# Active Contours Under Topology Control
# Genus Preserving Level Sets

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**Abstract.** We present a novel framework to exert topology control over a level set evolution. Level set methods offer several advantages over parametric active contours, in particular automated topological changes. In some applications, where some *a priori* knowledge of the target topology is available, topological changes may not be desirable. This is typically the case in biomedical image segmentation, where the topology of the target shape is prescribed by anatomical knowledge. However, topologically constrained evolutions often generate topological barriers that lead to large geometric inconsistencies. We introduce a topologically controlled level set framework that greatly alleviates this problem. Unlike existing work, our method allows connected components to merge, split or vanish under some specific conditions that ensure that no topological defects are generated. We demonstrate the strength of our method on a wide range of numerical experiments and illustrate its performance on the segmentation of cortical surfaces and blood vessels.

## 1 Introduction

Active contours constitute a general technique of matching a deformable model onto an image by means of energy minimization. Curves or surfaces deform within a 2-dimensional or 3-dimensional image subject to both internal and external forces and external constraints. Since their introduction by Kass et al. in [9], active contours have benefited many computer vision research areas.

Geometric active contours, which represents the manifold of interest as level sets of functions defined on higher-dimensional manifolds [13,3], offer many advantages over parametric approaches. In addition to their ease of implementation, level sets do not require any parameterization of the evolving contour. Self-intersections, which are costly to prevent in parametric deformable models, are naturally avoided and topological changes are automated. Also, the geometric properties of the contour, such as the normal or the curvature, are easily computed from the level set function.

The ability to automatically change topology is often presented as an advantage of the level set method over explicit deformable models. However, this behavior is not desirable in some applications. This is typically the case in biomedical image segmentation, where the topology of the target shape is prescribed by anatomical knowledge. In order to overcome this problem, a topology-preserving variant of the level set method has been proposed [7]. The level set function is iteratively updated with a modified

procedure based on the concept of *simple point* from digital topology [1]; the final mesh is obtained with a modified topology-consistent marching cubes algorithm. This method ensures that the resulting mesh has the same topology as the user-defined initial level set.

While such topological preservation is desired in some applications, it is often too restrictive. Because the different components of the contour are not allowed to merge or to split up, the number of connected components will remain constant throughout the evolution. This number must be known by the user *a priori* and the initial contour must be designed accordingly. Also, the sensitivity to initial conditions, which already limits the applicability and efficiency of active contour methods, is considerably increased. The initial contour must both have the same topology as the target shape and be close enough to the final configuration, otherwise the evolution is likely to be trapped in topological dead-ends including large geometric inconsistencies (*cf* Fig.1 b&e).

In this paper, we propose a method to exert a more subtle topological control on a level set evolution. Some *a priori* knowledge of the target topology can be integrated without requiring that the topology be known exactly. Our method greatly alleviates the sensitivity to initial conditions by allowing connected components to merge, split or vanish without introducing any topological defects (such as handles). For example, an initial contour with a spherical topology may split into several pieces, go through one or several mergings, and finally produce a certain number of contours, all of which will be topologically equivalent to a sphere. A subset of these components may then be selected by the user as the desired output (typically the largest component if one spherical contour is needed, the others being caused by noise).

Our approach is based on an extension of the concept of *simple point* to "multi-label" images, that we have called *multisimple point*. This criterion ensures that no topological defects are generated while splitting or merging the components of the object. This algorithm fills the gap between the original level set framework and topology-preserving level set methods. We apply our method to the segmentation of cortical surfaces in Magnetic Resonance Imaging (MRI) and of blood vessels in Magnetic Resonance Angiography (MRA).

## 2   Background

### 2.1   Topology

Topology is a branch of mathematics that studies the properties of geometric figures that are preserved through deformations, twistings and stretchings, hence without regard to size, absolute position.

Here, we focus on closed contours (i.e. compact surfaces) in three dimensions. Any compact connected orientable surface is homeomorphic to a sphere with some number of handles. This number of handles is a topological invariant called the *genus*. For example, a sphere is of genus 0 and a torus is of genus 1. The genus $g$ is directly related to another topological invariant called the *Euler characteristic* $\chi$ by the formula $\chi = 2 - 2g$. The Euler characteristic is of great practical interest because it can be calculated from any polyhedral decomposition of the surface by $\chi = V - E + F$,

where $V$, $E$ and $F$ denote respectively the number of vertices, edges and faces of the polyhedron.

Homeomorphisms are used to define the *intrinsic* topology of an object, independently of the embedding space. For example, a knotted solid torus has the same genus as a simple torus, or a hollow sphere as two spheres. In order to topologically differentiate these surfaces, one needs a theory that considers the embedding space. Homotopy, which defines two surfaces to be homotopic if one can be continuously transformed into the other, is such a theory that provides a measure of an object's topology (see [8] for an excellent course in algebraic topology).

### 2.2 Digital Topology

Digital topology provides an elegant framework, which transposes the continuous concepts of topology to discrete images. In this theory, binary images inherit a precise topological meaning. In particular, the concept of homotopic deformation, necessary to assign a topological type to a digital object, is clearly defined through the notion of *simple point*. An extensive discussion of these concepts can be found in the work of Mangin et al [12]. In this section, some basic notions of digital topology are presented. All definitions are from the work of G. Bertrand, which we refer to for more details [1].

A 3D binary digital image $I$ is composed of a foreground object $X$ and its inverse, the complement $\overline{X}$. We first need the concept of *connectivity*, which specifies the condition of adjacency that two points must fulfill to be regarded as connected. Three types of connectivity are commonly used in 3D: 6-, 18- and 26-connectivity. Two voxels are 6-adjacent if they share a face, 18-adjacent if they share at least an edge and 26-adjacent if they share at least a corner. In order to avoid topological paradoxes, different connectivities, $n$ and $\overline{n}$, must be used for $X$ and $\overline{X}$. This leaves us with four pairs of compatible connectivities, (6,26), (6,18), (18,6) and (26,6). We note that digital topology does not provide a consistent framework for multi-label images, since no compatible connectivities could be chosen for neighboring components of the same object. Digital topology is strictly limited to binary images.

We then go to the definition of a *simple point*. This concept is central to the method of [7] and to our method. A point of a binary object is *simple* if it can be added or removed without changing the topology of both the object and its background, i.e. without changing the number of connected components, cavities and handles of both $X$ and $\overline{X}$. A simple point is easily characterized by two *topological* numbers with respect to the digital object $X$ and a consistent connectivity pair $(n, \overline{n})$. These numbers, denoted $T_n(x, X)$ and $T_{\overline{n}}(x, \overline{X})$, have been introduced by G. Bertrand in [1] as an elegant way to classify the topology type of a given voxel. The values of $T_n(x, X)$ and $T_{\overline{n}}(x, \overline{X})$ characterize isolated, interior and border points as well as different kinds of junctions. In particular, a point is simple if and only if $T_n(x, X) = T_{\overline{n}}(x, \overline{X}) = 1$. Their efficient computation, which only involves the 26-neighborhood, is described in [2].

### 2.3 Topology-Preserving Level Sets

Han et al. [7] have used the concept of simple point to design a topology-preserving variant of the level set framework. The binary object of interest is the interior of the contour, i.e. the domain where the level set function is strictly negative: $X=\{x|\Phi(x) < 0\}$.

The digital topology of $X$ is preserved during the evolution by the means of a modified update procedure detailed in Algorithm 1, below. This approach prevents digital non-simple grid points from changing sign, therefore retaining the initial digital topology throughout the level set evolution.

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**Algorithm 1** Topology-preserving level sets. Han et al. [7]

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**for all** iterations **do**
  **for all** grid points **do**
    Compute the new value of the level set function
    **if** the sign does not change **then**
      Accept the new value
    **else** {sign change}
      Compute the topological numbers
      **if** the point is simple **then**
        Accept the new value
      **else**
        Discard the new value
        Set instead a small value of the adequate sign

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For this method to be useful, it must be complemented with a topology-consistent isocontour extraction algorithm. Standard marching squares or marching cubes algorithm [11] do not generate topologically consistent tessellations. In order to alleviate this problem, Han et al. have designed a modified connectivity-consistent marching contour algorithm, by building a specialized case table for each type of digital topology. Using the topology-preserving level set algorithm and the topology-consistent marching contour algorithm in conjunction, with the same digital topology $(n, \overline{n})$ throughout the process, guarantees that the output mesh is topologically equivalent to the user-defined initial level set.

## 3   Methods

The simple point condition is a very efficient way to detect topological changes during a level set evolution. However, in many applications, the topology-preserving level set method of Han et al. is too restrictive.

The primary concern is topological defects such as handles, which are difficult to retrospectively correct [6,4,5,10]. On the other hand, changes in the number of connected components during the evolution are less problematic. Different connected components are easily identified using standard region growing algorithms. A subset of them may be selected by the user as the final output, typically the largest one if a single component is needed, the others being imputable to noise in the input data. Unexpected cavities, which can be interpreted as background $\overline{n}$-connected components, might generate large geometrical inaccuracies in level set evolutions that prevent their formation. This might be of particular importance when dealing with noisy data or in medical imaging, when unexpected medical structures, such as tumors, might exist in the volume to be segmented.

We extend the concept of simple point to allow distinct connected components to merge and split while ensuring that no additional handle is generated in the object. For example, an initial contour with spherical topology may split into several pieces, generate cavities, go through one or several mergings, and finally produce a specific number of surfaces, all of which are topologically equivalent to a sphere.

### 3.1   From Simple Points to Multisimple Points

The different values of $T_n$ and $T_{\overline{n}}$ characterize the topology type of a given point $x$, providing important information with regards to its connectivity to the object $X$. In particular, isolated points are characterized by $T_n = 0$ and $T_{\overline{n}} = 1$, while different junctions by $T_n > 1$ and $T_{\overline{n}} = 1$. This additional information can be used to carefully design a multi-label digital framework, which allows different connected components to split, merge or vanish under topology control.

We say that a point is *multisimple* if it can be added or removed without changing the number of handles of the object $X$. Contrary to the case of simple points, the addition of a multisimple point may merge several connected components, and its removal may split a component into several parts. We note $C_n(x, X)$ the set of $n$-connected components of $X \setminus \{x\}$ that are $n$-adjacent to $x$. If the cardinality of this set is strictly greater than one, the addition or removal of $x$ involves a merge or a split respectively. By duality, the generation of one or several cavities in $X$ can be interpreted as a split of connected components of $\overline{X}$.

Formally, a point is said to be multisimple if and only if:

$$T_{\overline{n}}(x, \overline{X}) = 1$$
$$\forall\, C \in C_n(x, X),\ T_n(x, C) = T_{\overline{n}}(x, \overline{C}) = 1$$
$$\text{or}$$
$$T_n(x, X) = 1$$
$$\forall\, C \in C_{\overline{n}}(x, \overline{X}),\ T_n(x, \overline{C}) = T_{\overline{n}}(x, C) = 1$$

When merging or splitting connected components by adding or removing a multisimple point, the total genus (i.e. the total number of handles) of the different components is preserved. We note that, under this condition, an isolated point is a multisimple point, which allows components to be created or to disappear.

### 3.2   Level Set Evolution Under Topology Control

With the concept of multisimple point in hand, we are now ready to describe our new level set framework. Similarly to the approach of Han et al. [7], we exploit the binary nature of the level set function that partitions the underlying digital image into strictly negative inside points and positive outside points. During the evolution, we maintain a map $L$ of labels coding for the different connected components of $X$ and $\overline{X}$.

The update procedure for each grid point at each iteration is concisely described in Algorithm 2. The simple point condition, more restrictive, is checked first, because it is computationally cheaper. If the point is non-simple, then $C_n(x, X)$ and $C_{\overline{n}}(x, \overline{X})$ are computed in order to check the multisimple criterion. We refer to the technical report [14] for more details.

Interestingly, if $x$ is part of the background (resp. foreground) object and is a candidate for addition, $C_n(x, X)$ (resp. $C_{\overline{n}}(x, \overline{X})$) can be deduced directly from the map $L$. If $x$ is a candidate for removal in $X$ (resp. in $\overline{X}$), the complete set $C_n(x, X)$ (resp. $C_{\overline{n}}(x, \overline{X})$) must be computed. However, when dealing with components that do not possess any handles, the most common situation in practice, the computation of $C_n(x, X)$ and $C_{\overline{n}}(x, \overline{X})$ only involves local computations.

Some care must be taken in order to ensure that the map of labels $L$ is correctly updated. The more complex case is the removal of a multisimple point involving a split: in this case, some unused labels must be assigned to the new connected components. The first equation of each condition guarantees that sign changes do not generate ambiguous label changes. For each point $x \in X$ changing sign, only one single connected component $C$ of the inverse object $\overline{X}$ is adjacent to the point $x$; unambiguously, the newly assigned label of $x$ is the one of the component $C$.

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**Algorithm 2** Genus Preserving Level Sets

Compute the new value of the level set function
**if** the sign does not change **then**
  Accept the new value
**else** {sign change}
  Compute the topological numbers
  **if** the point is simple **then**
    Accept the new value
  **else** {non-simple point}
    **if** the point is multisimple **then**
      Accept the new value
      Update $L(x)$
    **else**
      Discard the new value
      Set instead a small value of the adequate sign

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The resulting procedure is an efficient level set model that prevents handles from being created during the evolution, allowing the number of connected components (including cavities) to vary. We insist on the fact that digital topology does not provide a consistent framework for multi-label images. However, by ensuring that no components of the same object $X$ or $\overline{X}$ are adjacent, topological inconsistencies can be eliminated.

## 4     Experiments and Applications

In this section, we present some experiments illustrating the performance of our approach and introduce some potential applications. We first apply our level set framework to two synthetic examples to demonstrate its ability to handle disconnected components and cavities. Next, two segmentation tasks are presented: the generation of cortical surfaces from MRI images and the extraction of blood vessels from MRA images.

### 4.1     Experiments

*'C' shape:* First, we consider the segmentation of a simple 'C' shape under two different initializations (*cf* Fig.1 *a-c* & *d-f*). Our method, columns *c,f,* is compared to the
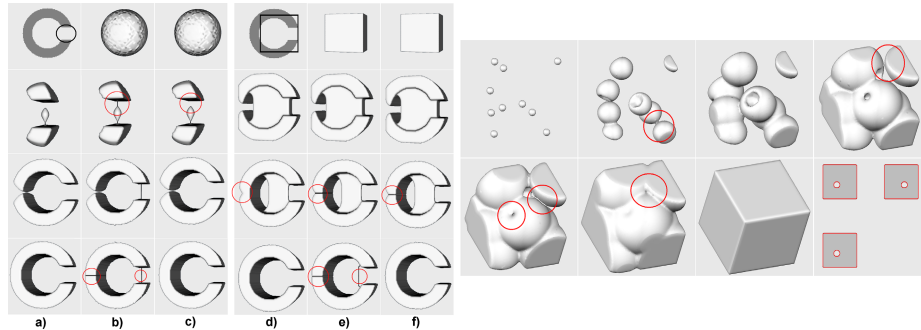
**Fig. 1.** Left: Segmentation of a 'C' shape using a spherical initialization a-c and a rectangular initialization d-f. The top row shows cuts of the 3D shape locating the initial component. a,d: Traditional level set evolution. b,e: Topology-preserving level set evolution. c,f: level set evolution under topology control. Differences of behavior are circled in the images. Note that in both cases, our level set framework is able to correctly segment the shape of interest, without generating invalid topology during the evolution. Right: Segmentation of a cubic shape containing 3 cavities. 10 initial seed points were randomly selected. Note how components split, merge and disappear during the evolution, and how the active contour encloses cavities.

original level set formulation, columns *a,d*, and the topology-preserving model introduced by Han et al. [7], columns *b,e*. In the case of this simple example, standard level sets produce accurate segmentations. However, they do not provide control over the final topology, as illustrated by the second initialization, column *d*. During the evolution, the level set automatically changes topology, temporarily generating a toroïdal shape.

On the other hand, topology-preserving level sets, while guaranteeing the correct topology, might easily be geometrically inaccurate (see Fig.1 *b,e*). Topological barriers, generated during the evolution to prevent topological changes, generate incorrect contour configurations which are difficult to retrospectively correct. We note that even close initializations (*cf* column *e*) can lead to inaccurate segmentations.

The behavior of our approach corresponds to a trade-off in between the previous two methods. Our level set framework, which offers more flexibility during the evolution, greatly alleviates the sensitivity to initialization. It provides a control over the topology of all connected components, without enforcing strong topological constraints. The first initialization (*cf* column *c*) produces a level set evolution, which splits the active contour into three distinct components with a spherical topology. While one of these components vanishes, the two other components merge, closing the 'C' shape. In this case, topologically controlled level sets behaved in exactly the same way traditional level sets do. Our method is still sensitive to topological barriers, in the sense that it will not allow component to generate handles. However, it is less likely to be trapped into topological dead-ends than a strict topology-preserving model. This is illustrated in Fig.1 *f*, where the spherical component is prevented from generating a handle until it gets separated into two distinct components.

*Formation of cavities:* The second experiment illustrates the ability of our approach to generate cavities during the evolution. The object to be segmented is a synthetic cube,

containing 3 large cavities. 10 seed points, randomly selected inside and outside the volume, were used to initialize the level set evolution reported in Fig.1-right. During the evolution, components merge, vanish and produce cavities, generating a final accurate representation constituted of 3 spherical cavities and the main object. We note that all the components are easily extracted, since they carry distinct labels that are iteratively updated during the evolution.

### 4.2   Medical Image Segmentation

Two segmentation tasks are presented that illustrate the potential benefits of our novel level set framework : the segmentation of cortical surfaces from MRI and the extraction of blood vessels from MRA data sets. The level set evolution is naively guided by a simple intensity-based velocity term and a curvature term:

$$\partial_t \phi(x,t) = [\alpha(I(x) - T(x)) - \kappa]|\nabla \phi|,$$

where $I(x)$ and $\kappa$ represents the image intensity and mean curvature respectively at location $x$, $T$ is a threshold computed locally from image intensities, $\alpha$ a weighting coefficient equal to $\alpha = 0.1$.

*Cortical segmentation:*   Excluding pathological cases, the cortex, which corresponds to a highly-folded thin sheet of gray matter, has the topology of a sphere. The extraction of accurate and topologically correct cortical representations is still an active research area [4,6]. In this example, the cortical surface is initialized with 55 spherical components, automatically selected in a greedy manner, such that every selected point is located at a minimal distance of $10mm$ from the previous ones (see Fig.2). During the evolution, the components progressively merge together and enclose cavities, resulting in a final surface composed of one single spherical component with 5 cavities. The same evolution without topology constraint produces a final cortical surface with 18 topological defects (i.e. handles).
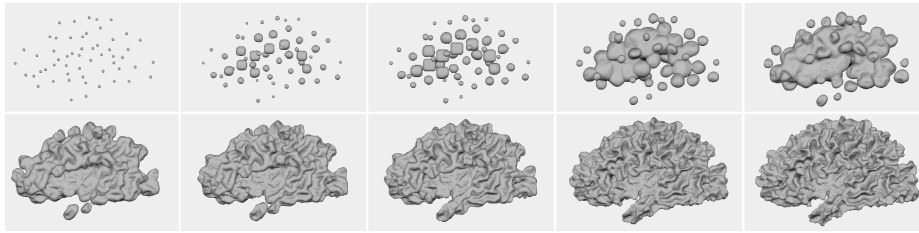


**Fig. 2.** Segmentation of the cortex from an anatomical MRI. The initial level set was constituted of 55 connected components. The final surface has a spherical topology, corresponding to an Euler number of 2. The same level set evolution without topological control results in a final surface with 18 topological defects (Euler number of $\chi = -34$).
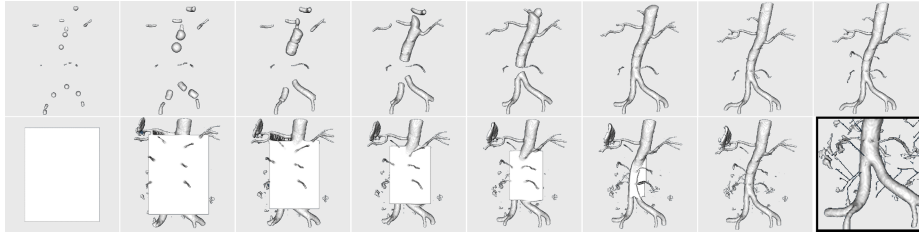
**Fig. 3.** Segmentations of blood vessels in a 3D angiography under two different initializations. Top row: 20 seed points were selected to initialize the active contour, which generates 3 components. Bottom row: An enclosing contour is used to initialize the level set. After 9 mergings and 99 splittings, the final segmentation is constituted of 91 components, 53 of which were due to random noise. The last image in the black frame shows the final segmentation produced by a topologically constrained evolution under the same initialization.

*Segmentation of blood vessels:*  Finally, we show how our method could be applied to the segmentation of blood vessels from Magnetic Resonance Angiography. Because these vessels do not split and merge, their topology is the one of several distinct components with no handles (i.e. each component has the topology of a sphere). While traditional level sets produce segmentations that could include topological defects, topologically constrained level sets would result in a slow and laborious segmentation. Since the simultaneous evolution of several components, which cannot be merged together, can easily be trapped in topological dead-ends, each component would need to be iteratively initialized, when the evolution of the previous one has terminated.

On the other hand, our method offers the possibility to concurrently evolve multiple components that can merge, split and . The initial contour can be initialized by a set of seed points, manually or automatically selected, or by a single enclosing component, without affecting much the final representation.

Figure 3 shows the segmentation of an angiography under two different initializations. In a first experiment (*cf* top row), 20 seed points were automatically selected at the brightest locations in the MRA. The level set evolution iteratively merged most of the components, generating a final segmentation with 3 spherical components. In the second experiment (*cf* bottom row), one single global component, enclosing most of the object of interest, was used to initialize the active contour. During the evolution, 9 components merged and 99 split producing a final segmentation composed of 91 components, 53 of which were single voxel components due to random noise in the imaging process.

## 5   Discussion and Conclusion

We introduce a new level set framework that offers control over the topology of the level set components during the evolution. Previous approaches either do not constrain the topology or enforce a hard topological constraint. Our method exerts a subtle control over the topology of each component to prevent the formation of topological defects. Distinct components can merge, split or disappear during the evolution, but no handles

will be generated. In particular, a contour solely composed of spherical components will only produce spherical components throughout the evolution. In this case, which is the most common situation in practice, all computations are local and the multisimple point checking can be done efficiently. The only computational complexity comes from the generation of new components, as new labels need to be assigned to each of them.

While the original level set model does not provide any topological control, topology-preserving level sets enforce a strong constraint that is often too restrictive. Our framework establishes a trade-off in between the two models. It offers a subtle topological control that alleviates most problems of methods that enforce a strong topological constraint (i.e. sensitivity to initialization and noise, simultaneous evolution of multiple components and speed of convergence). The experiments presented in this paper illustrate some potential applications that could greatly benefit from our approach.

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