Advanced Algorithms Deadline: November 28, 2017, 10:15 am

Graded Homework 2

Lecturer: Mohsen Ghaffari

1 PTAS for Sum of Squares Load Balancing (Williamson-Shmoys's Problem 3.8, 2 points)

Consider the setting of makespan minization job scheduling considered in the class, i.e., jobs with load p_1, p_2, ..., p_ℓ, which should be assigned to m machines. Devise a Polynomial-Time Approximation Scheme for the objective of minimizing the sum of squares of the loads assigned to different machines, i.e., $\sum_{i=1}^{m}(\sum_{j\in S_i} p_j)^2$, where S_i denotes the set of all jobs assigned to machine i.

2 FPTAS for Subset-sum ratio (Vazirani's Problem 8.7, 2 points)

Given n positive integers $a_1 < a_2 < \ldots < a_n$, the objective is to find two disjoint nonempty subsets $S_1, S_2 \subseteq \{1, 2, \ldots, n\}$ that minimize $\max\{\frac{\sum_{i\in S_1} a_i}{\sum_{i\in S_2} a_i}, \frac{\sum_{i\in S_2} a_i}{\sum_{i\in S_1} a_i}\}$. Devise a Fully Polynomial-Time Approximation Scheme (FPATS) for this objective.

3 Approximation for Modified MAX-SAT (Williamson-Shmoys's Problem 5.8, 2 points)

Consider the setting of the MAX-SAT problem, but now with an additional constraint that all variables appear positively in all clauses. Given this change, we could easily satisfy all clauses. But we now want to maximize a different measure instead: suppose the i^{th} clause has a weight w_i and the j^{th} variable has weight w'_j. We want to maximize the total weight of satisfied clauses plus the total weight of variables set to false. Give an Integer Programming formulation of this problem, and a randomized rounding for it that leads to a $2(\sqrt{2}-1)$ approximation.

4 Generalized Balanced Cut (Chekuri's Problem 5.5, 2 points)

Given a graph $G = (V, E)$ with edge-weights $c : E \to R^+$, you wish to partition G into three parts $G_1 = G[V_1]$, $G_2 = G[V_2]$, and $G_3 = G[V_3]$ such that for each $i \in \{1, 2, 3\}$, we have $\lfloor |V|/3 \rfloor \leq |V_i|| \leq \lceil |V|/3 \rceil$ and the cost of the edges between the partitions is minimized. Suppose OPT denotes this ideal cost. Provide an $O(\log n)$ pseudo-approximation for this problem, that is, an algorithm that partitions the graph into three parts $G'_1 = G[V'_1]$, $G'_2 = G[V'_2]$, and $G'_3 = G[V'_3]$ such that (A) for each $i \in \{1, 2, 3\}$, we have $|V|/c_1 \leq |V_i| \leq |V|/c_2$ for some constants constants $1 < c_1 < c_2$, and (B) the cost of the edges between the partitions is $O(\log n) \cdot OPT$. What constants c_1, c_2 can you guarantee?

5 L$_p$-Embedding (Vazirani's Problem 21.6, 2 points)

Prove that for any $p \geq 1$, any n-point metric $(V, \mathbf{d})$ has an $O(\log n)$-distortion ℓ_p-embedding in the $O(\log^2 n)$-dimensional space.