A Frequency Analysis of Light Transport

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Illumination effects

• Blurry reflections:

From [Ramamoorthi and Hanrahan 2001]
Illumination effects

- Shadow boundaries:

  Point light source

  Area light source

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Illumination effects

- Indirect lighting is usually blurry:

Complete lighting
Illumination effects

• Indirect lighting is usually blurry:
Frequency aspects of light transport

• Blurriness = frequency content
  – Sharp variations: high frequency
  – Smooth variations: low frequency

• All effects are expressed as frequency content:
  – Diffuse shading: low frequency
  – Shadows: introduce high frequencies
  – Indirect lighting: tends to be low frequency
Problem statement

• How does light interaction in a scene explain the frequency content?

• Theoretical framework:
  – Understand the frequency spectrum of the radiance function
  – From equations of light transport
Unified framework:

- Spatial frequency (e.g. shadows, textures)
- Angular frequency (e.g. blurry highlight)
Disclaimer: not Fourier optics

- We do **not** consider wave optics, interference, diffraction
- Only geometrical optics

From [Hecht]
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From [Hecht]
Overview

• Previous work
• Our approach:
  – Local light field
  – Transformations on local light field
• Case studies:
  – Diffuse soft shadows
  – Adaptive shading sampling
• Conclusions and future directions
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Previous work

- Vast body of literature:
  - Light field sampling
  - Perceptually-based rendering
  - Wavelets for Computer Graphics
  - Irradiance caching
  - Photon mapping
  - ...

- We focus on frequency analysis in graphics:
  - Light field sampling
  - Reflection as a convolution
Light field sampling

[Chai et al. 00, Isaksen et al. 00, Stewart et al. 03]

- Light field spectrum as a function of object distance
- No BRDF, occlusion ignored

From [Chai et al. 2000]
Signal processing for reflection

[Ramamoorthi & Hanrahan 01, Basri & Jacobs 03]

• Reflection on a curved surface is a convolution
• Direction only
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Our approach

• Light sources are input signal
• Interactions are filters/transforms
  – Transport
  – Visibility
  – BRDF
  – Etc.
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Light source signal

Transport

Occlusion

Signal 2
Our approach

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Our approach

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Local light field

- 4D light field, around a central ray
Local light field

• 4D light field, around a central ray
• We study its spectrum during transport
Local light field

- 4D light field, around a *central ray*
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Local light field

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Local light field

• We give explanations in 2D
  – Local light field is therefore 2D
• See paper for extension to 3D
Local light field parameterization

- Space and angle

Diagram showing space and angle with a central ray.
Local light field representation

• Density plot:

Area light source

Space

Angle
Local light field
Fourier spectrum

• We are interested in the Fourier spectrum of the local light field

• Also represented as a density plot
Local light field
Fourier spectrum

Spectrum of an area light source:

Angular frequency

Spatial frequency
Fourier analysis 101

• Spectrum corresponds to blurriness:
  – Sharpest feature has size $1/F_{\text{max}}$

• Convolution theorem:
  – Multiplication $\leftrightarrow$ convolution

• Classical spectra:
  – Box $\leftrightarrow$ sinc
  – Dirac $\leftrightarrow$ constant
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Example scene

Light source

Blockers

Receiver
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Transport in free space

Shear

Angle

Space

Angle

Space
Transport in free space

Shear

Spatial frequency

Angular freq.

Angular freq.
propagation
Transport $\rightarrow$ Shear

- This is consistent with light field spectra [Chai et al. 00, Isaksen et al. 00]

From [Chai et al. 2000]
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Occlusion: multiplication

- Occlusion is a multiplication in ray space
  - Convolution in Fourier space
- Creates new spatial frequency content
  - Related to the spectrum of the blockers
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BRDF integration

- Outgoing light:
  - Integration of incoming light times BRDF
BRDF integration

• Ray-space: convolution
  – Outgoing light: convolution of incoming light and BRDF
  – For rotationally-invariant BRDFs

• Fourier domain: multiplication
  – Outgoing spectrum: multiplication of incoming spectrum and BRDF spectrum
BRDF in Fourier: multiplication

- BRDF is bandwidth-limiting in angle
Example: diffuse BRDF

- Convolve by constant:
  - multiply by Dirac
  - Only spatial frequencies remain
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Curved receiver

• Reduce to the case of a planar surface:
  – “Unroll” the curved receiver

• Equivalent to changing angular content:
  – Linear effect on angular dimension
  – No effect on spatial dimension

• Shear in the angular dimension
<table>
<thead>
<tr>
<th></th>
<th>Radiance/Fourier</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transport</td>
<td>Shear</td>
<td></td>
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<tr>
<td>Occlusion</td>
<td>Multiplication/Convolution</td>
<td>Adds spatial frequencies</td>
</tr>
<tr>
<td>BRDF</td>
<td>Convolution/Multiplication</td>
<td>Removes angular frequencies</td>
</tr>
<tr>
<td>Curvature</td>
<td>Shear</td>
<td></td>
</tr>
</tbody>
</table>
More effects in paper

- Cosine term (multiplication/convolution)
- Fresnel term (multiplication/convolution)
- Texture mapping (multiplication/convolution)
- Central incidence angle (scaling)
- Separable BRDF
- Spatially varying BRDF (semi-convolution)

...and extension to 3D
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Diffuse soft shadows

- Decreasing blockers size:
  - First high-frequencies increase
  - Then only low frequency
  - Non-monotonic behavior
Diffuse soft shadows (2)

- Occlusion: convolution in Fourier
  - creates high frequencies
  - Blockers scaled down $\rightarrow$ spectrum scaled up

![Diagram showing Fourier space and blocker spectrum](image-url)
**Diffuse soft shadows (3)**

- **Transport after occlusion:**
  - Spatial frequencies moved to angular dimension

- **Diffuse reflector:**
  - Angular frequencies are cancelled
Diffuse soft shadows (3)

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- Diffuse reflector:
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Adaptive shading sampling

- Monte-Carlo ray tracing
- Blurry regions need fewer shading samples
Adaptive shading sampling

- Per-pixel prediction of max. frequency (bandwidth)
  - Based on curvature, BRDF, distance to occluder, etc.
  - No spectrum computed, just estimate max frequency
Adaptive shading sampling

• Per-pixel prediction of max. frequency (bandwidth)
  – Based on curvature, BRDF, distance to occluder, etc.
  – No spectrum computed, just estimate max frequency
Adaptive sampling

Adaptive sampling
20,000 samples
Uniform sampling

20,000 samples
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Conclusions

• Unified framework:
  – For frequency analysis of radiance
  – In both space and angle
  – Simple mathematical operators
  – Extends previous analyses

• Explains interesting lighting effects:
  – Soft shadows, caustics

• Proof-of-concept:
  – Adaptive sampling
Future work

• More experimental validation on synthetic scenes
• Extend the theory:
  – Bump mapping, microfacet BRDFs, sub-surface scattering…
  – Participating media
• Applications to rendering:
  – Photon mapping
  – Spatial sampling for PRT
  – Revisit traditional techniques
• Applications to vision and shape from shading
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  – Realreflect EU IST project
  – MIT-France
Solar oven

- Curved surface
- In: parallel light rays
- Out: focal point
Other bases?

• We’re not using Fourier as a function basis
  – Don’t recommend it, actually
  – Just used for analysis, understanding, predictions

• Results are useable with any other base:
  – Wavelets, Spherical Harmonics, point sampling, etc
  – Max. frequency translates in sampling rate

• Analysis relies on Fourier properties:
  – Especially the convolution theorem
Why *Local* Light Field?

- **Linearization:**
  - $\theta \approx \tan \theta$
  - Curvature

- **Local information is what we need:**
  - Local frequency content, for local sampling
  - Local properties of the scene (occluders, curv.)
Extension to 3D

• It works. See paper:

rotation around the x axis by $\alpha$

step 1, 2, 3, 4 central normal

local normal

incoming light field

Mirror direction (wrt local N)

step 5

(a) (b) (c)
Reflection on a surface: Full summary

- Angle of incidence
- Curvature
- Cosine/Fresnel term
- Mirror re-parameterization
- BRDF
- Curvature
Reflection on a surface: Full summary

- Angle of incidence: scaling
- Curvature: shear in angle
- Cosine/Fresnel term: multiplication/convolution
- Mirror re-parameterization
- BRDF: convolution/multiplication
- Curvature: shear in angle
Application: 3D scene

1. Spectrum of the source

Spatial frequencies
Application: 3D scene
Application: 3D scene
Application: 3D scene
Application: 3D scene