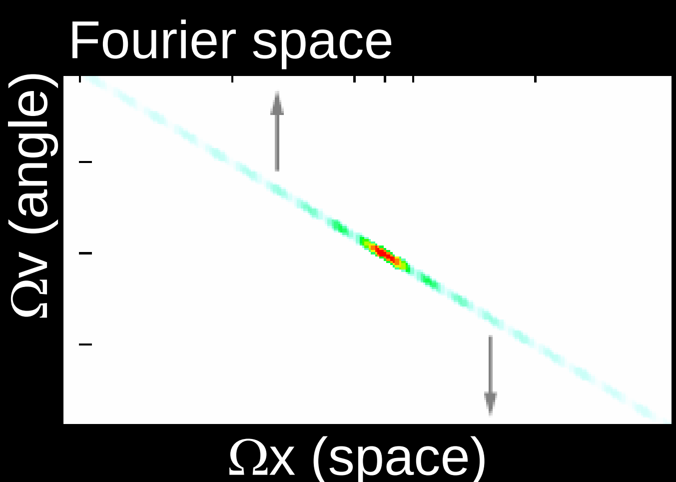
# Diffuse soft shadows

- Decreasing blockers size:
  - First high-frequencies increase
  - Then only low frequency
  - Non-monotonic behavior

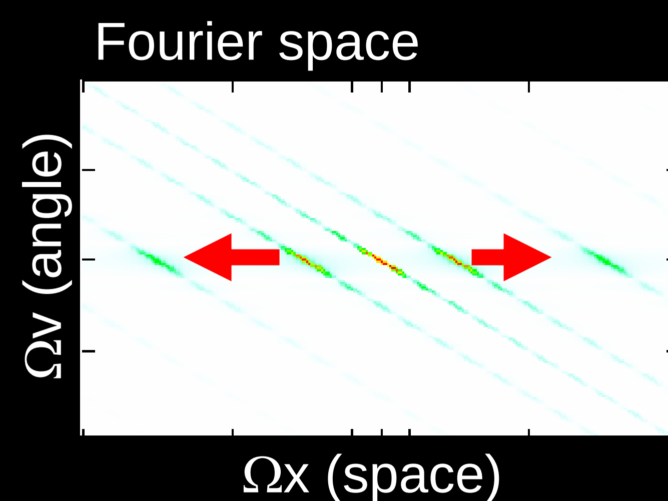


# Diffuse soft shadows (2)

- Occlusion : convolution in Fourier
  - creates high frequencies
  - Blockers scaled down → spectrum scaled up

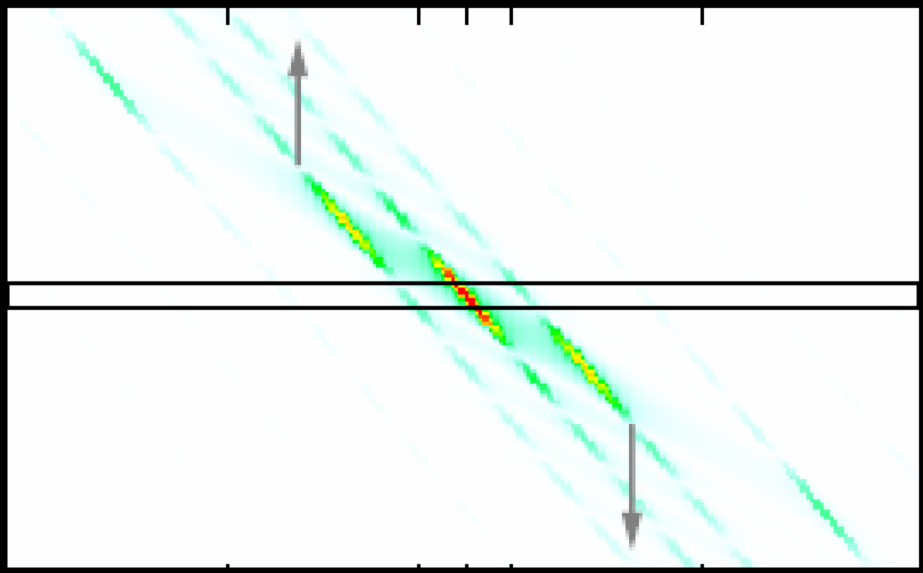






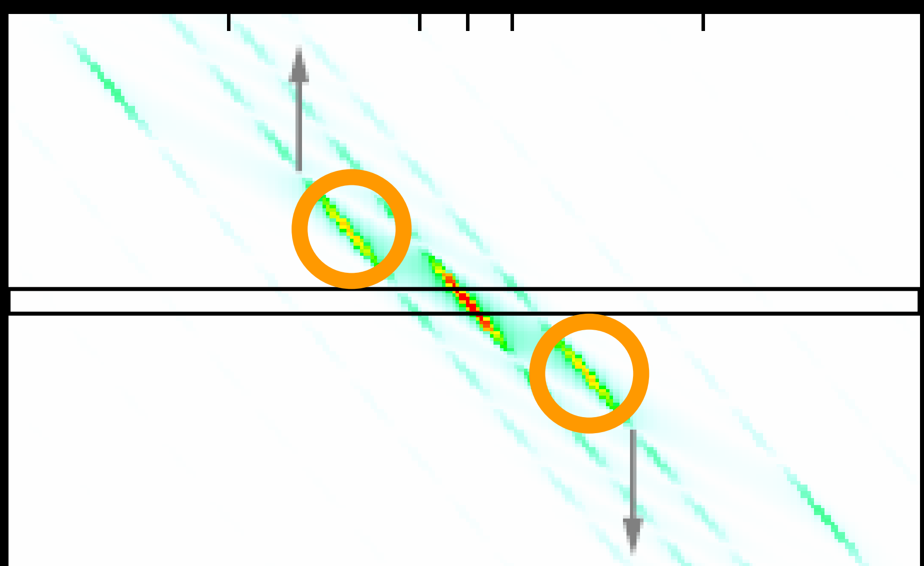
# Diffuse soft shadows (3)

- Transport after occlusion:
  - Spatial frequencies moved to angular dimension
- Diffuse reflector:
  - Angular frequencies are cancelled



# Diffuse soft shadows (3)

- Transport after occlusion:
  - Spatial frequencies moved to angular dimension
- Diffuse reflector:
  - Angular frequencies are cancelled



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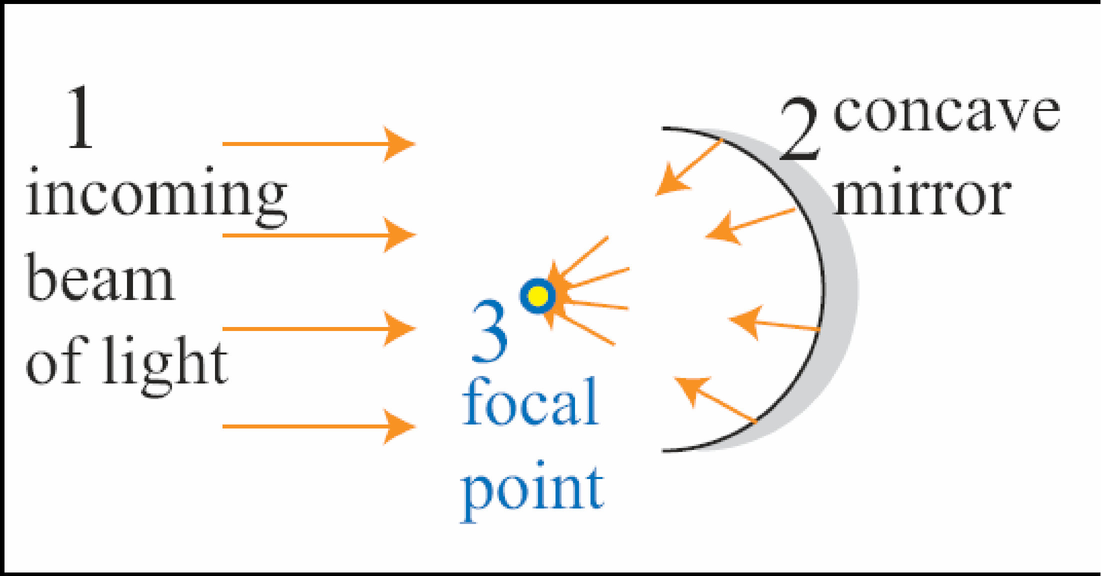
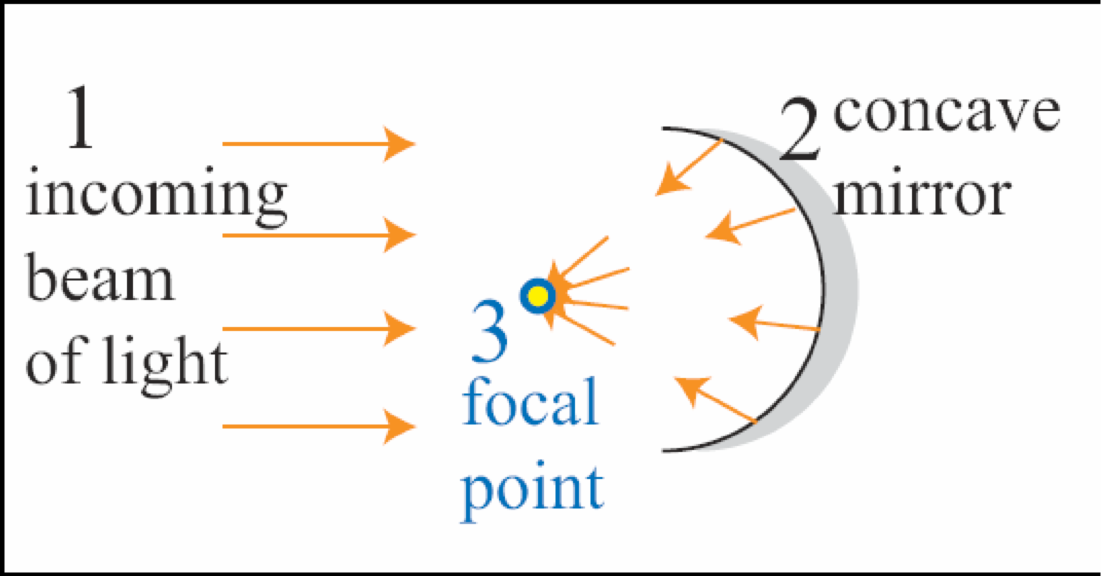
# Solar oven

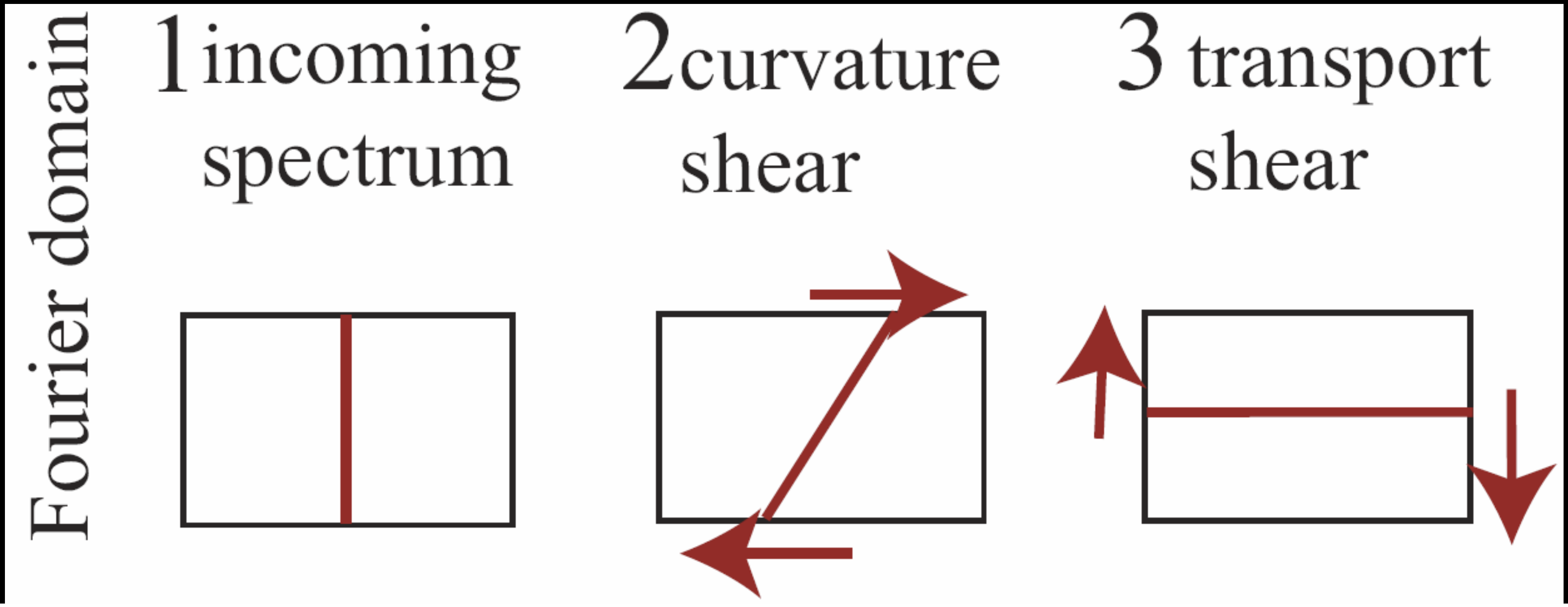
- Curved surface
- In: parallel light rays
- Out: focal point



# Solar oven

- Curved surface
- In: parallel light rays
- Out: focal point



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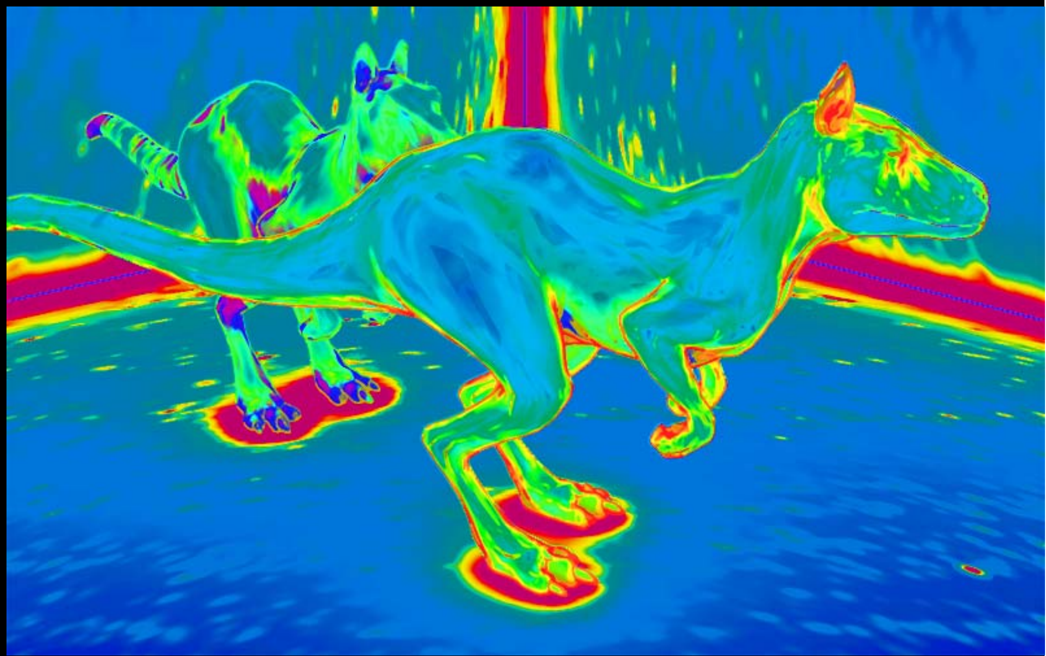
# Adaptive shading sampling

- Monte-Carlo ray tracing
- Blurry regions need fewer shading samples

# Adaptive shading sampling

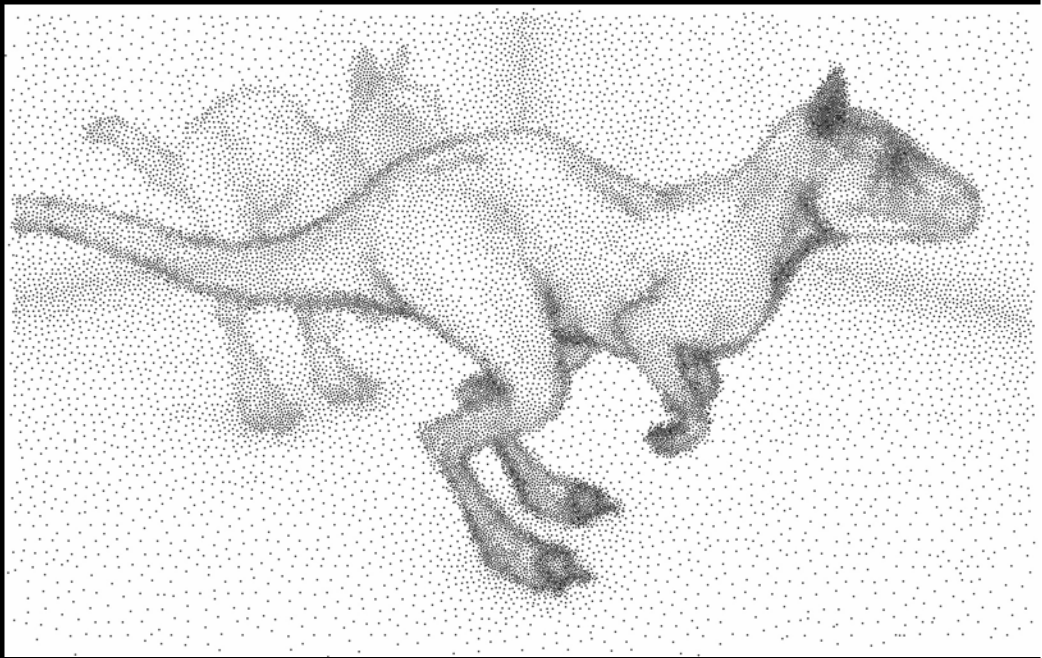
- Per-pixel prediction of max. frequency (bandwidth)
  - Based on curvature, BRDF, distance to occluder, etc.
  - No spectrum computed, just estimate max frequency



Per-pixel bandwidth criterion

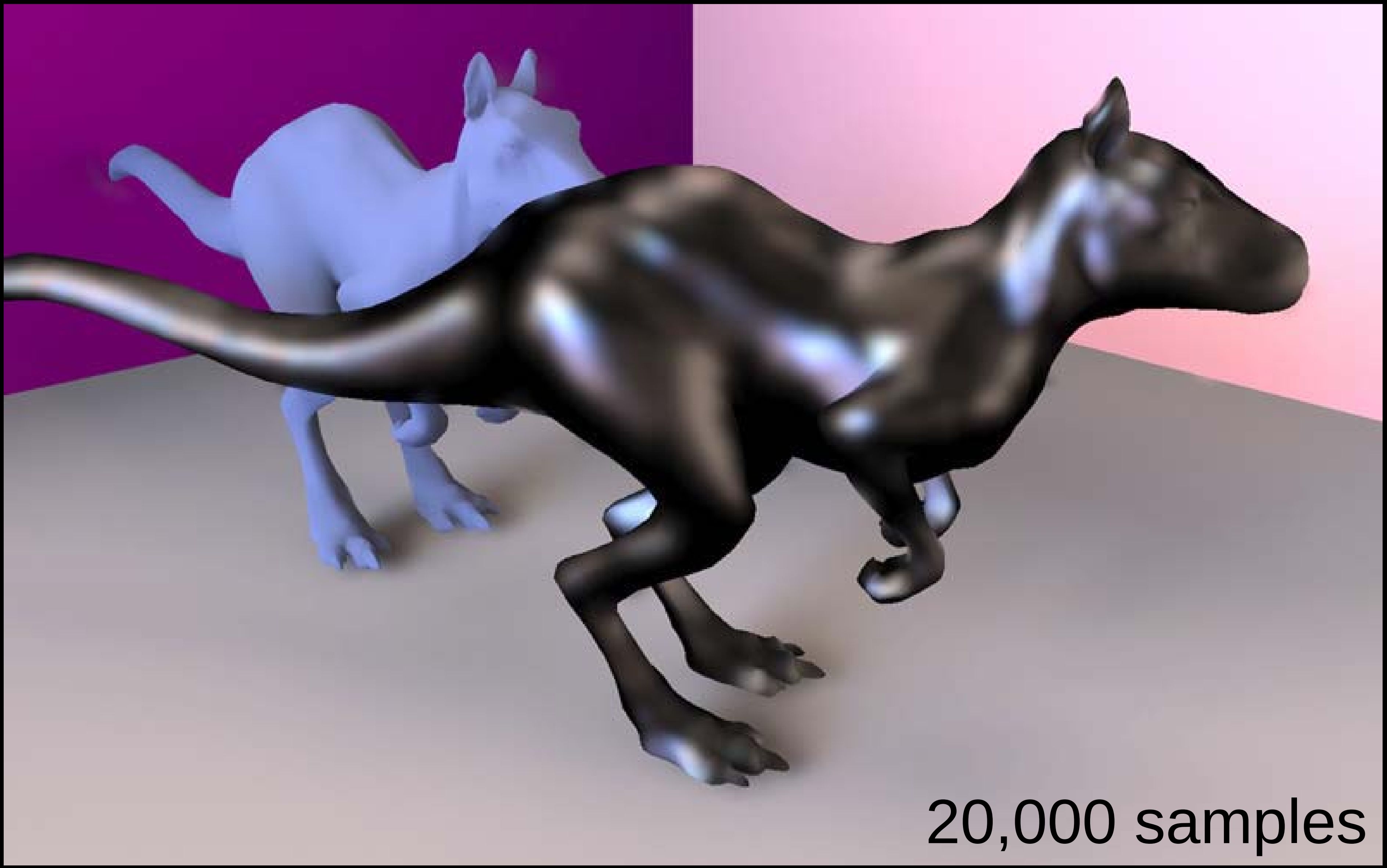
# Adaptive shading sampling

- Per-pixel prediction of max. frequency (bandwidth)
  - Based on curvature, BRDF, distance to occluder, etc.
  - No spectrum computed, just estimate max frequency

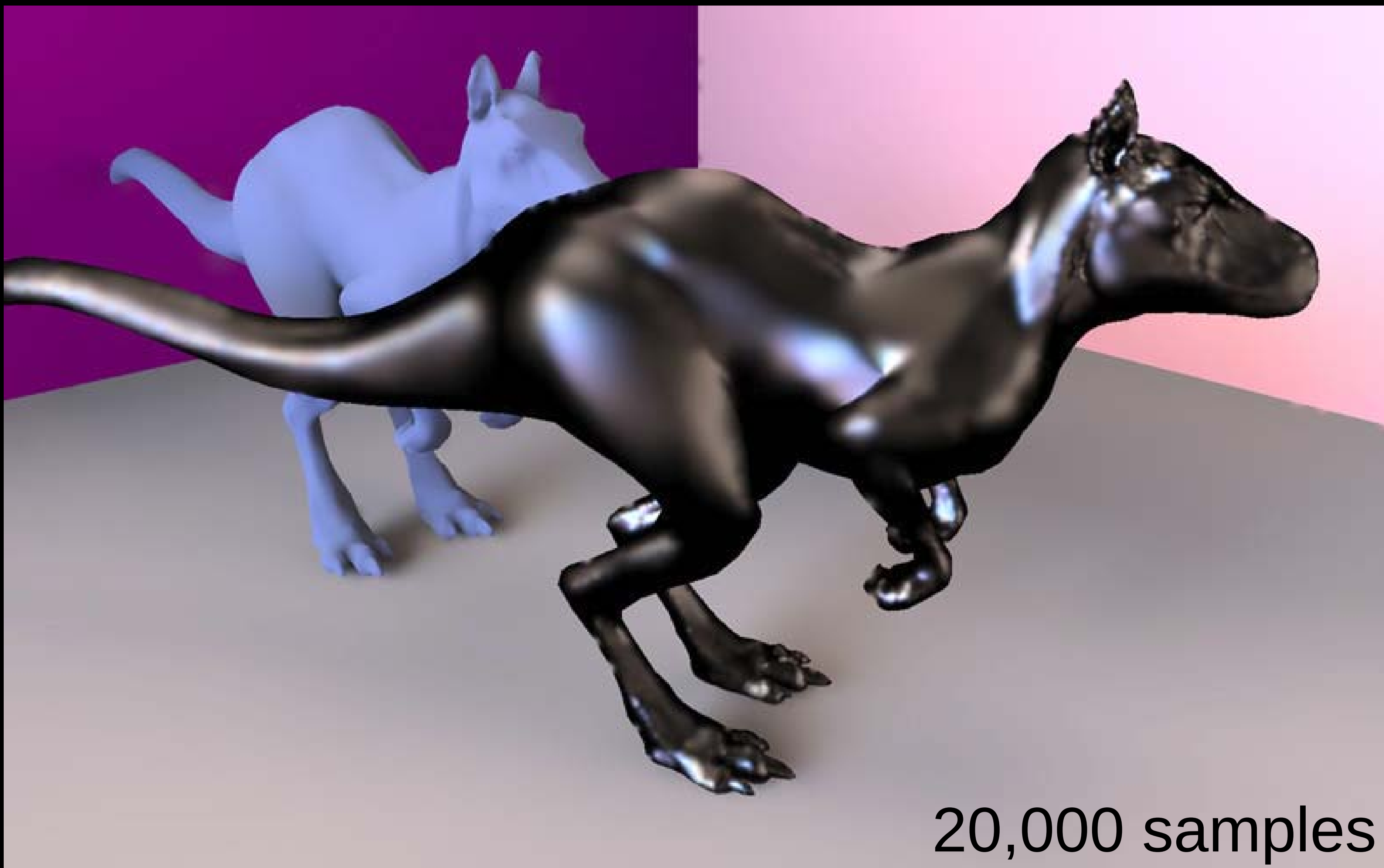


Shading samples

# Uniform sampling



# Adaptive sampling



# Overview

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# Conclusions

- Unified framework:
  - For frequency analysis of radiance
  - In both space and angle
  - Simple mathematical operators
  - Extends previous analyses
- Explains interesting lighting effects:
  - Soft shadows, caustics
- Proof-of-concept:
  - Adaptive sampling

# Future work

- More experimental validation on synthetic scenes
- Extend the theory:
  - Bump mapping, microfacet BRDFs, sub-surface scattering…
  - Participating media
  - Wave optics
- Applications to rendering:
  - Photon mapping, spatial sampling for PRT
  - Revisit traditional techniques
- Vision and shape from shading
- 3D displays aliasing
- Optics and computational photography

# Acknowledgments


- Jaakko Lehtinen
- Reviewers of the MIT and ARTIS graphics groups
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  - INRIA Équipe associée
  - Realreflect EU IST project
  - MIT-France

# Why *Local* Light Field?

- Linearization:
  - $\theta \approx \tan\theta$
  - Curvature
  - Extension to 3D
- *Local* information is what we need:
  - Local frequency content, for local sampling
  - Based on local properties of the scene (occluders)

# Other bases?

- We’re not using Fourier as a function basis
  - Don’t recommend it, actually
  - Just used for analysis, understanding, predictions
- Results are useable with any other base:
  - Wavelets, Spherical Harmonics, point sampling, etc
  - Max. frequency translates in sampling rate
- Analysis relies on Fourier properties:
  - Especially the convolution theorem

# Reflection on a surface: Full summary

- Angle of incidence
- Curvature
- Cosine/Fresnel term
- Mirror re-parameterization
- BRDF
- Curvature

# Reflection on a surface: Full summary

- Angle of incidence: scaling
- Curvature: shear in angle
- Cosine/Fresnel term: multiplication/convolution
- Mirror re-parameterization
- BRDF: convolution/multiplication
- Curvature: shear in angle