Exercise 07

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1 Analyze the Ball-Carving with Exponential Clocks

Consider a following process of Ball-Carving:

- every vertex v pick radius r_v according to *exponential distribution*, that is with probability density $\mathrm{EXP}(x) = \beta \cdot e^{-\beta x}$ for $x \geq 0$.
- every vertex u picks as its ball-center the vertex $v = \arg\max_x (r_x - d(u, x))$ (we say that $u \in B_v$)

(1) Show that each B_v is an induced subgraph of G.
(2) Give a reasonable upperbound on the diameter of each B_v (that holds w.h.p.).
(3) Give a reasonable lowerbound on the diameter of each B_v (that holds w.h.p.).

2 Min-Cost Communication Network

2.1 Small average stretch tree

Consider a problem of finding a *single* tree T such that the average stretch is small. More precisely, given weight function $w : V^2 \to \mathbb{R}^+$, we say that T has average stretch α if

$$\forall_{u,v \in V} d^G(u, v) \leq d^T(u, v)$$

$$\sum_{u,v \in V} w_{u,v} d^T(u, v) \leq \alpha \sum_{u,v \in V} w_{u,v} d^G(u, v).$$

Adapt the FRT tree embedding algorithm from the lecture to solve this problem (with the same stretch parameter $\alpha = \mathcal{O}(\log n)$.
Hint: This is a dual problem to the tree embedding problem.

2.2 Minimal cost communication network

In this problem, we are given n nodes $v_1, \ldots, v_n$ in a metric network, and a set of communication requirement $r_{i,j}, 1 \leq i, j \leq n$. The goal is to find a *spanning tree* T for nodes that minimizes $\sum_{i,j} d^T(v_i, v_j) \cdot r_{i,j}$.

3 Steiner Forest

Given edge weighted graph G, and a set of pairs of terminals $(s_1, t_1), \ldots, (s_k, t_k)$, consider problem of finding minimal cost E' such that s_i, t_i are connected in $G[E']$. Use FRT tree embedding algorithm to build $\mathcal{O}(\log n)$ approximate algorithm to this problem.
Hint: sample single tree T and solve the problem in T. Project solution back to G.