



On the Uniformization Principle. by Yoshindo Suzuki
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sists of: I. Axioms 0 through 4; II. $(qx)\phi \rightarrow (\exists^{\geq n}x)\phi$, where $\exists^{\geq n}$ is the first-order defined quantifier "there exist at least n ." The rules of inference are *modus ponens*, generalization, and the following version of an ω -rule: III. From $\neg(S)\neg(\exists^{\geq 1}x)\phi$, $\neg(S)\neg(\exists^{\geq 2}x)\phi$, \dots , $\neg(S)\neg(\exists^{\geq n}x)\phi$, \dots infer $\neg(S)\neg(qx)\phi$, where (S) is any string of the quantifiers \exists and Q .

The author discusses the role of the axiom of choice in his results. In this connection he introduces the notion of a regular set of cardinals, generalizing that of a regular cardinal. There are numerous applications to the metamathematics of set theory, to second-order number theory, and to the model theory of first-order languages. Among the latter is a simple proof of the Ehrenfeucht-Mostowski theorem on automorphisms and the result that a theory Γ formulated in L that allows uncountably many types in m variables, for some $m \in \omega$, has 2^{\aleph_1} non-isomorphic models of power \aleph_1 .

GEBHARD FUHRKEN

K. L. DE BOUVÈRE. *A mathematical characterization of explicit definability*. *Koninklijke Nederlandse Akademie van Wetenschappen, Proceedings*, series A, vol. 66 (1963), pp. 264–274; also *Indagationes mathematicae*, vol. 25 (1963), pp. 264–274.

These are variations on a well-known theme of Beth, from a model-theoretic point of view. The paper is a sequel to the author's dissertation on this subject. It is based on the fact that the implicit definability of a relation in a theory in the sense of Beth is equivalent to the unique realizability of the relation in any model of the theory. The reasoning is straightforward.

ABRAHAM ROBINSON

YOSHINDO SUZUKI. *On the uniformization principle*. *Proceedings of the Symposium on the Foundations of Mathematics, held at Katada, Japan, 1962*, Sponsored jointly by The Division of the Foundations of Mathematics of the Mathematical Society of Japan, The Sugaku Shinkokai, and The Toyo Spinning Company, Tokyo 1963, pp. 137–144.

This paper contains the first proof of the Novikov-Kondô-Addison uniformization theorem to appear in print. The somewhat complicated history of this theorem is reviewed below.

A subset of the plane is *uniform* if and only if it is the graph of a function; one set *uniformizes* another if and only if the first is a uniform subset of the second having the same projection. Since every set can be uniformized (by the axiom of choice), the foundational interest in uniformization lies in measuring the complexity of sets required to uniformize sets of a given logical complexity. When attention is limited to sets with unit projection, this problem reduces to a *basis* problem of whether certain definable sets contain elements definable in certain forms.

Luzin (*Mathematica*, vol. 4 (1930), pp. 54–66) studied the uniformizability of Borel and analytic sets, showing in particular that every Borel set could be uniformized by a Π_1^1 or complete analytic set. He also argued that Π_1^1 sets in general could not be uniformized by any definable sets, and that in fact it was impossible to "describe in a finite number of words" any point in an arbitrary non-empty Π_1^1 set. Novikov showed that this latter claim was false (Luzin and Novikov, *Fundamenta mathematicae*, vol. 25 (1935), pp. 559–560). Kondô applied Novikov's method simultaneously to all vertical cross-sections of a Π_1^1 set and showed, via a complicated evaluation, that every Π_1^1 set could be uniformized by a Π_1^1 set (*Japanese journal of mathematics*, vol. 15 no. 3 (1938), pp. 197–230).

Addison, while investigating classical and effective hierarchies in descriptive set theory (cf. XXXI 137), verified that the methods of Novikov and Kondô could be effectivized. He announced, in an invited address delivered to the International Symposium on the Foundations of Mathematics: Infinitistic Methods, held in Warsaw, 1959, that every set Π_1^1 in a given function of natural numbers could be uniformized by another set Π_1^1 in that function. As a corollary, he proved that the class of Δ_2^1 functions is a basis for the class of Σ_2^1 sets of functions, i.e., that every non-empty Σ_2^1 set of functions contains a Δ_2^1 element. Addison's proof, which was based on the construction of Novikov, but which used an evaluation different from that of Kondô, unfortunately never appeared in print. Suzuki's proof is also based on Novikov's method and a simplification of Kondô's evaluation due to Sampei (XXXV 146(3)).

We note in passing that Luzin's claim was not as far-fetched as it might seem. Indeed, Lévy (XXXIV 653) has shown that it is consistent relative to Zermelo-Fraenkel set theory that there is a non-empty Π_2^1 set of functions which contains no ordinal definable element.

STEPHEN J. GARLAND