

Constructing Symbolic Representations for High-Level Planning

George Konidaris, Leslie Pack Kaelbling and Tomas Lozano-Perez

MIT Computer Science and Artificial Intelligence Laboratory
32 Vassar Street, Cambridge MA 02139 USA
{gdk, lpk, tlp}@csail.mit.edu

Abstract

We consider the problem of constructing a symbolic description of a continuous, low-level environment for use in planning. We show that symbols that can represent the preconditions and effects of an agent’s actions are both necessary and sufficient for high-level planning. This eliminates the symbol design problem when a representation must be constructed in advance, and in principle enables an agent to autonomously learn its own symbolic representations. The resulting representation can be converted into PDDL, a canonical high-level planning representation that enables very fast planning.

Introduction

A core challenge of artificial intelligence is creating intelligent agents that can perform high-level learning and planning, while ultimately performing control using low-level sensors and actuators. Hierarchical reinforcement learning approaches (Barto and Mahadevan 2003) attempt to address this problem by providing a framework for learning and planning using temporally abstract high-level actions. One motivation behind such approaches is that an agent that has learned a set of high-level actions should be able to plan using them to quickly solve new problems, without further learning. However, planning directly in such high-dimensional, continuous state spaces remains difficult.

By contrast, high-level planning techniques (Ghallab, Nau, and Traverso 2004) are able to solve very large problems in reasonable time. These techniques perform planning using pre-specified symbolic *state descriptors*, and *operators* that describe the effects of actions on those descriptors. Although these methods are usually used for planning in discrete state spaces, they have been combined with low-level motion planners or closed-loop controllers to construct robot systems that combine high-level planning with low-level control (Nilsson 1984; Malcolm and Smithers 1990; Cambon, Alami, and Gravit 2009; Choi and Amir 2009; Dornhege et al. 2009; Wolfe, Marthi, and Russell 2010; Kaelbling and Lozano-Pérez 2011). Here, a symbolic state at the high level refers to (and abstracts over) an infinite set of low-level states. However, in all of these cases the robot’s symbolic representation was specified by its designers.

We consider the problem of *constructing a symbolic representation suitable for evaluating plans composed of sequences of actions in a continuous, low-level environment*. We can create such a representation by reasoning about potential propositional symbols via the sets of states they reference. We show that propositional symbols describing low-level representations of the preconditions and effects of each action are both sufficient and necessary for high-level planning, and discuss two classes of actions for which the appropriate symbols can be concisely described. We show that these representations can be used as a symbol algebra suitable for high-level planning, and describe a method for converting the resulting representation into PDDL (McDermott et al. 1998), a canonical high-level planning representation.

Our primary contribution is establishing a close relationship between an agent’s actions and the symbols required to plan to use them: *the agent’s environment and actions completely determine the symbolic representation required for planning*. This removes a critical design problem when constructing agents that combine high-level planning with low-level control; moreover, it in principle enables an agent to construct its own symbolic representation through learning.

Background and Setting

Semi-Markov Decision Processes

We assume that the low-level sensor and actuator space of the agent can be described as a fully observable, continuous-state semi-Markov decision process (SMDP), described by a tuple $M = (S, O, R, P, \gamma)$, where $S \subseteq \mathcal{R}^n$ is the n -dimensional continuous state space; $O(s)$ returns a finite set of temporally extended actions, or *options* (Sutton, Precup, and Singh 1999), available in state $s \in S$; $R(s', t|s, o)$ is the reward received when executing action $o \in O(s)$ at state $s \in S$, and arriving in state $s' \in S$ after t time steps; $P(s', t|s, o)$ is a PDF describing the probability of arriving in state $s' \in S$, t time steps after executing action $o \in O(s)$ in state $s \in S$; and $\gamma \in (0, 1]$ is a discount factor.

An option o consists of three components: an *option policy*, π_o , which is executed when the option is invoked; an *initiation set*, $I_o = \{s|o \in O(s)\}$, which describes the states in which the option may be executed; and a *termination condition*, $\beta_o(s) \rightarrow [0, 1]$, which describes the probability that an option will terminate upon reaching state s . The combi-

nation of reward model, $R(s', t|s, o)$, and transition model, $P(s', t|s, o)$, for an option o is known as an *option model*.

An agent that possesses option models for all of its options is capable of sample-based planning (Sutton, Precup, and Singh 1999; Kocsis and Szepesvári 2006), although doing so in large, continuous state spaces is very difficult.

High-Level Planning

High-level planning operates using symbolic states and operators. The simplest formalism is the *set-theoretic representation* (Ghallab, Nau, and Traverso 2004). Here, a planning domain is described by a set of propositions $\mathcal{P} = \{p_1, \dots, p_n\}$ and a set of operators $\mathcal{A} = \{\alpha_1, \dots, \alpha_m\}$. In a continuous problem, each proposition holds in an infinite number of low-level states; a high-level state is obtained by assigning a truth value to every $p_i \in \mathcal{P}$.

Each operator α_i is described by a tuple $\alpha_i = (\text{precond}_i, \text{effect}_i^+, \text{effect}_i^-)$, where $\text{precond}_i \subseteq \mathcal{P}$ lists all propositions that must be true in a state for the operator to be applicable at that state, and positive and negative effects, $\text{effect}_i^+ \subseteq \mathcal{P}$ and $\text{effect}_i^- \subseteq \mathcal{P}$, list the propositions set to true or false, respectively, as a result of applying the operator. All other propositions retain their values. A planning problem is obtained by augmenting the domain description with a start state, s_0 , and a set of goal states, S_g .

A more common formulation is the *classical representation* (Ghallab, Nau, and Traverso 2004), which uses a relational representation to more compactly describe the planning domain. For simplicity, we adopt the set-theoretic representation and leave parametrizing it to future work.

Problems in both formulations are typically described using the *Planning and Domain Definition Language* or PDDL (McDermott et al. 1998), which serves as the input format for most general-purpose planners. Although such methods have been widely applied—even in continuous problems—with few exceptions the set of propositions and their semantics must be supplied by a human designer.

Symbols and Plans

Given an SMDP, our goal is to build an abstract representation of the task for use in high-level planning. In this section we define a propositional symbol as a *test* that is true in some set of states. We then define a plan as a sequence of option executions from a set of start states, and the plan space as the set of all possible plans. This allows us to reason about the symbols required to evaluate the feasibility of any plan in the plan space. We begin with a working definition of the notion of a symbol:

Definition 1. A propositional symbol σ_Z is the name associated with a test τ_Z , and the corresponding set of states $Z = \{s \in S \mid \tau_Z(s) = 1\}$.

Symbol σ_Z is a name for a proposition, specified by both a test (or classifier) τ_Z defined over the state space, and an associated set $Z \subseteq S$ for which the test returns true. The set of states Z is referred to as the symbol’s *grounding*, and specifies the symbol’s semantics. A symbol’s grounding can be represented in two different ways: an *extensional representation* that lists the states in which the proposition holds—

impossible in our case, since our low-level state space is continuous—and an *intensional representation*, a classifier that determines membership in that set of states. The classifier allows us to express the set of states compactly, and also to learn a representation of them (if necessary). These representations both refer to the same set of states (and therefore are in some sense equivalent) but *we can perform task-based planning by manipulating and reasoning about intensional grounding representations*—we compute the results of logical operations on symbols by computing the resulting grounded classifier. This process is depicted in Figure 1.

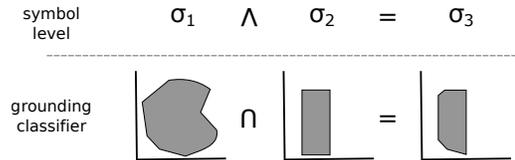


Figure 1: A logical operation on two symbols, along with the corresponding grounding sets.

We therefore require the grounding classifiers to support the ability to efficiently compute the following operations: 1) the union of two classifiers (equivalently, the `OR` of two symbols); 2) the intersection of two classifiers (equivalently, the `AND` of two symbols); 3) the complement of a classifier (equivalently, the `NOT` of a symbol); 4) whether or not a classifier refers to the empty set. (This allows us evaluate the subset operator: $A \subseteq B$ iff $\neg A \cap B$ is empty; consequently, we can test implication.) The first three of these operations take classifiers as inputs and return a classifier representing the result of a logical operation. This defines a *symbol algebra* (a concrete boolean algebra) that allows us to construct classifiers representing the results of boolean operations applied to our symbols. Classifiers that support these operations include, for example, decision trees (which we use throughout this paper for ease of visualization) and boolean ensembles of linear classifiers.

We next define the *plan space* of a domain. For simplicity, we aim to find a sequence of actions guaranteed to reach a goal state, without reference to cost.

Definition 2. A plan $p = \{o_1, \dots, o_{p_n}\}$ from a state set $Z \subseteq S$ is a sequence of options $o_i \in O$, $1 \leq i \leq p_n$, to be executed from some state in Z . A plan is feasible when the probability of being able to execute it is 1, i.e., $\exists \bar{S} = \{s_1, \dots, s_j\}$ such that $s_1 \in Z$, $o_i \in O(s_i)$ and $P(s_{i+1}|s_i, o_i) > 0, \forall i < j$, and $o_j \notin O(s_j)$ for any $j < p_n$.

Definition 3. The plan space for an SMDP is the set of all tuples (Z, p) , where $Z \subseteq S$ is a set of states (equivalently, a start symbol) in the SMDP, and p is a plan.

For convenience, we treat the agent’s goal set g —itself a propositional symbol—as an option o_g with $I_g = g$, $\beta_g(s) = 1, \forall s$, and a null policy.

Definition 4. A plan tuple is satisficing for goal g if it is feasible, and its final action is goal option o_g .

The role of a symbolic representation is to determine whether any given plan tuple (Z, p) is satisficing for goal

g , which we can achieve by testing whether it is feasible (since then determining whether it is satisficing follows trivially). In the following section, we define a set of propositional symbols (via their groundings) and show that they are sufficient to test feasibility in any SMDP.

Symbols for Propositional Planning

To test the feasibility of a plan, we begin with a set of possible start states, and repeatedly compute the set of states that can be reached by each option, checking in turn that the resulting set is a subset of the initiation set of the following option. We must therefore construct abstract representations for the initiation set of each option, and that enable us to compute the set of states an agent may find itself in as a result of executing an option from some start set of states. We begin by defining a class of symbols that represent the initiation set of an option:

Definition 5. *The precondition of option o is the symbol referring to its initiation set: $Pre(o) = \sigma_{I_o}$.*

We now define a symbolic operator that computes the consequences of taking an action in a set of states:

Definition 6. *Given an option o and a set of states $X \subseteq S$, we define the image of o from X as: $Im(X, o) = \{s' | \exists s \in X, P(s' | s, o) > 0\}$.*

The image operator $Im(X, o)$ computes the set of states that might result from executing o from some state in X .

Theorem 1. *Given an SMDP, the ability to represent the preconditions of each option and to compute the image operator is sufficient for determining whether any plan tuple (Z, p) is feasible.*

Proof. Consider an arbitrary plan tuple (Z, p) , with plan length n . We set $z_0 = Z$ and repeatedly compute $z_{j+1} = Im(z_j, p_j)$, for $j \in \{1, \dots, n\}$. The plan tuple is feasible iff $z_i \subseteq Pre(p_{i+1}), \forall i \in \{0, \dots, n-1\}$. \square

Since the feasibility test in the above proof is biconditional, any other feasibility test must express exactly those conditions for each pair of successive options in a plan. In some SMDPs, we may be able to execute a different test that evaluates to the same value everywhere (e.g., if every option flips a set of bits indicating which options can be run next), but we can employ an adversarial argument to construct an SMDP in which any other test is incorrect. Representing the image operator and precondition sets are therefore also necessary for abstract planning. We call the symbols required to name an option's initiation set and express its image operator that option's *characterizing symbols*.

The ability to perform symbolic planning thus hinges on the ability to symbolically represent the image operator. However, doing this for an arbitrary option can be arbitrarily hard. Consider an option that maps each state in its initiation set to a single (but arbitrary and unique) state in S . In this case we can do no better than expressing $Im(Z, o)$ as a union of uncountably infinitely many singleton symbols. Fortunately, however, we can concisely represent the image operator for at least two classes of options in common use.

Subgoal Options

One common type of option—especially prevalent in research on skill discovery methods—is the *subgoal option* (Precup 2000). Such an option reaches a set of states (referred to as its subgoal) before terminating, and the state it terminates in can be considered independent of the state from which it is executed. This results in a particularly simple way to express the image operator.

Definition 7. *The effect set of subgoal option o is the symbol representing the set of all states that an agent can possibly find itself in after executing o : $Eff(o) = \{s' | \exists s \in S, t, P(s', t | s, o) > 0\}$.*

For a subgoal option, $Im(Z, o) = Eff(o), \forall Z \subseteq S$. This is a strong condition, but we may in practice be able to treat many options as subgoal options even when they do not strictly meet it. This greatly simplifies our representational requirements at the cost of artificially enlarging the image (which is a subset of $Eff(o)$ for any given starting set).

A common generalization of a subgoal option is the *partitioned subgoal option*. Here, the option's initiation set can be partitioned into two or more subsets, such that the option behaves as a subgoal option from each subset. For example, an option that we might describe as *walk through the door you are facing*; if there are a small number of such doors, then the agent can be considered to execute a separate subgoal option when standing in front of each. In such cases, we must explicitly represent each partition of the initiation set separately; the image operator is then the union of the effect sets of each applicable partitioned option.

Unfortunately, subgoal options severely restrict the potential goals an agent may plan for: all feasible goals must be a superset of one of the effect sets. However, they do lead to a particularly simple planning mechanism. Since a subset test between the precondition of one option and the effects set of another is the *only* type of expression that need ever be evaluated, we can test all such subsets in time $O(n^2)$ for $O(n)$ characterizing sets, and build a directed plan graph G with a vertex for each characterizing set, and an edge present between vertices i and j iff $Eff(o_i) \subseteq Pre(o_j)$. A plan tuple (p, Z) where $p = \{o_1, \dots, o_{p_n}\}$ is feasible iff $Z \subseteq I_{o_1}$ and a path from o_1 to o_{p_n} exists in G .

For an option to be useful in a plan graph, its effects set should form a subset of the initiation set of any options the agent may wish to execute after it. The options should therefore ideally have been constructed so that their termination conditions lie inside the initiation sets for potential successor options. This is the principle underlying pre-image backchaining (Lozano-Perez, Mason, and Taylor 1984; Burridge, Rizzi, and Koditschek 1999), the LQR-Tree (Tadrake 2009) feedback motion planner, and the skill chaining (Konidaris and Barto 2009b) skill acquisition method.

Abstract Subgoal Options

A more general type of option—implicitly assumed by all STRIPS-style planners—is the *abstract subgoal option*, where execution sets some of the variables in the low-level state vector to a particular set of values, and leaves others unchanged. (As above, we may generalize abstract subgoal

options to *partitioned abstract subgoal options*.) The subgoal is said to be abstract because it is satisfied for any value of the unchanged variables. For example, an option to grasp an object might terminate when that grasp is achieved; the room the robot is in and the position of its other hand are irrelevant and unaffected.

Without loss of generality, we can write the low-level state vector $s \in S$ given o as $s = [a, b]$, where a is the part of the vector that changes when executing o (o 's *mask*), and b is the remainder. We now define the following operator:

Definition 8. Given an option o and a set of states $Z \subseteq S$, we define the projection of Z with respect to o (denoted $\text{Project}(Z, o)$) as: $\text{Project}(Z, o) = \{[a, b] \mid \exists a', [a', b] \in Z\}$.

Projection expands the set of states named by Z by removing the restrictions on the value of the changeable portion of the state descriptor. This operator cannot be expressed using set operations, and must instead be obtained by directly altering the relevant classifier.

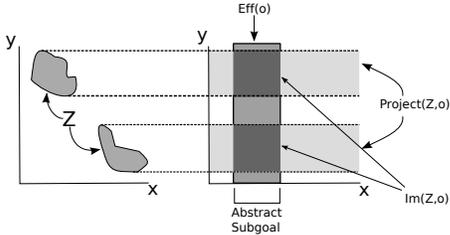


Figure 2: $\text{Im}(Z, o)$ for abstract subgoal option o can be expressed as the intersection of the projection operator and o 's effects set. Here, o has a subgoal in x , leaving y unchanged.

The image operator for an abstract subgoal option can be computed as $\text{Im}(Z, o) = \text{Project}(Z, o) \cap \text{Eff}(o)$ (see Figure 2). We can thus plan using only the precondition and effects symbols for each option.

Constructing a PDDL Domain Description

So far we have described a system where planning is performed using symbol algebra operations over grounding classifiers. We may wish to go even further, and construct a symbolic representation that allows us to plan without reference to any grounding representations at all.

A reasonable target is converting a symbol algebra into a set-theoretic domain specification expressed using PDDL. This results in a once-off conversion cost, after which the complexity of planning is no worse than ordinary high-level planning, and is independent of the low-level SMDP.

A set-theoretic domain specification uses a set of propositional symbols $\mathcal{P} = \{p_1, \dots, p_n\}$ (each with an associated grounding classifier $\mathcal{G}(p_i)$), and a state \mathcal{P}_t is obtained by assigning a truth value $\mathcal{P}_t(i)$ to every $p_i \in \mathcal{P}$. Each state \mathcal{P}_t must map to some grounded set of states; we denote that set of states by $\mathcal{G}(\mathcal{P}_t)$. A scheme for representing the domain using STRIPS-like operators and propositions must commit to a method for determining the grounding for any given \mathcal{P}_t so that we can reason about the correctness of symbolic constructions, even though that grounding is never used during

planning. We use the following grounding scheme:

$$\mathcal{G}(\mathcal{P}_t) = \bigcap_{i \in I} \mathcal{G}(p_i), \quad I = \{i \mid \mathcal{P}_t(i) = 1\}.$$

The grounding classifier of state \mathcal{P}_t is the intersection of the grounding classifiers corresponding to the propositions true in that state. One can consider the propositions set to true as “on”, in the sense that they are included in the grounding intersection, and those set to false as “off”, and not included.

The key property of a deterministic high-level domain description is that the operator descriptions and propositional symbols are sufficient for planning—we can perform planning using only the truth value of the propositional symbols, without reference to their grounding classifiers. This requires us to determine whether or not an option o_i can be executed at \mathcal{P}_t , and if so to determine a successor state \mathcal{P}_{t+1} , solely by examining the elements of \mathcal{P}_t .

Our method is based on the properties of abstract subgoal options: that only a subset of state variables are affected by executing each option; that the resulting effects set is intersected with the previous set of states after the those variables have been projected out; and that the remaining state variables are unaffected. Broadly, we identify a set of *factors*—non-overlapping sets of low-level variables that could be changed simultaneously by an option execution—and then use the option effects sets to generate a set of symbols corresponding to any reachable grounding of each set of factors. Finally, we use this set of symbols to produce an operator description for each option.

Defining Factors Given a low-level state representation $s = [s_1, \dots, s_n]$, we define a mapping from each low-level state variable to the set of options capable of changing its value: $\text{modifies}(s_i) = \{o_j \mid i \in \text{mask}(o_j), o_j \in O\}$. We next partition s into factors, f_1, \dots, f_m , each of which is a set of state variables that maps to the same unique set of options. An example of this process is shown in Figure 3.

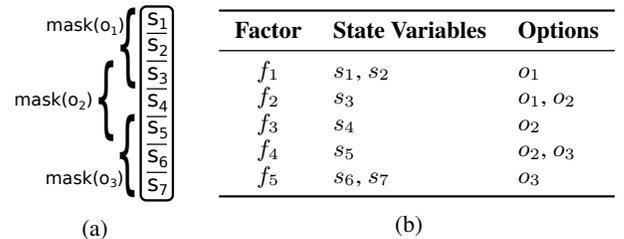


Figure 3: An example 7-variable low-level state vector, with three option masks (a). This is partitioned into 5 factors (b).

We denote the set of options modifying the variables in factor f_i as $\text{options}(f_i)$, and similarly the set of factors containing state variables that are modified by option o_j as $\text{factors}(o_j)$. In a slight abuse of notation, we also denote the factors containing variables used in the grounding classifier for symbol σ_k as $\text{factors}(\sigma_k)$.

Building the Symbol Set Executing option o_i projects the factors it changes out of the current state descriptor, which is then intersected with its effect set $\text{Eff}(o_i)$. Future execution of any option o_j where $\text{factors}(o_i) \cap \text{factors}(o_j) \neq \emptyset$ will

project the overlapping factors out of $\text{Eff}(o_i)$. We therefore require a propositional symbol for each option effects set with all combinations of factors projected out. However, we can represent the effect of projecting out a factor compactly if it obeys the following independence property:

Definition 9. Factor f_s is independent in effect set $\text{Eff}(o_i)$ iff $\text{Eff}(o_i) = \text{Project}(\text{Eff}(o_i), f_s) \cap \text{Project}(\text{Eff}(o_i), \text{factors}(o_i) \setminus f_s)$.

If f_s is independent in $\text{Eff}(o_i)$, we can break $\text{Eff}(o_i)$ into the two components $\text{Project}(\text{Eff}(o_i), f_s)$ and $\text{Project}(\text{Eff}(o_i), \text{factors}(o_i) \setminus f_s)$ and create a separate proposition for each. We repeat this for each independent factor, each time using the more general effect set we obtain by projecting out previous independent factors.

Let $\text{Eff}_r(o_i)$ denote the effect set that remains after projecting out all independent factors from $\text{Eff}(o_i)$, and $\text{factors}_r(o_i)$ denote the remaining factors. We will require a separate proposition for $\text{Eff}_r(o_i)$ with each possible subset of $\text{factors}_r(o_i)$ projected out. Therefore, we construct a vocabulary \mathcal{P} containing the following propositional symbols:

1. For each option o_i and factor f_s independent in $\text{Eff}(o_i)$, create a symbol for $\text{Project}(\text{Eff}(o_i), \text{factors}(o_i) \setminus f_s)$.
2. For each set of factors $f_r \subseteq \text{factors}_r(o_i)$, create a symbol for $\text{Project}(\text{Eff}_r(o_i), f_r)$.

We discard duplicate propositions (which have the same grounding set as an existing proposition), and propositions which correspond to the entire state space (as these have no effect on the grounding intersection). We now show that these symbols are sufficient for symbolically describing a problem by using them to construct a sound operator description for each option.

Constructing Operator Descriptions Executing operator o_i results in the following effects:

1. All symbols for each component of $\text{Eff}(o_i)$ that depends on a factor independent in $\text{Eff}(o_i)$, and an additional symbol for $\text{Eff}_r(o_i)$ if necessary, are set to true.
2. All symbols $\sigma_j \subseteq \text{Pre}(o_i)$ such that $\text{factors}(\sigma_j) \subseteq \text{factors}(o_i)$ are set to false.
3. All currently true symbols $\sigma_j \subseteq \text{Pre}(o_i)$, where $f_{ij} = \text{factors}(\sigma_j) \cap \text{factors}(o_i) \neq \emptyset$ but $\text{factors}(\sigma_j) \not\subseteq \text{factors}(o_i)$, are set to false. For each such σ_j , the symbol $\text{Project}(\sigma_j, f_{ij})$ is set to true.

Recall that we compute the image of an abstract subgoal option o_i from an existing set of states Z using the equation $\text{Im}(Z, o_i) = \text{Project}(Z, o_i) \cap \text{Eff}(o_i)$. The first effect in the above list corresponds to the second component of the image equation, and can be considered the primary effect of the option. The remaining two types of effects model the projection component of the image equation. The second type of effects removes propositions defined entirely using variables within $\text{mask}(o_i)$, and whose effects are thus eliminated by the projection operator. The third effects type models the side-effects of the option, where an existing classifier has the variables affected by the projection (those in $\text{mask}(o_i)$) projected out. This is depicted in Figure 4.

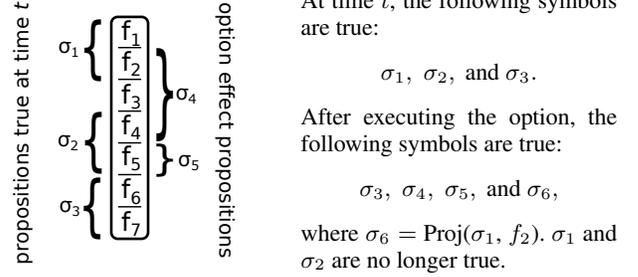


Figure 4: An example effect computation. At time t , σ_1 , σ_2 , and σ_3 are true. The option's effect set is represented by σ_4 and independent factor σ_5 . After execution, σ_3 remains true because its factors lie outside of the option mask, while σ_2 is set to false because its factors are covered by the mask. The factors used by σ_1 are only partially covered, so it is set to false and $\sigma_6 = \text{Proj}(\sigma_1, f_2)$ is set to true. σ_4 and σ_5 correspond to the option's direct effects and are set to true.

Theorem 2. Given abstract state descriptor \mathcal{P}_t , option o_j such that $\mathcal{G}(\mathcal{P}_t) \subseteq \text{Pre}(o_j)$, and \mathcal{P}_{t+1} as computed above:

$$\mathcal{G}(\mathcal{P}_{t+1}) = \text{Im}(\mathcal{G}(\mathcal{P}_t), o_j).$$

Proof. Recall our grounding semantics:

$$\mathcal{G}(\mathcal{P}_t) = \bigcap_{i \in I} \mathcal{G}(p_i), \quad I = \{i \mid \mathcal{P}_t(i) = 1\},$$

and the definition of the image operator:

$$\text{Im}(\mathcal{G}(\mathcal{P}_t), o_j) = \text{Project}(\mathcal{G}(\mathcal{P}_t), o_j) \cap \text{Eff}(o_j).$$

Substituting, we see that:

$$\begin{aligned} \text{Im}(\mathcal{G}(\mathcal{P}_t), o_j) &= \text{Project}(\bigcap_{i \in I} \mathcal{G}(p_i), o_j) \cap \text{Eff}(o_j) \\ &= \bigcap_{i \in I} \text{Project}(\mathcal{G}(p_i), o_j) \cap \text{Eff}(o_j). \end{aligned}$$

We can split I into three sets: I_u , which contains indices for symbols whose factors do not overlap with o_j ; I_r , which contains symbols whose factors are a subset of $\text{mask}(o_j)$; and I_o , the remaining symbols whose factors overlap. Projecting out $\text{mask}(o_j)$ leaves the symbols in I_u unchanged, and replaces those in I_r with the universal set, so:

$$\text{Im}(\mathcal{G}(\mathcal{P}_t), o_j) = \bigcap_{i \in I_u} \mathcal{G}(p_i) \cap \bigcap_{i \in I_o} \text{Project}(\mathcal{G}(p_i), o_j) \cap \text{Eff}(o_j).$$

For each $i \in I_o$, we can by construction find a k such that $\mathcal{G}(p_k) = \text{Project}(\mathcal{G}(p_i), o_j)$. Let K be the set of such indices. Then we can write:

$$\text{Im}(\mathcal{G}(\mathcal{P}_t), o_j) = \bigcap_{i \in I_u} \mathcal{G}(p_i) \cap \bigcap_{k \in K} \mathcal{G}(p_k) \cap \text{Eff}(o_j).$$

The first term on the right hand side of the above equality corresponds to the propositions left alone by the operator application; the second term to the third type of effects propositions; the third term to the first type of effects propositions; and I_r to the second type of (removed) effects propositions. These are exactly the effects enumerated by our operator model, so $\text{Im}(\mathcal{G}(\mathcal{P}_t), o_j) = \mathcal{G}(\mathcal{P}_{t+1})$. \square

Finally, we must compute the operator preconditions, which express the conditions under which the option can be executed. To do so, we consider the preimage $\text{Pre}(o_i)$

for each option o_i , and the factors it is defined over, $\text{factors}(\text{Pre}(o_i))$. Our operator effects model ensures that no two symbols defined over the same factor can be true simultaneously. We can therefore enumerate all possible “assignments” of factors to symbols, compute the resulting grounding classifier, and determine whether it is a subset of $\text{Pre}(o_i)$. If so, the option can be executed, and we can output an operator description with the appropriate preconditions and effects. We omit the proof that the generated preconditions are correct, since it follows directly from the definition.

Note that the third type of effects (expressing the option’s side effects) depends on propositions other than those used to evaluate the option’s preimage. We can express these cases either by creating a separate operator description for each possible assignment of relevant propositions, or more compactly via PDDL’s support for conditional effects.

Since the procedure for constructing operator descriptions given in this section uses only propositions enumerated above, it follows that \mathcal{P} is sufficient for describing the system. It also follows that the number of reachable symbols in a symbol algebra based on abstract subgoal options is finite.

Computing the factors requires time in $O(n|F||O|)$, where F , the set of factors, has at most n elements (where n is the dimension of the original low-level state space). Enumerating the symbol set requires time in $O(|O||F|^c)$ for the independent factors and $O(|O|2^{|F|}c)$ for the dependent factors in the worst case, where each symbol algebra operation is $O(c)$. This results in a symbol set of size $|\mathcal{P}| = O(F_I + 2^{F_D})$, where F_I indicates the number of symbols resulting from independent factors, and F_D the number of factors referred to by dependent effects sets. Computing the operator effects is in time $O(|\mathcal{P}||O|)$, and the preconditions in time $O(|O||\mathcal{P}|^{|F|})$ in the worst case.

Although the resulting algorithm will output a correct PDDL model for any symbol algebra with abstract subgoals, it has a potentially disastrous worst-case complexity, and the resulting model can be exponentially larger than the $O(|O|)$ source symbol algebra. We expect that conversion will be most useful in problems where the number of factors is small compared to the number of original state variables ($|F| \ll n$), most or all of the effects sets are independent in all factors ($F_D \ll F_I$), many symbols repeat (small $|\mathcal{P}|$), and most precondition computations either fail early or succeed with a small number of factors. These properties are typically necessary for a domain to have a compact PDDL model in general.

Planning in the Continuous Playroom

In the continuous playroom domain (Konidaris and Barto 2009a), an agent with three effectors (an eye, a hand, and a marker) is placed in a room with five objects (a light switch, a ball, and red and green buttons) and a monkey; the room also has a light (initially off) and music (also initially off). The agent is given options that allow it to move a specified effector over a specified object (always executable, resulting in the effector coming to rest uniformly at random within 0.05 units of the object in either direction), plus an “interact” option for each object (for a total of 20 options).

The effectors and objects are arranged randomly at the start of every episode; one arrangement is depicted in Figure 5.

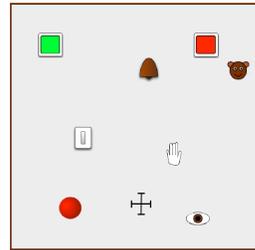


Figure 5: An instance of the continuous playroom domain.

Interacting with the buttons or the bell requires the light to be on, and the agent’s hand and eye to be placed over (within 0.05 units in either direction) the relevant object. The ball and the light switch are brightly colored and can be seen in the dark, so they do not require the light to be on. Interacting with the green button turns the music on; the red button turns it off. Interacting with the light switch turns the lights on or off. Finally, if the agent’s marker is on the bell and it interacts with the ball, the ball is thrown at the bell and makes a noise. If this happens when the lights are off and the music is on, the monkey cries out, and the episode ends. The agent’s state representation consists of 33 continuous state variables describing the x and y distance between each effector and each object, the light level (0 when the light is off, and dropping off from 1 with the squared distance from the center of the room when the light is on), the music volume (selected at random from the range $[0.3, 1.0]$ when the green button is pressed), and whether the monkey has cried out.

We first construct the appropriate symbolic descriptions for the continuous playroom by hand, following the theory developed above. First, we partition the option initiation sets to obtain partitioned abstract subgoal options; we then construct models of the relevant masks and effects sets. For example, Figure 6 shows the two characterizing sets for moving the eye over the ball. The light level perceived by the agent changes with eye movement if the light is on and remains unchanged if it is not, so we partition the initiation set based on the existing light level.

Converting these symbolic descriptions to PDDL was completed in approximately 0.4 seconds. This resulted in 6 factors, three containing variables describing the distance between an effector and all of the objects, and one each for the music, the lights, and the monkey’s cry. The factors and the symbol algebra operator descriptors generated 20 unique symbol groundings—all effects sets resulted in independent factor assignments. An example operator description with its symbol groundings is shown in Figure 7.

We tested planning on three goals, each starting from the set of all states where the monkey is not crying out and the lights and music are off. We implemented a breadth-first symbol algebra planner in Java, and also used the automatically generated PDDL description as input to the FF planner (Hoffmann and Nebel 2001). Timing results, given in Table 1, show that both systems can solve the resulting search problem very quickly, though FF (a modern heuristic

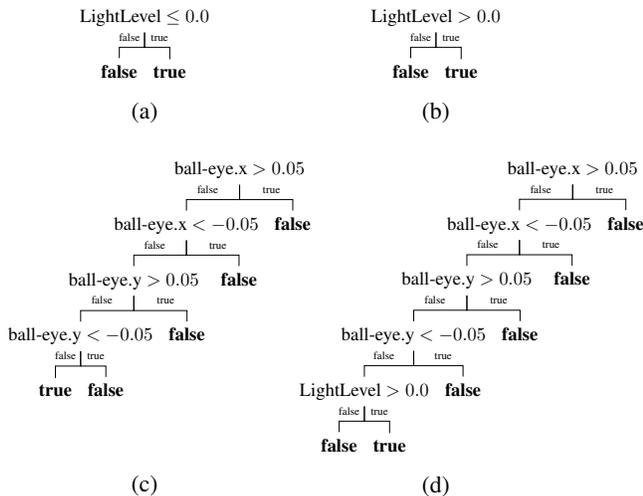


Figure 6: The two characterizing sets of the option for moving the eye over the ball. The precondition classifier of the first characterizing set (a) includes states where the light is off. Its effect mask does not include `LightLevel`, and its effects set (c) indicates that the relationship between the eye and the ball is set to a specific range. The precondition classifier for the second characterizing set (b) includes only states where the light is on. Its effects mask includes the `LightLevel` variable, which its effect set (d) indicates could change to any positive value after option execution, as the position of the eye affects the amount of light received.

planner) can solve it roughly a thousand times faster than a naive planner using a symbol algebra. The resulting plans are guaranteed to reach the goal for any domain configuration where the start symbol holds. We know of no existing sample-based SMDP planner with similar guarantees.

Goal	Depth	Symbol Algebra (BFS)		PDDL (FF)	
		Visited	Time (s)	Visited	Time (s)
Lights On	3	199	1.35	4	0.00199
Music On	6	362	2.12	9	0.00230
Monkey Cry	13	667	3.15	30	0.00277

Table 1: The time required, and number of nodes visited, for the example planning problems. Results were obtained on a 2.5Ghz Intel Core i5 processor and 8GB of RAM.

To demonstrate symbol acquisition from experience, we gathered 5,000 positive and negative examples¹ of each option’s precondition and effects set by repeatedly creating a new instance of the playroom domain, determine which options could be executed at each state, and then sequentially executing options at random. For the effects set, we used option termination states as positive examples and states encountered during option execution but before termination as negative examples. We used the WEKA toolkit (Hall et al.

¹This number was chosen arbitrarily and does not reflect the difficulty of learning the relevant sets.

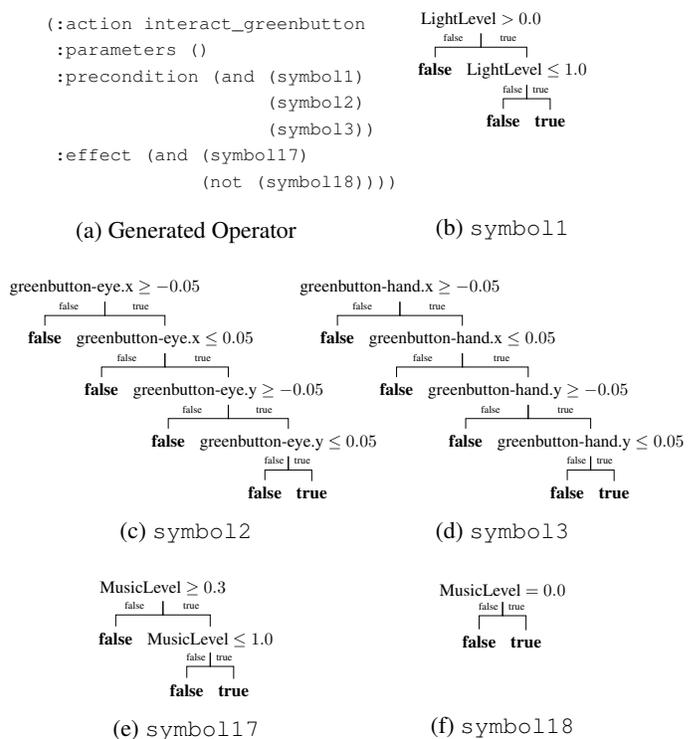


Figure 7: The automatically generated PDDL descriptor for interacting with the green button (a) with the groundings of the automatically generated symbols it mentions (b–f). The operator symbolically expresses the precondition that *the light is on and the hand and eye are over the green button*, and that, as a result of executing the operator, *the music is on, and the music is no longer off*.

2009) C4.5 decision tree (Quinlan 1993) for symbol learning. A representative learned precondition classifier for interacting with the green button is given in Figure 8.

A close look at the numerical values in Figure 8 suggests that, in practice, a learned set-based representation will always be approximate due to sampling, even in deterministic domains with no observation noise. Thus, while our formalism allows an agent to learn its own symbolic representation in principle, its use in practice will require a generalization of our symbol definition to probability distributions that account for the uncertainty inherent in learning. Additionally, it is unrealistic to hope for plans that are guaranteed to succeed—consequently future work will generalize the notion of feasibility to the probability of plan success.

Related Work

Several systems have learned symbolic models of the preconditions and effects of pre-existing controllers for later use in planning (Drescher 1991; Schmill, Oates, and Cohen 2000; Džeroski, De Raedt, and Driessens 2001; Pasula, Zettlemoyer, and Kaelbling 2007; Amir and Chang 2008; Kruger et al. 2011; Lang, Toussaint, and Kersting 2012; Mourão et al. 2012). In all of these cases, the high-level

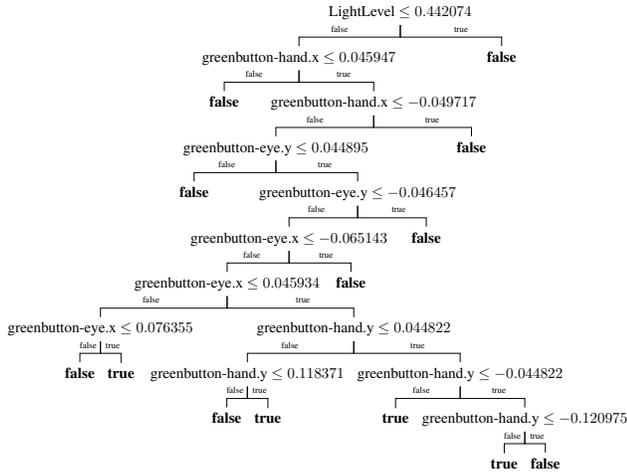


Figure 8: The decision tree representing the learned initiation set symbol for interacting with the green button. It approximately expresses the set of states we might label *the hand and eye are over the green button, and the light is on*.

symbolic vocabulary used to learn the model were given; our work shows how to construct it.

We know of very few systems where a high-level, abstract state space is formed using low-level descriptions of actions. The most closely related work is that of Jetchev, Lang, and Toussaint (2013), which uses the same operational definition of a symbol as we do. Their method finds relational, probabilistic STRIPS operators by searching for symbol groundings that maximize a metric balancing transition predictability with model size. They are able to find small numbers of symbols in real-world data, but are hampered by the large search space. We completely avoid this search by directly deriving the necessary symbol groundings from the actions.

Huber (2000) describes an approach where the state of the system is described by the (discrete) state of a set of controllers. For example, if a robot has three controllers, each one of which can be in one of four states (cannot run, can run, running, completed), then we can form a discrete state space made up of three attributes, each of which can take on four possible values. Hart (2009) used this formulation to learn hierarchical manipulation schemas. This approach was one of the first to construct a symbolic description using motor controller properties. However, the resulting symbolic description only fulfills our requirements when all of the low-level states in which a controller can converge are equivalent. When that is not the case—for example, when a grasp can converge in two different configurations, only one of which enables a subsequent controller to execute—it may incorrectly evaluate the feasibility of a plan.

Modayil and Kuipers (2008) show that a robot can learn to distinguish objects in its environment via unsupervised learning, and then use a learned model of the motion of the object given an action to perform high-level planning. However, the learned models are still in the original state space. Later work by Mugan and Kuipers (2012) use *qualitative distinctions* to discretize a continuous state space to acquire

a discrete model for planning. Discretization is based on the ability to predict the outcome of action execution.

Other approaches to MDP abstraction have focused on discretizing large continuous MDPs into abstract discrete MDPs (Munos and Moore 1999) or minimizing the size of a discrete MDP model (Dean and Givan 1997).

Discussion and Conclusions

An important implication of the ideas presented here is that actions must play a central role in determining the representational requirements of an intelligent agent: *a suitable symbolic description of a domain depends on the actions available to the agent*. Figure 9 shows two examples of the same robot navigating the same room but requiring different representations because it uses different navigation options.

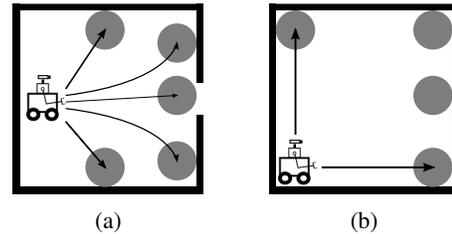


Figure 9: a) A robot navigating a room using options that move it to the centroid of each wall. Its effect sets (gray) might be described as *AtLeftWall*, *AtRightWall*, etc. b) The same robot using wall-following options. Its effect sets could be described as *LowerLeftCorner*, *LowerRightCorner*, etc.

The combination of high-level planning and low-level control—first employed in Shakey (Nilsson 1984)—is critical to achieving intelligent behavior in embodied agents. However, such systems are very difficult to design because of the immense effort required to construct and interface matching high-level reasoning and low-level control components. We have established a relationship between an agent’s actions and the symbolic representation required to plan using them—a relationship so close that *the agent’s environment and actions completely determine an appropriate symbolic representation*, thus both removing a critical obstacle to designing such systems, and enabling agents to autonomously construct their own symbolic representations.

Acknowledgements

We thank Greg Lewis, Siddharth Srivastava, and LIS for insightful conversations. GDK was supported in part by an MIT Intelligence Initiative Fellowship. This work was supported in part by the NSF under Grant No. 1117325. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation. We also gratefully acknowledge support from ONR MURI grant N00014-09-1-1051, from AFOSR grant FA2386-10-1-4135 and from the Singapore Ministry of Education under a grant to the Singapore-MIT International Design Center.

References

- Amir, E., and Chang, A. 2008. Learning partially observable deterministic action models. *Journal of Artificial Intelligence Research* 33:349–402.
- Barto, A., and Mahadevan, S. 2003. Recent advances in hierarchical reinforcement learning. *Discrete Event Dynamic Systems* 13:41–77.
- Burrige, R.; Rizzi, A.; and Koditschek, D. 1999. Sequential composition of dynamically dextrous robot behaviors. *International Journal of Robotics Research* 18(6):534–555.
- Cambon, S.; Alami, R.; and Gravat, F. 2009. A hybrid approach to intricate motion, manipulation and task planning. *International Journal of Robotics Research* 28(1):104–126.
- Choi, J., and Amir, E. 2009. Combining planning and motion planning. In *Proceedings of the IEEE International Conference on Robotics and Automation*, 4374–4380.
- Dean, T., and Givan, R. 1997. Model minimization in Markov decision processes. In *In Proceedings of the Fourteenth National Conference on Artificial Intelligence*, 106–111.
- Dornhege, C.; Gissler, M.; Teschner, M.; and Nebel, B. 2009. Integrating symbolic and geometric planning for mobile manipulation. In *IEEE International Workshop on Safety, Security and Rescue Robotics*.
- Drescher, G. 1991. *Made-Up Minds: A Constructivist Approach to Artificial Intelligence*. MIT Press.
- Džeroski, S.; De Raedt, L.; and Driessens, K. 2001. Relational reinforcement learning. *Machine learning* 43(1):7–52.
- Ghallab, M.; Nau, D.; and Traverso, P. 2004. *Automated planning: theory and practice*. Morgan Kaufmann.
- Hall, M.; Frank, E.; Holmes, G.; Pfahringer, B.; Reutemann, P.; and Witten, I. 2009. The WEKA data mining software: An update. *SIGKDD Explorations* 11(1):10–18.
- Hart, S. 2009. *The Development of Hierarchical Knowledge in Robot Systems*. Ph.D. Dissertation, University of Massachusetts Amherst.
- Hoffmann, J., and Nebel, B. 2001. The FF planning system: Fast plan generation through heuristic search. *Journal of Artificial Intelligence Research* 14:253–302.
- Huber, M. 2000. A hybrid architecture for hierarchical reinforcement learning. In *Proceedings of the 2000 IEEE International Conference on Robotics and Automation*, 3290–3295.
- Jetchev, N.; Lang, T.; and Toussaint, M. 2013. Learning grounded relational symbols from continuous data for abstract reasoning. In *Proceedings of the 2013 ICRA Workshop on Autonomous Learning*.
- Kaelbling, L., and Lozano-Pérez, T. 2011. Hierarchical planning in the Now. In *Proceedings of the IEEE Conference on Robotics and Automation*.
- Kocsis, L., and Szepesvári, C. 2006. Bandit based Monte-Carlo planning. In *Proceedings of the 17th European Conference on Machine Learning*, 282–293.
- Konidaris, G., and Barto, A. 2009a. Efficient skill learning using abstraction selection. In *Proceedings of the Twenty First International Joint Conference on Artificial Intelligence*.
- Konidaris, G., and Barto, A. 2009b. Skill discovery in continuous reinforcement learning domains using skill chaining. In *Advances in Neural Information Processing Systems* 22, 1015–1023.
- Kruger, N.; Geib, C.; Piater, J.; Petrick, R.; Steedman, M.; Wörgötter, F.; Ude, A.; Asfour, T.; Kraft, D.; Omrčen, D.; Agostini, A.; and Dillmann, R. 2011. Object-action complexes: Grounded abstractions of sensory-motor processes. *Robotics and Autonomous Systems* 59:740–757.
- Lang, T.; Toussaint, M.; and Kersting, K. 2012. Exploration in relational domains for model-based reinforcement learning. *Journal of Machine Learning Research* 13:3691–3734.
- Lozano-Perez, T.; Mason, M.; and Taylor, R. 1984. Automatic synthesis of fine-motion strategies for robots. *International Journal of Robotics Research* 3(1):3–24.
- Malcolm, C., and Smithers, T. 1990. Symbol grounding via a hybrid architecture in an autonomous assembly system. *Robotics and Autonomous Systems* 6(1-2):123–144.
- McDermott, D.; Ghallab, M.; Howe, A.; Knoblock, C.; Ram, A.; Veloso, M.; Weld, D.; and Wilkins, D. 1998. PDDL—the planning domain definition language. Technical Report CVC TR98003/DCS TR1165, Yale Center for Computational Vision and Control.
- Modayil, J., and Kuipers, B. 2008. The initial development of object knowledge by a learning robot. *Robotics and Autonomous Systems* 56(11):879–890.
- Mourão, K.; Zettlemoyer, L.; Patrick, R.; and Steedman, M. 2012. Learning STRIPS operators from noisy and incomplete observations. In *Proceedings of Conference on Uncertainty in Artificial Intelligence*.
- Mugan, J., and Kuipers, B. 2012. Autonomous learning of high-level states and actions in continuous environments. *IEEE Transactions on Autonomous Mental Development* 4(1):70–86.
- Munos, R., and Moore, A. 1999. Variable resolution discretization for high-accuracy solutions of optimal control problems. In *Proceedings of the Sixteenth International Joint Conference on Artificial Intelligence*, 1348–1355.
- Nilsson, N. 1984. Shakey the robot. Technical report, SRI International.
- Pasula, H.; Zettlemoyer, L.; and Kaelbling, L. 2007. Learning symbolic models of stochastic domains. *Journal of Artificial Intelligence Research* 29:309–352.
- Precup, D. 2000. *Temporal Abstraction in Reinforcement Learning*. Ph.D. Dissertation, Department of Computer Science, University of Massachusetts Amherst.
- Quinlan, J. 1993. *C4.5: programs for machine learning*, volume 1. Morgan Kaufmann.
- Schmill, M.; Oates, T.; and Cohen, P. 2000. Learning planning operators in real-world, partially observable environments. In *Proceedings of the Fifth International Conference on Artificial Intelligence Planning and Scheduling*, 245–253.
- Sutton, R.; Precup, D.; and Singh, S. 1999. Between MDPs and semi-MDPs: A framework for temporal abstraction in reinforcement learning. *Artificial Intelligence* 112(1-2):181–211.
- Tedrake, R. 2009. LQR-Trees: Feedback motion planning on sparse randomized trees. In *Proceedings of Robotics: Science and Systems*, 18–24.
- Wolfe, J.; Marthi, B.; and Russell, S. J. 2010. Combined Task and Motion Planning for Mobile Manipulation. In *International Conference on Automated Planning and Scheduling*.